Model updating incorporating measured response uncertainties and confidence levels of tuning parameters

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Abstract: Automated model updating of real civil engineering structures is often very challenging due to the presence of different degrees of uncertainty in measured responses and confidence levels of the tuning parameters used. To address this issue, this paper presents a hybrid model updating procedure for large-scale civil engineering structures which incorporate these variations by means of data scatter for both measured responses and tuning parameters as a logical extension to the conventional automated model updating procedures. Scatters in the measured responses are derived through statistically analysing ambient vibration test data, while confidence levels of tuning parameters are derived based on the engineering judgement. The results of applying this hybrid model automated model updating procedure to a ten story building show a significant improvement in obtaining more realistic updated models, against its conventional counterpart that was done previously on the same structure.

Keywords: automated model updating; uncertainty of measured data; confidence of tuning parameters; ambient vibration; finite element modelling.

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1 Introduction

With the recent advancements in computer technology and development of powerful software finite element (FE) modelling has become a popular numerical modelling method to develop physics based analytical models. However, even with the presence of highly developed computer technology, development of a complete FE model is still a very challenging task, mainly because of the uncertainties in the FE modelling such as simplifying assumptions of geometrical and material properties and uncertain boundary conditions (Aktan and Brownjohn, 2013; Jaishi and Ren, 2005). Hence, it is important to calibrate and update the initial FE model, before using in civil engineering applications such as damage assessment, rehabilitation design and load bearing assessment. Model updating can be defined as the process of rectifying the modelling errors of the initial FE model to obtain a better correlation of dynamic and/or static behaviour with the actual structure (Liu et al., 2014).

Model updating methods can be broadly classified as global methods which attempt to directly reconstruct the global mass and stiffness matrices from test data and local methods which try to change values of physical parameters and minimise the discrepancies between FE model and test data. Compared to global methods, local methods are often better in-terms of producing reliable updated models if the updating parameters and responses are chosen appropriately. Most popular local methods of model updating are based on sensitivity analysis, and these methods firstly identify the uncertain parameters through a comprehensive sensitivity analysis and subsequently modify the parameters to minimise the discrepancies between FE model and test data (Živanović et al., 2007). These sensitivity-based model updating methods can be categorised as manual methods where the tuning parameters are changed manually and automated methods in which case it is often conducted in an iterative manner. Several successful studies had been reported on real structures both using manual model updating procedures (Daniell and Macdonald, 2007; Saudi et al., 2009; Votsis et al., 2012) and

automated model updating procedures (Brownjohn and Xia, 2000; Brownjohn et al., 2003; Cismaşiu et al., 2015; Ding and Li, 2008; Fei et al., 2007; Kim et al., 2013; Park et al., 2012; Ventura et al., 2001, 2005; Wu and Li, 2004; Zhang et al., 2001; Živanović et al., 2007). Even though, most of these case studies were based on the assumption that test data are accurate and reliable, and in real structures test data can be subjected to numerous uncertainties such as environmental effects and measurement errors. This might possibly affect the quality of the measured data and hence the model updating procedure, which leads to updated models that may not represent the true behaviour of the actual structure (Mottershead and Friswell, 1993). In addition, there are a number of tuning parameters used in the sensitivity-based model updating of real structures with different confidence levels, which will vary depending on the nature of the parameters, such as for a concrete structure mass density of concrete might have a higher confidence level compared to Young's modulus of concrete. Only a few researchers in the past identified the importance of these variations in sensitivity-based automated model updating of real structures. For instance, Brownjohn and Xia (2000) analysed the results of an automated sensitivity-based model updating of a curved cable stayed bridge and identified that quality of test data is more critical for updating the higher modes. Živanović et al. (2007) incorporated a confidence factor of ten times lower for MAC values of the mode shapes than the measured natural frequencies to account for the lower reliability of the mode shapes in comparison to measured natural frequencies, in an automated model updating of a foot bridge structure. However, there has been a lack of comprehensive studies on dealing with the actual variation of measured responses and the selection of confidence levels for the tuning parameters in the automated model updating process for real civil engineering structures.

Hence, this paper presents a hybrid automated model updating procedure for large-scale civil engineering structures by incorporating the actual uncertainties in the measured responses and appropriately selected confidence levels for tuning parameters to the sensitivity-based automated model updating of real structures to obtain more reliable updated FE models to represent actual behaviour of the structure. The real structure investigated in this paper is a ten-story building located at Queensland University of Technology (QUT), Brisbane. The structure contains an innovative vibration sensing system operating in a continuous monitoring manner to capture the ambient vibration responses. The previous research work at QUT (Nguyen et al., 2014, 2015) provide more details about the output-only modal analysis (OMA) procedure and modal properties about the case study considered in this research. First five natural frequencies and associated mode shapes obtained from the experimental OMA are used for the automated modal updating of the structure. The uncertainties of the measured responses and confidence levels of the tuning parameters were incorporated to the automated model updating procedure by means of statistical scatter. To compute the scatter of the measured responses, results of 60 datasets obtained in various days were statistically analysed prior to the model updating process, while engineering judgement is used for selecting the confidence levels of the model tuning parameters. To demonstrate the importance of the automated model updating study presented in this paper, the results of this case study are compared against the results of a previous conventional sensitivity-based automated model updating exercise conducted by the authors on the same structure (Kodikara et al., 2016).

2 Test structure description and OMA results

For the sake of completeness, a brief description about the test structure and its experimental analysis results are reported in this paper. More details can be found in previous publications of the authors (Kodikara et al., 2016; Nguyen et al., 2015). The case study considered in this paper is a ten story concrete building (commonly known as P block) in the newly constructed Science and Engineering Centre complex of QUT. The structure is comprised of a reinforced concrete (RC) frame with post-tensioned slabs and it has a common level configuration with four-semi underground bases. Dimensions for the lower floor levels are approximately 75 m × 65 m whereas in the upper floor levels it has a smaller floor area with approximate dimensions of 65 m × 45 m. Floor to floor height of the building varies in the range of 2.7 m to 4.5 m and the total height of the building is 42 m from the formation level of the building. The building consists of three main shear walls (two at the eastern side of the building and one at the western side) to resist the torsional forces induced by potential wind and earthquake loads. Even though the structure is considered to have a common overall configuration, for interior structure detailing it has a number of variations in terms of slab thicknesses, slab openings, column sizes and orientations, which needs to be considered in the development of initial FE models and in the model updating process.

As illustrated in Figure 1, P block contains a vibration monitoring system with six analogue tri-axial accelerometers and two single axis accelerometers which operates in a continuous monitoring manner. Acceleration data obtained from the sensors due to ambient vibration were sampled at a frequency of 2,000 Hz and then split into 30-minute subsets to allow sufficient undisrupted data acquisition length and total number of 60 such datasets obtained in various days over a period of three months are used for modal analysis purposes. To process vibration monitoring data, Data driven Stochastic Subspace Identification (SSI-data) has been used as the main OMA technique as this technique was proven to be efficient for processing large number of datasets as well as robust against data uncertainties (Nguyen et al., 2015). Figure 2 illustrates a typical SSI-data stabilisation diagram for OMA of the building and animation views of the first five modes extracted from a particular dataset (detailed description about the mode shapes is provided in Table 1).

Figure 1 View of p block and sensor locations (see online version for colours)

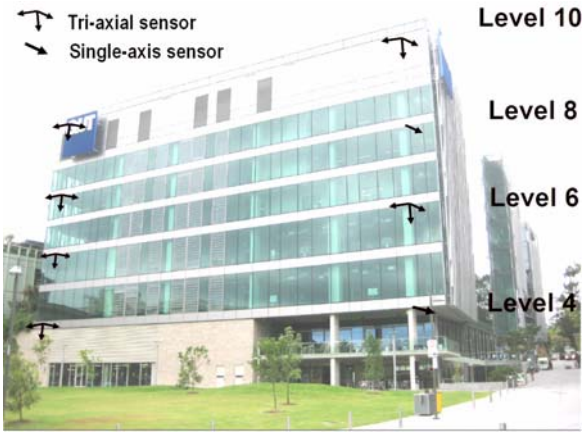
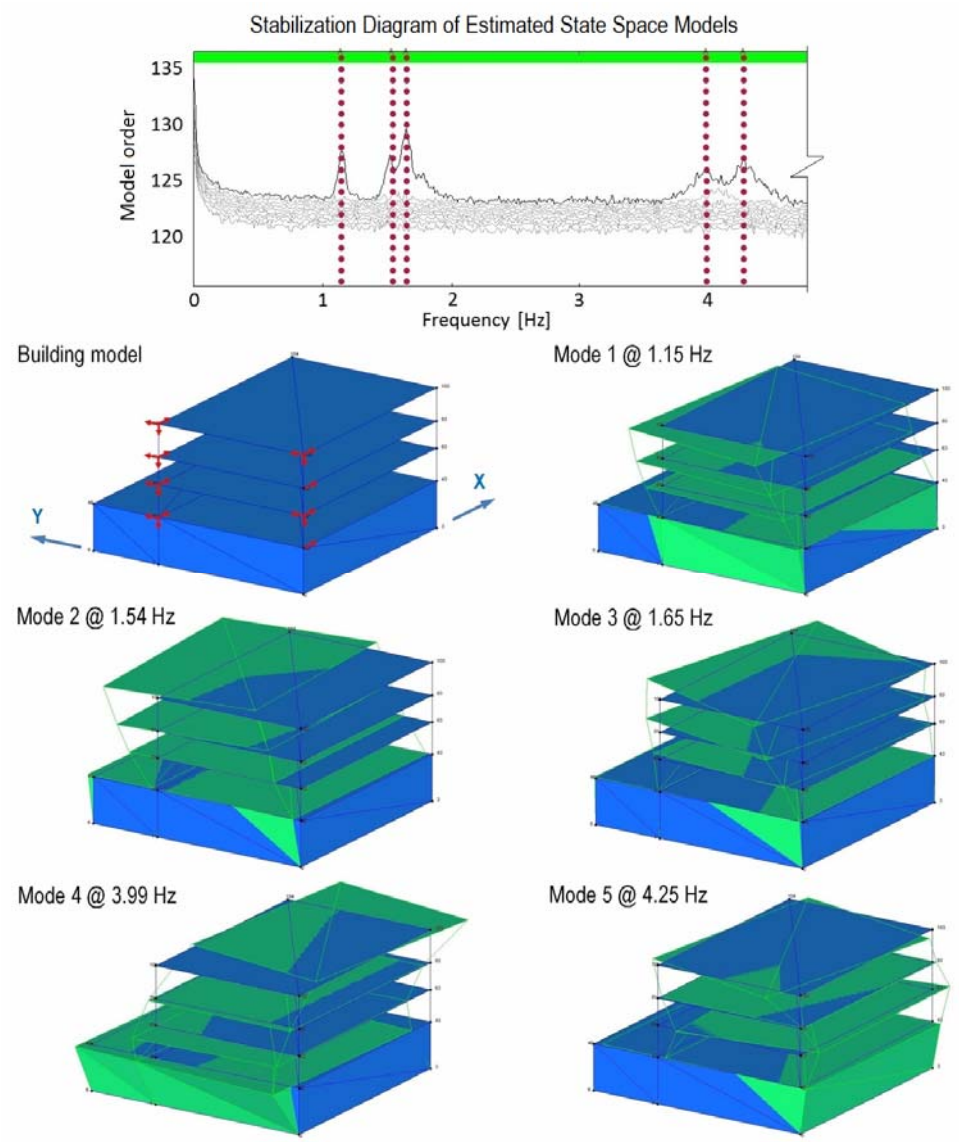


Figure 2 Typical stabilisation diagram and animation views of first five modes (see online version for colours)



3 Model updating

3.1 Initial FE modelling and correlation analysis

The same initial FE model developed for the previous conventional model updating study (Kodikara et al., 2016) is used for the case study presented in this paper. Since the main aim of this paper is to demonstrate the importance of using hybrid model updating

process in real structures, only a brief description about the development of initial FE model is included in this paper for the purpose of completeness.

The detailed initial FE model was developed using the commercially available software package SAP2000 nonlinear version 15.2.0 (Computers and Structures, 2014). Some of the particular considerations taken during the development of initial FE model are:

1 detailed modelling was carried out for shear cores taking minor openings and internal thin walls into account to enable the torsional behaviour of the FE model to be as close as possible to the real structure

2 non-structural components such as light weight partitions and glazed claddings were not included in the initial FE model, since the effect of mass and stiffness of these elements are negligible in the dynamic characteristics of the structure

3 average slab thicknesses are considered in the initial FE model due to high amount of internal variation in slab thicknesses.

The developed FE model consists of 9,400 local elements with 1,400 frame elements (for columns) and 8,000 shell elements (for floor slabs and shear walls). Since the entire floor system of the building consists of only post-tensioned slabs (without beams) all frame elements in the FE model represent columns in the building. As illustrated in Figure 3, all the first five modes are global varying in the range of 0.990 Hz to 4.972 Hz and three modes are translational (mode1, mode 2 and mode 4) and other two modes are torsional (mode 3 and mode 5). Table 1 shows the correlation of the FEM results with a representative test dataset (which has values close to the mean of 60 datasets) in terms of the natural frequencies and associated mode shapes. The modal assurance criterion (MAC), which is a correlation criterion used in the statistics was used for the correlation of associated mode shapes.

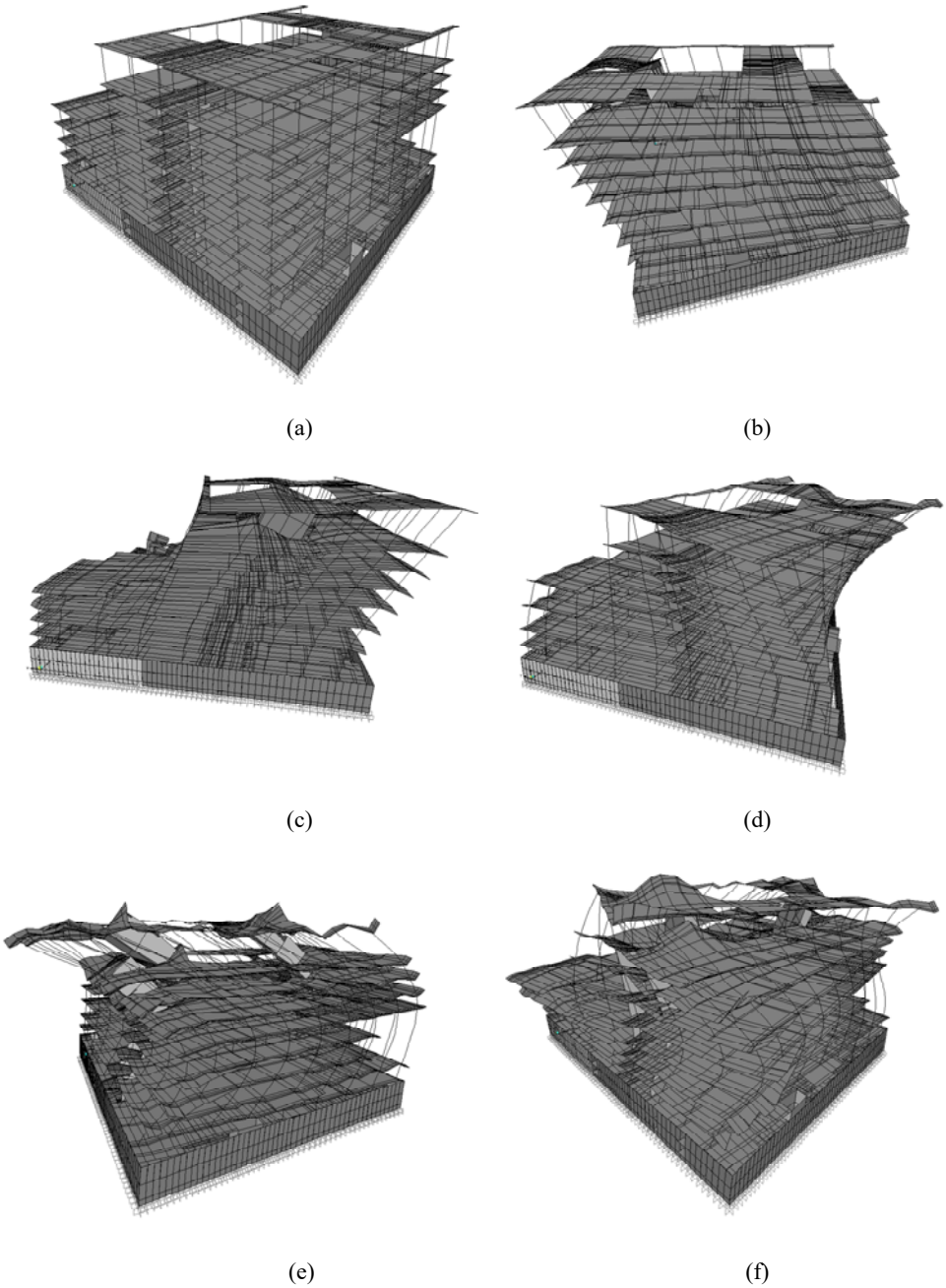
$$MAC\left(\Psi_a, \Psi_e\right) = \frac{\left|\left(\{\Psi_a\}^T \{\Psi_e\}^T\right|^2}{\left(\{\Psi_a\}^T \{\Psi_a\}\right)\left(\{\Psi_e\}^T \{\Psi_e\}\right)} \tag{1}$$

where Ψ_a and Ψ_e are analytical and experimental mode shape vectors, respectively.

Table 1 Correlation between initial FE model and OMA results

Mode	*OMA frequency (Hz)*	*Initial FE model (Hz)*	*Error (%)*	*MAC (%)*	*Mode shape*
1	1.147	0.990	–13.69	89.9	1st translational – X direction
2	1.544	1.452	–5.96	50.5	1st translational – Y direction
3	1.653	1.678	1.51	42.5	1st torsional
4	3.989	3.680	–7.75	63.2	2nd translational
5	4.254	4.972	16.88	68.4	2nd torsional

Figure 3 Initial FE models of the building and its first five modes, (a) FE model (b) Mode 1 @ 0.990 Hz (c) Mode 2 @ 1.452 Hz (d) Mode 3 @ 1.678 Hz (e) Mode 4 @ 3.680 Hz (e) Mode 5 @ 4.972 Hz



3.2 Sensitivity analysis and model updating procedure

In order to compare the results of this case study with the previous model updating study which was carried out using conventional model updating procedures, the same tuning parameters were selected for the case study presented in this paper. Normalised relative sensitivities are used for the sensitivity analysis since the parameters used for the sensitivity analysis are of different types. The relative sensitivity matrix $[S_r]$ is a rectangular matrix of order $m \times n$ where m and n are the number of target responses and parameters, respectively.

$$[S_r] = [S_{ij}] = \left[\frac{\delta R_i}{\delta P_j}\right][P_{ij}] \tag{2}$$

S_{ij} is the sensitivity of the target response R_i due to the change in tuning parameter value P_j and the operator δ represents the change in the variable while $[P_{ij}]$ is the diagonal square matrix holding the tuning parameter values. The forward finite difference approach has been implemented to compute the derivatives in equation (2).

$$\frac{\delta R_i}{\delta P_j} = \frac{R_i\left(P_j + \Delta P_j\right) - R_i\left(P_j\right)}{\Delta P_j} \tag{3}$$

$R_i(P_j)$ is the ith response value for the parameter value P_j and $R_i(P_j + \Delta P_j)$ is the response of the ith response value when the parameter value changes by ΔP_j. Then, the sensitivity matrix is normalised with respective to the response value as shown in equation (4).

$$[S_n] = [R_i]^{-1}[S_r] = [R_i]^{-1}\left[\frac{\delta R_i}{\delta P_j}\right][P_j] \tag{4}$$

$[S_n]$ normalised relative sensitivity matrix

$[R_i]$ a diagonal, square matrix holding the response values.

In order to calculate normalised relative sensitivities, the relevant target responses and tuning parameter should be selected and in this study the natural frequencies of the first five measured global modes and the MAC values of the mode shapes pairs between the first five measured modes and the associated FE model modes are used as the target responses. For tuning parameters, initially all possible parameters are used for the sensitivity analysis and only the parameters with high sensitivities and the parameters that can be systematically coped in an automatic model updating process are chosen as the tuning parameters for the model updating process. At the end of the sensitivity analysis process tuning parameters are identified with their respective total number of FEs (parameter space) to be used in the automatic model updating. For example, for the tuning parameter Young's modulus of concrete, the parameter space is all shell elements and frame elements in the FE model (9,400 FEs). Once the tuning parameters and parameter spaces are selected for the model updating, the identified tuning parameters are grouped to generate the parameter sets in order to make the model updating more realistic and meaningful. For example, for the selected tuning parameters for columns (frame elements), sets are defined so that in the model updating process, the magnitude of the tuning parameter will be constant for these columns. The selected parameters with their parameter space and defined parameter sets for the model updating are tabulated in

Table 2. The parameter sets are defined for all the parameters except the shell thickness where variations in local shell elements were allowed in the model updating process. Since the average slab thicknesses are used in the initial FE model and slab thicknesses vary significantly in small portions in the actual structure, not defining parameter sets for shell thicknesses is justifiable (Kodikara et al., 2016).

Table 2 Parameter space and parameter sets for the model updating

Tuning parameter	*Parameter space*		*Parameter set*
	Element type	*Number of elements*	
Young's modulus (E)	All elements	9,400	[For frame elements] 1 Individual columns [For shell elements] 2 Individual walls 3 Slabs in each level
Mass density – ρ	All elements	9,400	[For frame elements] 1 Individual columns [For shell elements] 2 Individual walls 3 Slabs in each level
Cross section area – AX	Frame elements	1,400	Individual columns
Torsional stiffness – IX	Frame elements	1,400	Individual columns
Bending moment of Inertia about Y – IY	Frame elements	1,400	Individual columns
Bending moment of inertia about Z – IZ	Frame elements	1,400	Individual columns
Shell thickness – H	Shell elements (floor slabs only)	5,680 (slabs only) 30,080 (total)	None

In the conventional model updating (Kodikara et al., 2016), the updating process was carried out using pseudo-inverse parameter estimation as the updating algorithm, where there were no measures available to account for the different degrees of uncertainty in measured responses and confidence levels of tuning parameters. Hence, it may lead to updated models that may not fully represent the actual behaviour of the real structure. To address this issue, in the hybrid approach presented in this paper, model updating algorithm includes the use of weighting coefficients on both the tuning parameters and measured responses. To implement the updating process FEMtools (2012), which is a multi-functional computer-aided program for FE model updating has been used in this study. A weighted error (E_R) is derived which includes the differences in the target responses and numerical responses as well as the differences in tuning parameters in two successive iterations. Further, both the target responses and tuning parameters are coupled with weighting matrices based on their confidence levels which are determined

using either statistical methods or judgement of the analyst and experimentalists (FEMtools, 2014):

$$E_R = \{\Delta R\}^t [C_R] \{\Delta R\} + \{\Delta P\}^t [C_P] \{\Delta P\} \tag{5}$$

ΔR difference of the responses

ΔP difference of the parameters

$[C_R]$ diagonal weighting matrix for responses

$[C_P]$ diagonal weighting matrix for parameters (see Section 3.3 for more detail of these two matrices).

The linear term of Taylor's expansion series is used to approximate the target response vector R_e using the vectors R_0 (original response vector), P_0 (original parameter vector) and P_u (updated parameter vector).

$$R_e \approx R_0 + S(P_u - P_0) \tag{6}$$

Hence, using the above linear relationship between target responses and tuning parameters, parameter difference ΔP can be expressed as follows.

$$\Delta P = P_u - P_0 = G(R_e - R_0) \tag{7}$$

$[G]$ gain matrix.

Matrix G is derived in such a way to minimise the error function and when there are more responses than parameters it is calculated as:

$$[G] = \left([C_p] + [S_n][C_R][S_n]\right)^{-1} [S_n]^t [C_R] \tag{8}$$

And when there are more parameters than responses matrix G is calculated as:

$$[G] = \left([C_p]\right)^{-1} [S_n]^t \left([C_R]^{-1} + [S_n]\left([C_P]\right)^{-1} [S_n]^t\right)^{-1} \tag{9}$$

$[S_n]$ normalised relative sensitivity matrix.

Updated parameter vector can be obtained through a re-arranged equation (7) and the new response vector corresponding to the new updating parameter P_u is calculated from the modal analysis.

$$\{P_u\} = \{P_o\} + [G]\{-\Delta R\} \tag{10}$$

The resulting response vector and the updated parameters will be the starting vectors R_0 and P_0 for the next iteration. This iteration process is carried out until the error function is minimised to a pre-determined tolerance (FEMtools, 2014).

3.3 Derivation of weighting matrices for responses and parameters

As mentioned above in the equation (5), $[C_R]$ represents a diagonal weighting matrix expressing the confidence in the model parameters and $[C_P]$ represents the diagonal weighting matrix relating to the confidence in test data. The inverse of $[C_R]$ and $[C_P]$ are the co-variance matrices of measured responses and tuning parameters respectively.

Since, the parameters and the responses used in this model updating study are of different order and magnitude the standard deviation values should be normalised before calculating the covariance matrices and then the weighting matrices. Hence, in this study the scatter values are obtained to calculate the co-variance matrices. Since the scatter values are normalised with respect to the mean value, it is independent of the type and magnitude of the parameters used in the model updating process. A scatter value is defined as the ratio of the standard deviation (σ) to the mean value (μ) for a given set of data samples, which is also called coefficient of variation in statistical terms.

$$Scatter = \frac{\sigma}{\mu} \tag{11}$$

For a tuning parameter if the scatter value is high (low confidence level), it will result in low C_p values and parameter term in the objective function [equation (5)] to be small. Hence, the objective function will be controlled by the response term and this will result in large parameter changes in the model updating process. Similarly, if a tuning parameter has low scatter (high confidence level), it will only change if majority of the responses have an effect upon the change of that parameter which leads to small parameter changes. Equivalently, measured responses with low uncertainty values will have low scatter values and drive the parameter changes during the updating process while responses with high uncertainties will only make an impact on the updating process if the responses with low scatter values also change the parameters in a similar way. Hence, it is important to identify the appropriate scatter values for both tuning parameters and measured responses to make the model updating process more realistic and meaningful.

3.4 Scatter in measured responses and tuning parameters

Sixty datasets obtained from OMA results in various days are used to compute the scatter values of the responses. The distribution of the natural frequencies of the first five modes for 60 test samples used in this study are illustrated as bar chart plots in Figure 4. These bar charts clearly show that the test data for the natural frequencies has different levels of variation such as some modes are concentrated to one single value (mode 1 and mode 3) and other modes show a range of values (mode 2, mode 4 and mode 5). Hence it is important to incorporate the different degrees of uncertainties in the model updating process to obtain updated models that represent the behaviour of the real structure. The scatter values of the measured natural frequencies and MAC values of associated mode shapes derived from the statistical analysis of 60 samples are tabulated in Table 3. To calculate the MAC values correlation analysis is carried out for each sample with the developed initial FE model. According to the data samples analysed lowest scatter in terms of natural frequencies is mode 3 and mode 1 for the MAC values of the mode shapes, while mode 2 has the highest scatter value for both natural frequencies and MAC values. Further, the results also revealed that MAC values have higher degree of uncertainty compared to the measured natural frequencies since the scatter values are much higher in the MAC values compared to the measured frequencies. Also, considering the combined effect of the natural frequencies and MAC values mode 1 has the lowest uncertainty which should have the highest priority in the model updating and mode 2 has the highest uncertainty and the lowest priority in the model updating process.

Figure 4 Variation of measured natural frequencies for the first five modes (60 datasets) (see online version for colours)

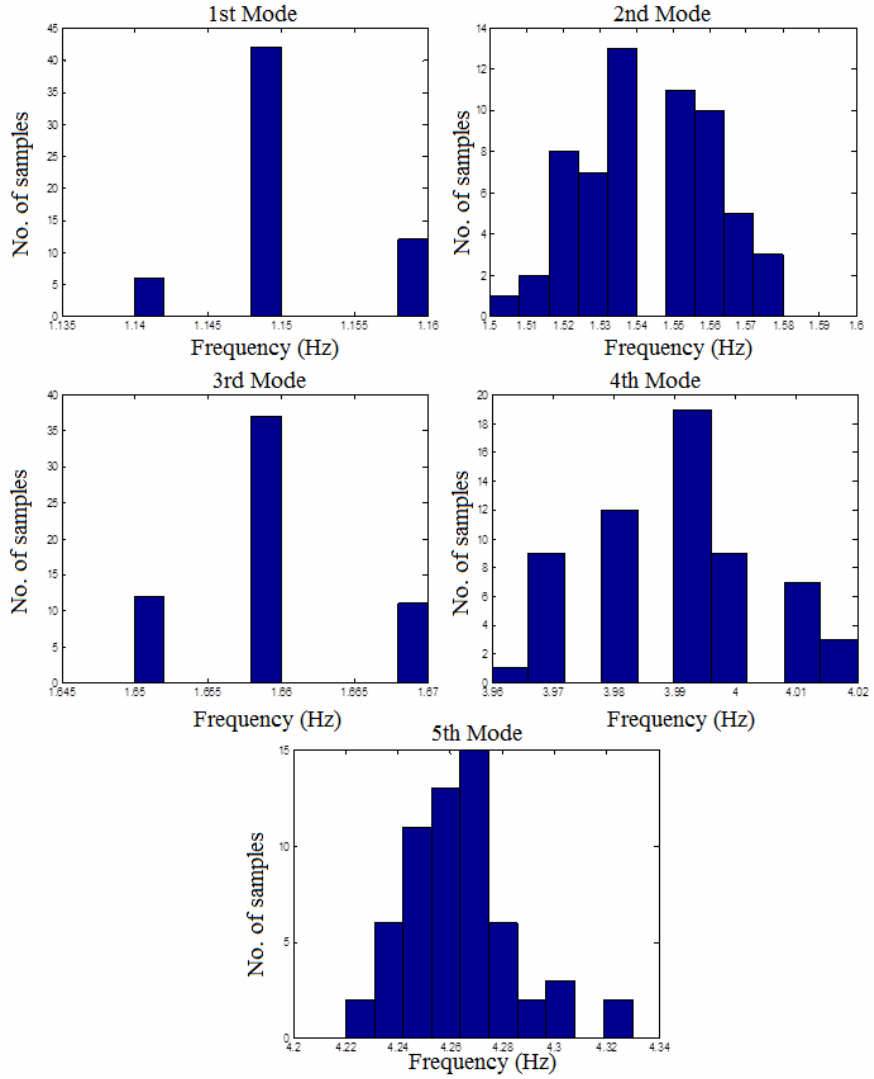


Table 3 Scatter values of measured frequencies and MAC values of associated mode shapes

Mode	*Measured frequencies*			*MAC values of the associated mode shapes*		
	Mean (Hz)	*St. dev.*	*Scatter (%)*	*Mean (%)*	*St. dev.*	*Scatter (%)*
1st mode	1.150	0.005	0.437	91.1	1.255	1.378
2nd mode	1.542	0.018	1.200	55.5	6.845	12.330
3rd mode	1.660	0.006	0.333	45.5	4.033	8.869
4th mode	3.989	0.014	0.351	59.3	2.801	4.726
5th mode	4.268	0.020	0.474	66.1	3.464	5.242

Since there is no actual testing data available for the derivation of scatter values for the tuning parameters, these values have been estimated based on the engineering judgement. For example, according to Australian Standard (2001) for concrete structures the E-value of concrete can have a scatter up to 20% from the design values, hence 20% value was chosen as the scatter for the tuning parameter E-value of concrete. In contrast, the parameters such as ρ-value of concrete normally has a relatively low scatter from the designed values, hence 10% was used in the model updating process. Since the shell thickness has a relatively high scatter for this particular case study a scatter value of 30% was used in this study. The scatter values chosen for each tuning parameter is listed in Table 4. In order to compare the results with the previous study conducted by the authors without considering the measurement uncertainties and confidence levels of tuning parameters, the same upper/lower bounds (tabulated in Table 4) for the model tuning parameters and the stopping criterion for the model updating algorithm have been used in the presented case study. The stopping criteria set for the model updating algorithm are as follows;

- minimum residue value – 0.1%
- minimum improvement between two consecutive iterations – 0.01%
- maximum number of iterations – 100.

Table 4 Parameters selected for the model updating and the implemented limits

Parameter	*Scatter*	*Minimum limit*	*Maximum limit*
Young's modulus (E)	20%	–15%	+15%
Mass density (ρ)	10%	–15%	+15%
Cross sectional area (AX)	10%	–15%	+15%
Torsional stiffness (IX)	10%	–15%	+15%
Bending moment of inertia about Y (IY)	10%	–15%	+15%
Bending moment of inertia about Z (IZ)	10%	–15%	+15%
Shell thickness (H)	30%	–30%	+30%

3.5 Model updating results

In the model updating procedure to minimise the weighted error E_R, and hence to improve the response prediction of the model, the following correlation function (C_f) is used as the optimisation algorithm which includes the weighted relative difference between natural frequencies and average MAC values, as reflected in the following equation.

$$C_f = \left[\frac{1}{C_R}\sum_{i=1}^{N} C_{R_i}\frac{\Delta f_i}{f_i}\right] + \left[1 - \frac{1}{C_R}\sum_{i=1}^{N} C_{R_i} MAC_i\right] \tag{12}$$

where

$$C_R = \sum_{i=1}^{N} C_{R_i};$$

C_{R_i} weighing values of responses.

N is the total number of target responses (frequencies/MAC responses) considered; Δf_i and f_i are the frequency error and target frequency respectively; and MAC_i corresponds to the MAC values of each mode shape pair.

The automated model updating procedure stopped after 33 iterations due to the minimum improvement between two consecutive improvements dropped below the set up value of 0.01%. Table 5 summarises the OMA frequencies and the FE model frequencies both before and after updating for the first five natural modes. The results of the previous work conducted without considering the scatter for the responses and tuning parameters (Kodikara et al., 2016) are also included in Table 5 for comparison purposes. The results clearly show a significant improvement in the natural frequency of the first mode in comparison to the previous case study with the error reduced from –4.62% to –1.83% by including measurement uncertainties in the model updating process. The frequency of the second mode shows notable decline compared to the previous study while for the other three modes the results are in the same region for both studies since the error is relatively low (less than 0.5%) in all those cases. These results are in-line with the measurement uncertainties presented in Section 3.3 where the first mode has the highest confidence which governed the model updating process and the second mode has the lowest confidence and less priority in the model updating process.

Table 5 Natural frequencies of the updated models of P block – with and without response and parameter uncertainties

Mode number	*OMA frequency*	*Initial FE model*		*Updated FE model – present study*		*Updated FE model – previous study*	
		Frequency	*Error*	*Frequency*	*Error*	*Frequency*	*Error*
1	1.147 Hz	0.990 Hz	–13.69%	1.126 Hz	–1.83%	1.094 Hz	–4.62%
2	1.544 Hz	1.452 Hz	–5.96%	1.521 Hz	–1.49%	1.555 Hz	0.71%
3	1.653 Hz	1.678 Hz	1.51%	1.650 Hz	–0.18%	1.657 Hz	0.24%
4	3.989 Hz	3.680 Hz	–7.75%	3.977 Hz	–0.30%	3.988 Hz	–0.03%
5	4.254 Hz	4.972 Hz	16.88%	4.251 Hz	–0.07%	4.258 Hz	0.09%

For the comparison of the results, Table 6 presents the MAC values of the associated mode shapes with OMA results of the first five modes for the initial FE model, present case study in this research and the previous case study. The results of the present study show an improvement in the results of 1st mode, 3rd mode and 4th mode and weaken the results of 2nd mode and 5th mode, in comparison to the MAC values of the previous study. Similar to the updated frequencies, results are compatible with the uncertainties in the measured responses, where the mode shape pairs with low scatter values (mode 1 and mode 4) shows noticeable improvement while the mode shape pairs with high scatter values (mode 2 and mode 3) are less prioritised in the model updating process and resulted in lower MAC values. Hence, the MAC values of the present case study reflect better the behaviour of the actual structure compared to the previous study, where there was no meaningful way to relate the improvements in the MAC values. A graphical comparison of the mode shapes of updated FE model and OMA results for the associated mode shapes of first five modes is shown in Figure 5.

Table 6 Comparison of MAC values for mode shape pairs before and after model updating

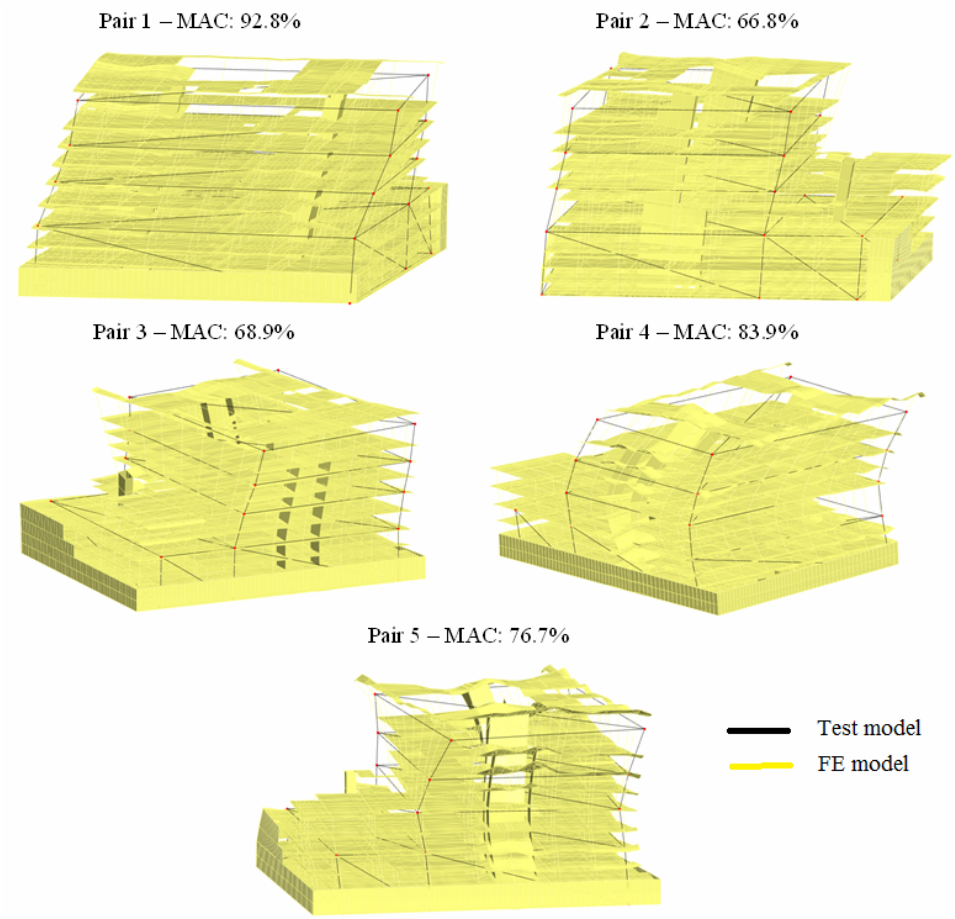
Mode shape pair	*Initial FE model*	*Updated model – present study*	*Updated model – previous study*
1	89.9%	92.8%	88.6%
2	50.5%	66.8%	89.4%
3	42.5%	68.9%	62.7%
4	63.2%	83.9%	62.6%
5	68.4%	76.7%	84.4%

Table 7 summarises the maximum and minimum parameter changes after the model updating for the present case study and the previous case study conducted by the authors without incorporating the confidence levels of the tuning parameter and measured response uncertainties in the model updating process. The Young's modulus and the shell thickness are assigned with relatively high scatter values (20% and 30% respectively) compared to other tuning parameters and it is reflected in the parameter changes where the maximum/minimum allowable limits are achieved for both the aforementioned tuning parameters in the present case study. Even though the same boundary limits are used for the other tuning parameters, these parameters achieved much less variation from the original values since these parameters are identified as high confidence tuning parameters in the model updating process. In contrast, in the previous model updating study conducted, there was no meaningful way to relate the parameter changes with their confidence levels although the parameter variation is limited by implementing upper/lower bounds to make the parameter changes realistic and meaningful. In both the studies a higher variation limit was adopted to the shell thickness to account for the simplifying assumptions made in the development of initial FE models. Since it was impossible to model the actual variation of the slab thicknesses, average values has been used in most of the occasions and in some areas actual thickness variation was 30% from the values used in the initial FE model.

Table 7 Parameter changes of the updated models for the present study and previous study

Parameter	*FE model – present study*		*FE model – previous study*	
	Max. % difference	*Min. % difference*	*Max. % difference*	*Min. % difference*
E	+15	–15	+15	–15
ρ	+10.11	–11.23	+15	–15
AX	+9.72	–8.78	+8.34	–9.61
IX	+5.63	–5.11	+1.31	–1.51
IY	+8.56	–9.72	+14.3	–15
IZ	+7.80	–4.62	+10.7	–4.35
H	+30	–30	+30	–30

Figure 5 Correlated mode shape pairs of the updated FE model and OMA modes of P block (see online version for colours)



4 Conclusions

A successful model updating study has been carried out on the P block structure located at QUT premises, using a hybrid model updating procedure to incorporate the different degrees of uncertainty in the measured responses and diverse confidence levels of tuning parameters in the conventional sensitivity-based automated model updating procedures. Uncertainty in the measured responses and confidence levels of the tuning parameters were included as statistical scatter in the hybrid model updating procedure. By comparing the results of the case study presented in this paper with the previous model updating study conducted by the authors on the P block structure using conventional model updating procedures, the paper demonstrated the efficacy of incorporating these scatter values in the model updating process to obtain more realistic updated FE models. The results showed significant improvement in the responses of the updated model which has low level of uncertainty in the measured data (natural frequency of mode 1, MAC value

of mode shape pair 4) and weaken the results of the responses with higher degree of uncertainty (natural frequency and MAC values of mode 2) compared to the previous study, which ensure the updated FE model is more realistic and meaningful in predicting the dynamic behaviour of the actual structure. In addition, inclusion of the statistical scatter for the tuning parameters based on different confidence levels provides a more meaningful way to interpret the parameter changes in the automated model updating process. For example, the E value of concrete and shell thickness have relatively high scatter value (20%, 30% respectively) compared to the other tuning parameters (10%) and the parameter changes in the updated FE model represent this by achieving maximum variation limit allowable for the E values of concrete (15%) and the shell thickness (30%) and a lesser variation than allowable for other tuning parameters as opposed to the previous study which did not have any meaningful way to interpret the parameter changes even-though the variation was limited by implementing boundary limits.

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