Ponder this!

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Problem set 5

This issue gives readers the opportunity to make a closer acquaintance with a few examples suggested by Gregory Galperin, who kindly gave his permission for problems to appear in the journal. As usually the purpose of the section is to supply teachers and students with a selection of interesting problems. At this time we do not focus on any particular topic and bring a set of miscellaneous problems to readers' consideration.

1. How many sides of a convex polygon can have the same length as its longest diagonal?

2. What is the maximum number of points can be located on the plane such that any triangle with vertices at these points would not have obtuse angle? Consider the same question for the three-dimensional case.

3. N people are not familiar with each other. It is required to make some acquaintances between them in such a way that any three people have the same number of people they are acquainted with. Prove that it is possible to do for any arbitrary N.

4. From the sequence $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$ we can distinguish an arithmetic progression $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$ that consists of three terms. Is it possible to find an arithmetic progression that consists of:
   (a) four terms?
   (b) five terms?
   (c) k terms, where k is an arbitrary positive integer?

5. Seven 7-digit numbers are composed from 1, 2, 3, 4, 5, 6, 7 taken in different orders. Prove that the sum of the seventh powers of any number of such 7-digit numbers is not equal to the sum of the seventh powers of the other 7-digit numbers.

Solutions to this set of problems for publication should be submitted to:
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Solutions to this set will be made available on the AAMT website (www.aamt.edu.au) after 1 October 2010.