Institut für Numerische Simulation
Rheinische Friedrich-Wilhelms-Universität Bonn

FOURTH INTERNATIONAL WORKSHOP
MESHFREE METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS

BOOK OF ABSTRACTS

DATE:	SEPTEMBER 17–20, 2007
LOCATION:	BONN, GERMANY
SPONSOR:	SONDERFORSCHUNGSBEREICH 611
ORGANIZERS:	Prof. Ivo Babuška (University of Texas, Austin, USA)
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The numerical treatment of partial differential equations with meshfree discretization techniques has been a very active research area in recent years. While the fundamental theory of meshfree methods has been developed and considerable advances of the various methods have been made, many challenges in the mathematical analysis and practical implementation of meshfree methods remain.

Meshfree methods, particle methods, and generalized finite element methods have undergone substantial development since the mid 1990s. The growing interest in these methods is in part due to the fact that they are very flexible numerical tools and can be interpreted in a number of ways. For instance, meshfree methods can be viewed as a natural extension of classical finite element and finite difference methods to scattered node configurations with no fixed connectivity. Furthermore, meshfree methods have some advantageous features which are especially attractive when dealing with multiscale phenomena: A-priori knowledge about particular local behavior of the solution can be introduced easily in the meshfree approximation space, and an enrichment of a coarse scale approximation with fine scale information is possible in a seamless fashion. The implementation of meshfree methods and their parallelization however requires special attention, for instance with respect to numerical integration.

This symposium aims to promote collaboration among engineers, mathematicians, and computer scientists and industrial researchers to address the development, mathematical analysis, and application of meshfree and particle methods especially to multiscale phenomena. While contributions in all aspects of meshfree methods are invited, some of the key topics to be featured are

- Coupling of meshfree methods, finite element methods, particle methods, and finite difference methods
- Coupling of multiple scales, e.g. continuum models to discrete models
- Application of meshfree, generalized/extended finite element methods
- Parallel computation in meshfree methods
- Mathematical theory of meshfree, generalized finite element, and particle methods
- Fast and stable domain integration methods
- Enhanced treatment of boundary conditions
- Identification and characterization of problems where meshfree methods have clear advantage over classical approaches

Program Committee

- Prof. Ivo Babuška (University of Texas, Austin, USA)
- Prof. Ted Belytschko (Northwestern University, USA)
- Prof. Michael Griebel (Universität Bonn, Germany)
- Prof. Antonio Huerta (Universitat Politècnica de Catalunya, Spain)
- Prof. Wing Kam Liu (Northwestern University, USA)
- Prof. Harry Yserentant (Technische Universität Berlin, Germany)

Local Organizers

- Prof. Michael Griebel
- Dr. Marc Alexander Schweitzer
- Konstantin Fackeldey

Acknowledgement

Special thanks to the Rektorat of the University of Bonn for their support and to the Rektor Rektor Prof. Dr. Matthias Winiger for the openings remarks.

Special thanks to the mayor's office and the city of Bonn (`http://www.bonn.de`) for the reception at the old town hall.

Special thanks also to Prof. Bernhard Korte and to Dr. Ina Prinz for the guided tour through the Arithmeum (`http://www.arithmeum.uni-bonn.de`) of the Research Institute for Discrete Mathematics.

Special thanks also to the Sonderforschungsbereich 611 and the Deutsche Forschungsgemeinschaft for funding the financial support.

Timetable

Time	Monday 17.09.2007		Tuesday 18.09.2007		Wednesday 19.09.2007		Thursday 20.09.2007	
8:30–8:45	Opening Remarks							
8:45–9:30	Belytschko		Hackbusch		Banerjee			
9:30–10:15	Luskin		Xu		Bonet		Schweitzer	
10:15–11:00	Arndt		Bank		Rosswog		Kirkilionis	
11:00–11:30	Coffee Break							
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11:30–12:00	Chinesta	Prakash	Liu	(11:30–12:15)	Kuhnert	May-Duy	Fasshauer	(11:30–12:15)
12:00–12:30	Daum	Emmrich	Noguchi	(12:15–13:00)	Prakash	Štiavnický	Wendland	(12:15–13:00)
12:30–13:00	Avila	Tiwari			Nogueira	Petritshenko		
13:00–14:30	Lunch Break							
			Room A	*Room B*				
14:30–15:00	Chen	(14:30–15:15)	Lehocq	Randrian.	Orkisz	(14:30–15:15)		
15:00–15:30	Robbins	(15:15–16:00)	Avila	Suciu	Huerta	(15:15–16:00)		
15:30–16:00			Kolpakov	Diyankov				
16:00–16:30	Coffee Break							
	Room A	*Room B*			*Room A*	*Room B*		
16:30–17:00	Fackeldey	Arroyo			Seibold	Belinha		
17:00–17:30	George	Simkins			Arora	Joyot		
17:30–18:00	Chatelain	Kolesnikov						
18:15			Arithmeum		Town Hall			
19:00					Dinner			

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Thu 9:30–10:15, Room A, M. A. Schweitzer. *TBA*.

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Thu 12:15–13:00, Room A, H. Wendland. *Hybrid Discretization Methods for Aeroelastic Problems*.

3.5 A meshless technique based on integrated radial basis function networks for elliptic partial differential equations

Wed 11:30–12:00, Room B

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This paper presents a global meshless technique based on integrated radial basis function networks (RBFNs) for solving second- and fourth-order elliptic partial differential equations (PDEs). RBFNs are known to have the property of universal approximation, i.e. they are able to approximate any continuous function to any degree of accuracy. An important feature of RBFNs is that the networks rely on the set of points, which may be randomly distributed, to represent the approximate function and its derivatives. The present technique employs integrated RBFNs to approximate the field variable and point collocation to discretize the PDE. The use of integration to construct the RBF approximations has several advantages over the use of conventional differentiation: (i) to avoid the reduction of convergence rate caused by differentiation, and (ii) to provide a more effective way to incorporate normal derivative boundary conditions. The technique is applied to the solution of several linear and nonlinear elliptic problems. Both structured and unstructured points are employed to discretize the problem domain. Numerical results show that the present technique yields a high level of accuracy.

Keywords:
Radial basis functions, Meshless discretization, Integral collocation formulation, Multiple boundary conditions, High-order PDEs.