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# Comparison of subsidy schemes for reducing waiting time: special focus on smart home care for elderly people

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## Abstract

With the aging of the population, community-based elderly care service becomes popular in China. To improve the well-being of elderly people and provide high-quality service, smart products are used in home care and community-based elderly care service, such as remote monitor system, health monitor device and intelligent home appliances. Chinese government also carries out some fiscal policies to promote smart home care and reduce service waiting time. While various subsidy schemes are proven effective in healthcare and other fields, it remains ambiguous which scheme is more efficient in elderly care services. In this socio-economic context, we formulate stylized queuing models within a game-theoretic framework to compare two types of subsidy schemes: investment subsidy for service providers and price subsidy for elders using the service. The results show that investment subsidy is more cost-effective given the same waiting time threshold. Government should choose different subsidy schemes under different market condition, and use subsidy schemes and waiting time threshold in combination. This study provides insights for smart home care service management and offers implications for government in subsidy scheme selection.

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## 1. Introduction

With aging populations increases dramatically, aging problem has been exerting more and more pressure on many countries. At the same time, the appeal of the modern elderly gradually changes from life support focusing on extending life span to quality support that aims to improve life quality. Elderly care services, whether provided in institutions or the community are essential to the well-being of elderly and disabled people with limitations in performing daily activities [1]. At present, home care, community-based care and institutional care are the three main modes of elderly care around the world. Many studies find that community-based care service is more efficient than institutional care [2]. Compared to institutional care, most elderly people prefer home care and community-based services [3].

Thanks to advances in technology, many smart devices and applications are designed and applied for elderly people,

especially for those who are in poor health status or disabled. To help the elders live quality life, numerous smart devices and technologies are applied in their homes, which is also known as smart homes. Among these homes, devices like remote monitor system, health monitor device and intelligent home appliances, not only make the elders' daily life easier, but also make it convenient for service providers to support the elders. That is because the providers can better access elders' routine life data with the help of intelligent devices and detect possible emergencies or urgent service requirements of an elder. Meanwhile, a community-based service provider is able to support everyday life of the elders. For instance, they can offer personal care and in favor of elders' daily living activities as doing cleanings, cooking and health examination.

In smart homes context, community-based service providers should provide pension products, such as smart devices, but more importantly, they need to respond to the

elders service demand quickly and organize service resources for elders. However, among those public service systems with limited service capacity, people have to wait until being served. With the improvement of the modern economic level and the increase of elderly population, the demand for elderly care services has greatly increased. This also means elders served by community-based service providers may encounter longer waiting times after they propose a service requirement [4].

Here, we define the waiting time in a community-based service context as follows. The waiting time that we focus on in this study is a periodical delay between an elder puts forward his service requirement and is served by community-based service provider. Due to the capability of the provider, this kind of waiting time varies from the elders. As for the pension product and service providers, they can invest on their facilities by employing more professional clients, expanding service coverage and applying advanced technologies. These methods equips the provider to serve the elders more flexible and even reduce the waiting times.

For governments, subsidization policy can shorten the waiting time and improve service experience in healthcare systems [5]. Many countries have taken measures to control public waiting time in healthcare systems. Sweden has introduced a waiting time guarantee scheme subsidy mechanism for elective surgeries [6]. The Australian government introduced a 30% health insurance rebate to the patients who purchase the private care service [7]. Meanwhile, some provincial government in Canada are considering to provide some unconditional subsidies to hospitals [8]. Chinese government also conducted out some fiscal policies to support the smart home care and community-based elderly care service [9]. But a key question for government is how to bring the best of a subsidy scheme efficiently in a service system. There are some studies which compare different subsidies to design the optimal subsidy strategy, but the results are varied in different field [10].

In this paper, we want to compare two types of subsidy schemes. The first one is investment subsidy, government subsidy the service provider when they invest in reducing waiting time or improving the quality of their service. The second is the price subsidy. In this scheme, the government will give a subsidy for each unit of service when the service provider sell the service to a patient. Accordingly, the service provider will decide whether to follow the policy and adjust its service price and/or capacity. We try to answer the following questions: 1) what factors affect decisions of government and the service provider? 2) which subsidy policy should the government choose to reduce the waiting time?

To answer these questions, this paper combines the stylized queuing models and Stackelberg game to analyze subsidy policy in healthcare systems. In the next section, we first explain the assumptions and analyze the benchmark. Section 3 and Section 4 study the investment subsidy and price subsidy respectively. Section 5 makes further comparison and discussion. In Section 6, we make a conclusion and describe the future possible research directions.

## 2. Assumptions and benchmark

**Assumptions 1.** The service system is modelled as parallel M/M/1 queues.

Elders arrive according to a Poisson process at the rate  $\lambda$ . Service rate  $\mu$  represents the service capacity of the service provider. The expected waiting time (queuing time) in an M/M/1 queue is  $W = 1/(\mu - \lambda)$ . We assume that  $\mu > \lambda > 1$  which ensures a stable queuing system.

**Assumptions 2.** All elders have the same service utility  $v$ , and  $p$  represents the service price. Elders' waiting cost parameter is denoted as  $\theta$ , which is assumed to be uniformly distributed over  $[0, 1]$ .

This represents that elders are heterogeneous in waiting time sensitivity. The expected elder waiting cost is computed as  $\theta W$ , an elder's net utility (called the utility surplus) is  $U = v - p - \theta W(\lambda, \mu)$ . Since  $v > p$  and  $p > c$ , we can get  $v > c$ . Elder will buy the service when the function  $U \geq 0$ , otherwise, he or she will leave the market. We assume that  $v - p - \bar{\theta}W(\lambda, \mu) = 0$ , then  $\bar{\theta} = (v - p)/W$ , each elder belongs to  $[0, \bar{\theta}]$  will buy the service. The effective arrive rate is  $\lambda = \int_0^{\bar{\theta}} \lambda_0 d\theta = \bar{\theta}\lambda_0$ , recall that  $W = 1/(\mu - \lambda)$ . Also, the arrival rate of the elder is  $\lambda(p, \mu) = \varphi(p)\mu$ . Let  $1/\lambda_0 = a$  and  $\varphi(p) = (v - p)/(a + v - p)$ , then  $\partial\varphi/\partial p = -a/(a + v - p)^2 < 0$  and  $\partial^2\varphi/\partial p^2 = -2a/(a + v - p)^3 < 0$ .

Table 1. Glossary of main notations.

System parameters	Definition
$\theta$	Waiting cost parameter of the elders, $\theta \in [0, 1]$
$v$	Utility of the elder who gets the service
$c$	The capacity cost per unit of service capacity
$\lambda_0$	Total arrival rate of the service request
$\lambda$	Effective arrival rate of the service request
$\Gamma$	Government objective threshold of waiting time
$\pi$	Profit of the service provider
$W$	Waiting time of the elders
$U$	Net utility of the elder who gets the service
$TU$	Total utility of elders
$Exp$	Expenditure of government subsidy
Decision variables	Definition
$t$	Investment subsidy parameter
$s$	Price subsidy parameter
$p$	Price of the elderly care service
$\mu$	Service capacity of the service provider

As a benchmark, this section first considers the case that government does not give any subsidy to the service provider, or the service provider chooses not to cooperate with the government. This can happen when the government's proposal is not practical or harmful to the profit of the provider. The provider decides the service price and service rate to maximize his profit. Elders decide whether to buy the service or not, which is reflected by the effective arrival rate. The profit of the provider can be modeled as follows:

$$\text{Max } \pi_0(p, \mu) = p\lambda - c\mu \tag{1}$$

As  $\lambda(p, \mu) = \varphi(p)\mu$ , we can know that  $\pi_0$  is increasing in  $\mu$ , so we first study the provider's price decision given a fixed capacity determined by  $\mu$ . This assumption is applicable as the capacity of the public service system is not easy to adjust in a short run. The provider's optimization problem (1) can be explicitly written as

$$\text{Max } \pi_0(p | \mu) = p\varphi(p)\mu - c\mu \quad (2)$$

Take the partial derivative of  $\pi_0$  with respect to  $p$  yields  $\partial\pi_0/\partial p = \mu(va - 2vp - 2pa + v^2 + p^2)/(a + v - p)^2$  and  $\partial^2\pi_0/\partial p^2 = -2\mu a(a + v)/(a + v - p)^3 < 0$ , so the optimal solution exists, and  $p_0^* = v + a - \sqrt{va + a^2}$ . Substituting the optimal solution into equation (1), the profit function of the provider will be

$$\pi_0^*(\mu | p^*(\mu)) = (v - c)\mu \quad (3)$$

The effective arrival rate will be  $\lambda = \mu - \sqrt{a/(a + v)}$ , as the effective arrival rate is no more than the total arrival rate  $\lambda \leq \lambda_0$ , then we can get  $\mu \leq \lambda_0 + \sqrt{1/(1 + v\lambda_0)}$ , so

$$\mu_0^* = \lambda_0 + \sqrt{1/(1 + v\lambda_0)}$$

$$p_0^* = v + \frac{1}{\lambda_0} - \frac{1}{\lambda_0} \sqrt{v\lambda_0 + 1}$$

$$\pi_0^* = (v - c)(\lambda_0 + \sqrt{1/(1 + v\lambda_0)})$$

$$W_0 = \sqrt{1 + v\lambda_0}$$

**Proposition 1.** *If the service provider enters the market, then (a)  $\partial W_0/\partial v > 0$ ,  $\partial p_0^*/\partial v > 0$ ; (b)  $\partial W_0/\partial \lambda_0 > 0$ ,  $\partial p_0^*/\partial \lambda_0 < 0$ .*

As the service utility  $v$  increase, the waiting time and price of the service increases. A higher total arrival rate means a larger market, and the provider prefers to offer service in low quality and price, which results in a relatively long waiting time. Consequently, when the utility is higher for the elders, more people are willing to purchase the service, and the waiting time will be longer. So, it is necessary for the government to take some measures to reduce the waiting time of those kinds of service.

### 3. Investment subsidy cases

This subsection considers the case that government gives investment subsidy if the service provider invests to reduce their waiting time below the threshold  $\Gamma$ . The timing of this game is as follows. In the first stage, the government declares the subsidy method and subsidy parameter. In the second stage, service providers determine whether to reduce the waiting time of their service system and decide the service price as well as their service capacity. In the third stage, the government will give the firm subsidy according to its capacity. To approximate equilibrium solution for the game problem, herein we adopt the backward induction approach. We first explore tentative optimal solutions for the service provider and then solve the problem of the government.

The service provider's aim is to optimize his profit

$$\text{Max } \pi_1(p, \mu) = p\lambda - c\mu + t\mu \quad (4)$$

*s.t.*  $W \leq \Gamma$

#### 3.1. Price decision with fixed capacity

With the case of fixed service rate or service rate is not easy to change, the provider can only get a fixed subsidy and he will reduce the waiting time through adjusting the service price. The service provider's optimization problem (3) can be explicitly written as

$$\text{Max } \pi_1(p | \mu) = p\lambda - c\mu + t\mu \quad (5)$$

*s.t.*  $W \leq \Gamma$

By substituting  $\lambda(p, \mu)$  into (5), we can get the optimal solution in Proposition 2.

**Proposition 2.** *For the case of investment subsidy with fixed capacity, there exist equilibrium solutions, where*

$$(a) \text{ if } \Gamma \geq \sqrt{1 + v\lambda_0} / \mu, \text{ then } p_1^* = p_0^*;$$

$$(b) \text{ if } \Gamma < \sqrt{1 + v\lambda_0} / \mu, \text{ then } p_1^* = a + v - a\Gamma, \lambda_1^* = \mu - 1/\Gamma.$$

With Proposition 2, we can see if  $\Gamma < \sqrt{1 + v\lambda_0} / \mu$ , the provider tend to reduce the waiting time by adjusting the price. We know that  $\Gamma < W_0 = \sqrt{1 + v\lambda_0}$  must be met as the objective waiting time is shorter than the case with no subsidy. In this solution, the constraint is stricter as  $\sqrt{1 + v\lambda_0} / \mu < W_0$ . When the service rate is low, the service provider will get a small amount of subsidy allowances. This suggests that if the objective waiting time is just a little lower than before, the provider is unwilling to reduce elders' waiting time due to a low service rate.

#### 3.2. The joint decision of price and capacity

To obtain maximize profits, the service provider may also improve its capacity. Since the price decision is a function of  $\mu$ , i.e.,  $p^*(\mu)$ , we can substitute  $p^*(\mu)$  into  $\pi_1$ . We can get

$$\pi_1^*(\mu | p^*(\mu)) = (a + v - \Gamma\mu a)(\mu - 1/\Gamma) - c\mu + t\mu \quad (6)$$

**Proposition 3.** *For the case of investment subsidy with joint decision of price and capacity, there exist equilibrium solutions, where*

$$\mu_1^* = (2a + v - c + t)/2\Gamma a$$

$$p_1^* = (v - t + c)/2$$

$$\pi_1^* = [v^2 + (t - c)^2 + 2(2a + v)(t - c)]/4\Gamma a$$

With Proposition 3, we can easily know that the investment subsidy parameter influence both the price and capacity, but the threshold of the waiting time will not affect the service price directly. So, the government can both adjust the investment subsidy parameter and threshold of the waiting time to have better control.

#### 3.3. Government decision of investment subsidy

The government is to achieve its objective waiting time with minimum expenditure. Similar government objectives can refer to [11]. Some study also proved that minimizing government cost and maximizing social welfare are equivalent [12][13]. The minimum government expenditure is easy to measure and interpret, so it more realistic and practical approach for the decision maker. Thus, government expenditure minimizing model could be expressed as

$$\text{Min } \text{Exp}_1 = t\mu \tag{7}$$

Substitute  $\mu_t^*$  into equation (7), we can get the optimal solution in Proposition 4.

**Proposition 4.** For the case of investment subsidy, to make sure the firm will cooperate with the government,  $t$  has the minimum value, which is

$$t^* = c - \frac{2}{\lambda_0} - v + \frac{2}{\lambda_0} \sqrt{1 + v\lambda_0 + \Gamma(v-c)\lambda_0(\lambda_0 + \sqrt{1/(1+v\lambda_0)})}$$

Government's expenditure is

$$\text{Exp}_1^* = (c - v - \frac{2}{\lambda_0}) \frac{1}{\Gamma} \sqrt{1 + v\lambda_0 + \Gamma(v-c)\lambda_0(\lambda_0 + \sqrt{1/(1+v\lambda_0)})} + \frac{2}{\Gamma\lambda_0} + \frac{2v}{\Gamma} + 2(v-c)\lambda_0 + 2(v-c)\sqrt{1/(1+v\lambda_0)}$$

With Proposition 4, we can know when to decide the investment subsidy parameter, the government should consider not only the objective waiting time but also the market condition.  $TU$  is the consumer total net utility, it can be obtained as

$$TU = \int_0^{\bar{\theta}} (v - p - \theta W) d\theta \tag{8}$$

By substituting the optimal solution into equation (8), we can get

$$TU_1^* = \frac{1}{2\Gamma\lambda_0^2} (\sqrt{1 + v\lambda_0 + \Gamma(v-c)\lambda_0(\lambda_0 + \sqrt{1/(1+v\lambda_0)})} - 1)^2$$

#### 4. Price subsidy cases

For the price subsidy, the government will give a rebate for every unit service the provider sold. The timing of this game is: In the first stage, the government declares the subsidy method and subsidy parameter. In the second stage, firms determine whether to reduce their waiting time and decide the service price and capacity. In the third stage, the government will give the firm subsidy after he provides the services. The service provider's aim is to maximize the profit

$$\text{Max } \pi_2(p, \mu) = (p + s)\lambda - c\mu \tag{9}$$

*s.t.*  $W \leq \Gamma$

##### 4.1. Price decision with fixed capacity

The service provider's optimization problem (4) can be explicitly written as

$$\text{Max } \pi_2(p | \mu) = (p + s)\lambda - c\mu \tag{10}$$

*s.t.*  $W \leq \Gamma$

By substituting  $\lambda(p, \mu)$  into (10), we can get the optimal solution in Proposition 5.

**Proposition 5.** For the case of price subsidy with fixed capacity, there exist equilibrium solutions, where

- (a) if  $\Gamma \geq \sqrt{1 + v\lambda_0 + s\lambda_0} / \mu$ , then  $p_2^* = p_0^*$ ;
- (b) if  $\Gamma < \sqrt{1 + v\lambda_0 + s\lambda_0} / \mu$ , then  $p_2^* = a + v - a\mu\Gamma$ ,  $\lambda_2^* = \mu - 1/\Gamma$ .

With proposition 5 we can know that for the price subsidy with fixed capacity, the objective of waiting time can be higher than the investment subsidy which requires  $\Gamma < \sqrt{1 + v\lambda_0} / \mu$ . As  $s$  increasing, the value of the  $\Gamma$  also increases. This demonstrates that if the government want to

reduce the waiting time slightly, implementing the price subsidy is more acceptable by the provider.

##### 4.2. Joint decision of price and capacity

Substitute  $p^*(\mu)$  into  $\pi_2$ . Then, we can get

$$\pi_2^*(\mu | p_2^*(\mu)) = (a + v + s - \Gamma\mu a)(\mu - 1/\Gamma) - c\mu \tag{11}$$

**Proposition 6.** For the case of investment subsidy with joint decision of price and capacity, there exist equilibrium solutions, where

$$\begin{aligned} \mu_2^* &= (2a + v - c + s) / 2\Gamma a \\ p_2^* &= (v - s + c) / 2 \\ \pi_2^* &= [(v + s)^2 - 2c(2a + v + s) + c^2] / 4\Gamma a \end{aligned}$$

Compare Proposition 3 and Proposition 6, we can know the investment subsidy and the price subsidy have a similar structure in service price and service rate. Given that  $\partial\pi_1^*/\partial t = (t + 2a + v) / 2\Gamma a$ , we can know the profit of provider increases with  $t$ . For the price subsidy scheme, when  $s < c$ ,  $\partial\pi_2^*/\partial s = (s - c) / 2\Gamma a < 0$ . That means if the price subsidy parameter is lower than the capacity unit cost, the service provider will not cooperate with the government.

##### 4.3. Government decision of price subsidy

The government is to minimize its expenditure to make the service provider improve their service quality and reduce waiting time.

$$\text{Min } \text{Exp}_2(s) = s\lambda \tag{12}$$

Substitute  $\lambda_s^* = (v + s - c) / 2\Gamma a$  into equation (12), we can get the optimal solution in Proposition 7.

**Proposition 7.** For the case of price subsidy, to make sure the firm will cooperate with the government,  $s$  has the minimum value, that is

$$s^* = c - v + \frac{2}{\lambda_0} \sqrt{c\lambda_0 + \Gamma(v-c)\lambda_0(\lambda_0 + \sqrt{1/(1+v\lambda_0)})}$$

Government's expenditure and total utility are as below.

$$\begin{aligned} \text{Exp}_2^* &= \frac{c-v}{\Gamma} \lambda_0 \sqrt{\frac{c}{\lambda_0} + \Gamma(v-c) + \Gamma(v-c) \frac{1}{\lambda_0} \sqrt{1/(1+v\lambda_0)}} \\ &\quad + \frac{2c}{\Gamma} + 2\lambda_0(v-c) + 2(v-c)\sqrt{1/(1+v\lambda_0)} \\ TU_2^* &= \frac{c + \Gamma(v-c)(\lambda_0\sqrt{1+v\lambda_0} + 1)}{2\Gamma\lambda_0\sqrt{1+v\lambda_0}} \end{aligned}$$

#### 5. Results and discussion

In this section, we compare the investment subsidy and price subsidy to find which is beneficial for the government and the service provider. Besides, we also consider the interests of elders and compare the price as well as elders' utility.

**Proposition 8.** For the consumer, (a) if  $\Gamma \leq \frac{(1 + v\lambda_0)^2 - 1 - v\lambda_0}{\lambda_0(v-c)(\lambda_0 + \sqrt{1/(1+v\lambda_0)})}$ , then  $\partial TU_1 / \partial \Gamma \leq 0$ , if  $\Gamma > \frac{(1 + v\lambda_0)^2 - 1 - v\lambda_0}{\lambda_0(v-c)(\lambda_0 + \sqrt{1/(1+v\lambda_0)})}$ , then  $\partial TU_1 / \partial \Gamma > 0$ ; (b)  $\partial TU_2 / \partial \Gamma < 0$ .

Proposition 8(a) indicates that with investment subsidy, as

the waiting time threshold increasing, the total utility of patient first decreases then increases. This is because when the threshold is low, the waiting time is very short and the service provider only targets a high-end market. Increasing the waiting time threshold will reduce the utility of this part of rich elders drop dramatically. But when the restrictions continue to be loose, more elders request the service, the total utility increases. But for price subsidy, the total utility strictly decreases. Hence, the government should adjust the threshold under different subsidy schemes. For the investment subsidy, the government should set a relatively higher or lower waiting time threshold according to the budget. But for the price subsidy, the government should set a low threshold as possible as they can afford.

**Proposition 9.** Government's expenditure under investment subsidy and price subsidy satisfies  $Exp_1^* < Exp_2^*$ .

Proposition 3 implies that to achieve the same objective waiting time, investment subsidy is more cost-effective compared to price subsidy. That is because the investment subsidy is more concentrated on reducing waiting time, while price subsidy caters to elders. So, for the price subsidy, to accomplish the same target, the government needs to set a larger budgets.

**Proposition 10.** When the service provider cooperates with the government, (a) if  $\Gamma < \frac{4c + (v-c)^2 \lambda_0}{4(v-c)\lambda_0(1 + \sqrt{1+v\lambda_0})} \sqrt{1+v\lambda_0}$ , then

$\mu_1^* > \mu_2^*$ ,  $p_1^* < p_2^*$ ,  $\pi_1^* = \pi_2^* = \pi_0^*$ ,  $W_1 = W_2 = \Gamma$ ,  $TU_1 > TU_2$ ; (b) if  $\Gamma \geq \frac{4c + (v-c)^2 \lambda_0}{4(v-c)(1 + \lambda_0 \sqrt{1+v\lambda_0})} \sqrt{1+v\lambda_0}$ , then  $\mu_1^* < \mu_2^*$ ,  $p_1^* > p_2^*$ ,  $\lambda_1^* < \lambda_2^*$ ,  $\pi_1^* = \pi_2^* = \pi_0^*$ ,  $W_1 = W_2 = \Gamma$ ,  $TU_1 < TU_2$ .

Let  $\Gamma_0 = \frac{4c + (v-c)^2 \lambda_0}{4(v-c)(1 + \lambda_0 \sqrt{1+v\lambda_0})} \sqrt{1+v\lambda_0}$ , from Proposition 10,

we can see that if  $\Gamma$  is less than  $\Gamma_0$ , the investment subsidy scheme gets a higher total consumer utility. When  $\Gamma$  exceeds  $\Gamma_0$ , the price subsidy method achieves a higher total consumer utility. Therefore, we can see that the optimal subsidy schemes varies according to different markets. For a market with high-income and small elderly populations, the government should choose the investment subsidy. For a market with a low-income and large elderly population, the government should choose the price subsidy.

## 6. Concluding remarks and future research

Community-based elderly care service grows popular in China, which paves the way of smart products designed for home care and community-based elderly care services. For pension product and pension service providers, it is important to balance the service quality and operations cost. To reduce waiting times of elders using smart home care, government needs to make appropriate subsidy policy to incentive the service provider. This study formulates stylized queuing models within a game-theoretic framework to compare two types of subsidy schemes: investment subsidy for service providers and price subsidy for elders using the service. The results show that the government and the service provider's decision can be influenced by market conditions like market scale and elders' financial level. The government should

choose different subsidy schemes under different market condition. That is to say, for the market with high-income and small elderly populations, implementing the investment subsidy and setting a low waiting time threshold is more beneficial. For the market with a low-income and large population, the government should choose the price subsidy, and combining the subsidy schemes and waiting time threshold is better. For future research, we may extend our study to a non-profit service provider and even consider the competition between different service providers.

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## Appendix

### Proof of Proposition 2.

$$L(p|\mu) = p\phi(p)\mu - c\mu + t\mu + \gamma(\Gamma - 1/(\mu - \mu\phi(p)))$$

When  $\gamma > 0$ ,

$$\partial L/\partial p = \frac{va - 2vp - 2pa + v^2 + p^2}{(a+v-p)^2} \mu + \frac{\gamma}{\mu a}$$

$$\gamma(\Gamma - \frac{1}{\mu} - \frac{v-p}{a} \frac{1}{\mu}) = 0$$

So, we can get  $p_1^* = a + v - a\mu^*$ ,  $\gamma_1^* = (a+v)/\Gamma^2 - a\mu^2$ ,  $\lambda_1^* = \mu - 1/\Gamma$ .

If  $\gamma_1^* > 0$ , then  $\Gamma < \sqrt{1+v\lambda_0}/\mu < W_0$ .

The profit of the service provider is

$$\pi_1^* = (a+v - \Gamma\mu a)(\mu - 1/\Gamma) - c\mu + t\mu$$

If  $\Gamma \geq \sqrt{a+v}/\sqrt{a}\mu$ , the provider's time is already less than it, they do not need to reduce the waiting time to get the subsidy,

which is not in conformity with the reality.

**Proof of Proposition 3.**

$$\partial \pi_1^* / \partial \mu = 2a + v - 2\Gamma \mu a - c + t,$$

$$\partial^2 \pi_1^* / \partial \mu^2 = -2\Gamma a < 0$$

The optimal solution gets when

$$\mu_1^* = \frac{v-c+t}{2\Gamma a} + \frac{1}{\Gamma}, \quad p_1^* = \frac{v-t+c}{2}, \quad \lambda_1^* = \frac{v+t-c}{2\Gamma a}.$$

$$\text{So, } \pi_1^* = \frac{v^2 + (t-c)^2 + 2(2a+v)(t-c)}{4\Gamma a}.$$

**Proof of Proposition 4.**

Substitute  $\mu_1^*$  into Equation (7), we can get

$$\text{Exp}_1 = t((v-c+t)/2\Gamma a + 1/\Gamma), \text{ and } \partial \text{Exp}_1 / \partial t > 0, \partial^2 \text{Exp}_1 / \partial t^2 > 0. \text{ Hence, } \text{Exp}_1 \text{ is strictly increasing with } t.$$

To incentive the service provider to reduce the waiting time, the government should make sure that the service provider's profit with investment subsidy is no less than the benchmark.

So,  $\pi_1^* \geq \pi_0^*$  should be met, then we can get

$$t \geq c - v - \frac{2}{\lambda_0} + \frac{2}{\lambda_0} \sqrt{1 + v\lambda_0 + \Gamma(v-c)\lambda_0(\lambda_0 + \sqrt{1/(1+v\lambda_0)})}.$$

When  $t^* = c - \frac{2}{\lambda_0} - v + \frac{2}{\lambda_0} \sqrt{1 + v\lambda_0 + \Gamma(v-c)\lambda_0(\lambda_0 + \sqrt{1/(1+v\lambda_0)})}$ , the

government gets the optimal solution

$$\text{Exp}_1^* = (c - v - \frac{2}{\lambda_0}) \frac{1}{\Gamma} \sqrt{1 + v\lambda_0 + \Gamma(v-c)\lambda_0(\lambda_0 + \sqrt{1/(1+v\lambda_0)})}$$

$$+ \frac{2}{\Gamma \lambda_0} + \frac{2v}{\Gamma} + 2(v-c)\lambda_0 + 2(v-c)\sqrt{1/(1+v\lambda_0)}$$

**Proof of Proposition 5.**

$$L(p | \mu) = (p+s)\varphi(p)\mu - c\mu + \gamma(\Gamma - 1/(\mu - \mu\varphi(p)))$$

When  $\gamma > 0$ ,

$$\partial L / \partial p = \frac{va - sa - 2vp - 2pa + v^2 + p^2}{(a+v-p)^2} \mu + \frac{\gamma}{\mu a} = 0$$

$$\gamma(\Gamma - \frac{1}{1-\varphi(p)} \frac{1}{\mu}) = 0$$

So, we can get  $p_2^* = a + v - a\mu\Gamma$ ,  $\gamma_2^* = (a + v + s) / \Gamma^2 - a\mu^2$ ,

$$\lambda_2^* = \mu - 1/\Gamma.$$

$$\gamma_2^* > 0, \text{ that is } \Gamma < \frac{1}{\mu} \sqrt{\frac{a+v+s}{a}}.$$

The profit of the service provider is

$$\pi_2^* = (a + v + s - \Gamma \mu a)(\mu - \frac{1}{\Gamma}) - c\mu$$

**Proof of Proposition 6**

$$\partial \pi_2^* / \partial \mu = 2a + v - 2\Gamma \mu a - c + s$$

$$\partial^2 \pi_2^* / \partial \mu^2 = -2\Gamma a < 0$$

when  $\mu_2^* = \frac{v-c+s}{2\Gamma a} + \frac{1}{\Gamma}$ , the objective function gets the optimal

$$\text{solution } \pi_1^* = \frac{(v+s)^2 + c^2 - 2c(2a+v+s)}{4\Gamma a},$$

$$p_2^* = \frac{v-s+c}{2}, \mu_2^* = \frac{v-c+s}{2\Gamma a} + \frac{1}{\Gamma}, \lambda_2^* = \frac{v+s-c}{2\Gamma a}.$$

**Proof of Proposition 7**

Substitute  $\lambda_2^* = (v+s-c)/2\Gamma a$  into Equation (12), we can get

$$\text{Exp}_2(s) = s(v+s-c)/2\Gamma a, \text{ and } \partial \text{Exp}_2 / \partial s > 0, \partial^2 \text{Exp}_2 / \partial s^2 > 0. \text{ Hence, } \text{Exp}_2 \text{ is strictly increasing in } s.$$

The service provider's profit is no less than the benchmark,

$$\pi_2^* \geq \pi_0^*, \text{ that is } s \geq c - v + 2\sqrt{ca + \Gamma(v-c) + \Gamma(v-c)\frac{1}{\lambda_0}\sqrt{1/(1+v\lambda_0)}}, \text{ so}$$

$$s^* = c - v + \frac{2}{\lambda_0} \sqrt{c\lambda_0 + \Gamma(v-c)\lambda_0(\lambda_0 + \sqrt{1/(1+v\lambda_0)})}. \text{ The government gets}$$

the optimal solution:

$$\text{Exp}_2^* = \frac{c-v}{\Gamma} \lambda_0 \sqrt{\frac{c}{\lambda_0} + \Gamma(v-c) + \Gamma(v-c)\frac{1}{\lambda_0}\sqrt{1/(1+v\lambda_0)}}$$

$$+ \frac{2c}{\Gamma} + 2\lambda_0(v-c) + 2(v-c)\sqrt{1/(1+v\lambda_0)}$$

**Proof of Proposition 9.**

$$\text{Exp}_1^* - \text{Exp}_2^* = -\frac{v-c}{\Gamma} [\sqrt{1 + v\lambda_0 + \Gamma(v-c)\lambda_0(\lambda_0 + \sqrt{1/(1+v\lambda_0)})}$$

$$- \sqrt{c\lambda_0 + \Gamma(v-c)\lambda_0(\lambda_0 + \sqrt{1/(1+v\lambda_0)})}]$$

$$- \frac{2}{\Gamma \lambda_0} (\sqrt{1 + v\lambda_0 + \Gamma(v-c)\lambda_0(\lambda_0 + \sqrt{1/(1+v\lambda_0)})} - 1) < 0$$

The expenditure of investment subsidy is lower.