Local buckling of steel plates in concrete-filled thin-walled steel tubular beam-columns

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Abstract

The availability of high strength steels and concrete leads to the use of thin steel plates in concrete-filled steel tubular beam-columns. However, the use of thin steel plates in composite beam-columns gives a rise to local buckling that would appreciably reduce the strength and ductility performance of the members. This paper studies the critical local and post-local buckling behavior of steel plates in concrete-filled thin-walled steel tubular beam-columns by using the finite element analysis method. Geometric and material nonlinear analyses are performed to investigate the critical local and post-local buckling strengths of steel plates under compression and in-plane bending. Initial geometric imperfections and residual stresses presented in steel plates, material yielding and strain hardening are taken into account in the nonlinear analysis. Based on the results obtained from the nonlinear finite element analyses, a set of design formulas are proposed for determining the critical local buckling and ultimate strengths of steel plates in concrete-filled steel tubular beam-columns. In addition, effective width formulas are developed for the ultimate strength design of clamped steel plates under non-uniform compression. The accuracy of the proposed design formulas is established by comparisons with available solutions. The proposed design formulas can be used directly in

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the design of composite beam-columns and adopted in the advanced analysis of concrete-filled thin-walled steel tubular beam-columns to account for local buckling effects.

Keywords: effective width; finite element analysis; local buckling; post-local buckling; steel plates; strength.

Nomenclature

a_1, a_2, a_3, a_4	coefficients for determining the critical local buckling stress of plates
A	coefficients given by Eq. (18)
b	inner width of a steel box column
b_e	total effective width of a steel plate
b_{e1}, b_{e2}	parts of the effective width of a plate
B	outer width of a steel box column
B_0	coefficients given by Eq. (19)
c_1, c_2, c_3, c_4	coefficients for determining the ultimate strength of plates
C	coefficients given by Eq. (20)
D	outer depth of a steel box column
d	inner depth of a steel box column
E	Young's modulus of steel
$E_{0.7}$	secant modulus of steel, $E_{0.7} = 0.7E$
f_y	yield strength of steel
k	elastic local buckling coefficient
k_n	buckling coefficient for clamped plates under non-uniform compression
n	knee factor that defines the sharpness of the stress-strain curve for steel

t	thickness of a steel plate
w	lateral deflection at the plate centre
w_0	initial out-of-plane deflection at the plate centre
α	stress gradient coefficient, $\alpha = \sigma_2 / \sigma_1$
β	width-to-thickness ratio parameter
β_n	width-to-thickness ratio parameter calculated using k_n
β_0	coefficient given by Eq. (17)
λ	coefficient given by Eq. (16)
γ	coefficient given by Eq. (21)
ε	strain
ν	Poisson's ratio
σ_1, σ_2	maximum and minimum edge stress
$\sigma_{0.7}$	stress corresponding to $E_{0.7} = 0.7E_s$
σ_{1c}	critical local buckling stress of steel plates with imperfections
σ_{1u}, σ_{2u}	maximum and minimum edge stress at the ultimate state
σ_{cr}	critical elastic local buckling stress
σ_{1p}	post-local buckling reserve strength of a steel plate
σ_{r}	compressive residual stress
σ_{u}	ultimate stress of plate under uniform compression
ϕ	coefficient, $\phi = 1 - \alpha$

1. Introduction

Concrete-filled thin-walled steel tubular beam-columns are efficient structural members that have been widely used in high rise buildings, bridges and offshore structures. A concrete-filled steel tubular beam-column is constructed by filling concrete into a square or rectangular steel hollow tubular column, as depicted in Fig. 1. In a concrete-filled steel tubular beam-column, steel plates under compression can only buckle outward locally due to the restraints of the concrete core. This buckling mode leads to a considerable increase in the critical local buckling strength of the steel box as well as the load-carrying capacity of the composite column. The steel box completely encases the concrete core so that the ductility of the encased concrete is remarkably improved. Steel plates also serve as longitudinal reinforcement and permanent formwork for the concrete core, which results in rapid construction and significant savings in materials. This type of composite columns offers excellent structural performance, such as high strength, high ductility and large energy absorption capacity.

The availability of high strength structural steels and high strength concrete leads to the use of thin steel plates in concrete-filled steel tubular beam-columns. However, this gives rise to the local instability problem of thin steel plates under compression and in-plane bending, which may be encountered in concrete-filled steel tubular beam-columns. Local buckling of thin steel plates with initial geometric imperfections and welding residual stresses will lead to an appreciable reduction in the strength and ductility performance. The local stability of thin plates subjected to compression and in-plane bending has been an active research area for many years. Walker [1] studied the local bucking behavior of plates under eccentric loading using the Galerkin's method to solve the governing nonlinear simultaneous differential equations. Rhodes and Harvey [2] investigated the effects of eccentricity of load on the local and post-local buckling behavior of flat plates simply supported on their loaded edges and subjected to various support conditions on their unloaded edges. Rhodes et al. [3] reported the load-carrying capacity of initially imperfect plates under linearly varying displacement. Usami [4,5] used an energy method and the nonlinear finite element method to study the post-local buckling strength of simply supported steel plates in compression and bending and proposed effective width formulas for predicting the ultimate strength of steel plates. Narayanan and Chan [6] conducted analytical and experimental studies on the elastic critical local buckling and post-local buckling strengths of plates containing holes under linearly varying edge displacements. Shanmugam et al. [7] investigated the ultimate loads of thin-walled steel box beam-columns including the local buckling of simply supported steel plates under compression and in-plane bending. It should be noted that in these studies mentioned above, plates are allowed to buckle freely in two lateral directions.

Thin steel plates in contact with concrete are constrained to buckle locally in a unilateral direction when subjected to edge compression. This unilateral buckling behavior of steel plates has increasingly attracted the attentions of researchers. Ge and Usami [8] performed nonlinear finite element analyses on short concrete-filled thin-walled steel box columns and proposed ultimate strength formula for steel plates in uniform compression. Wright [9,10] investigated the local buckling characteristics of thin steel plates in contact with concrete using an energy method and derived the limiting width-to-thickness ratios for proportioning steel plates in contact with concrete. The ultimate load behavior of concrete-filled thin-walled steel box columns with local buckling effects has been studied experimentally by Uy and Bradford [11], Bridge et al. [12] and Uy [13]. Liang and Uy [14] investigated the local and post-local buckling behavior of steel plates in concrete-filled steel box columns under axial compression using the finite element method and proposed effective width formulas for the ultimate strength design of steel plates in such columns. Liang et al. [15] has incorporated these effective width formulas into their fiber element analysis programs for the advanced analysis of concrete-filled thin-walled steel box columns to account for local buckling effects. Moreover, Liang et al. [16,17] proposed buckling and ultimate strength interaction formulas for the design of steel plates in double skin composite panels under biaxial compression and shear.

However, most of the studies on the local buckling of steel plates in contact with concrete reported in the literature were concerned with steel plates under uniform edge compression. The local and post-local buckling behavior of steel plates under non-uniform compression and in-plane bending in concrete-filled thin-walled steel tubular beam-columns has not been reported thus far in the literature. This paper extends the previously cited work [14] to steel plates under non-uniform compression and in-plane bending. Geometric and material nonlinear finite element analyses are undertaken to predict the critical local and post-local buckling strengths of unilaterally restrained steel plates under non-uniform compression and in-plane bending. Based on the results obtained from the nonlinear finite element analyses, a set of design formulas are proposed for quantifying the critical local buckling and ultimate strengths of steel plates in concrete-filled steel box beam-columns. Moreover, effective width formulas are developed for the ultimate strength predictions of clamped steel plates under non-uniform compression. The proposed design formulas are examined against available design formulas reported in the literature.

2. Finite element analysis

2.1 General

The finite element code STRAND7 [18] was utilized in the present study to investigate the critical local and post-local buckling strengths of steel plates in concrete-filled thin-walled steel tubular beam-columns. The four edges of a web or flange in a concrete-filled steel tubular beam-column were assumed to be clamped owing to the restraint provided by the concrete core as suggested by Liang and Uy [14]. Square steel plates clamped at four edges yield the minimum local buckling load so that they were used to represent the strength of flanges and webs of a concrete-filled thin-walled tubular beam-column. Geometric and material nonlinear analyses on steel plates with initial imperfections were undertaken. The von Mises yield criterion was adopted in the nonlinear analysis to treat the material plasticity of steel plates. An eight-node quadrilateral plate/shell element was employed in all analyses. A 10 × 10 mesh was used in all analyses and was found to be economic and adequate to yield accurate results for use in engineering practice.

2.2 Initial imperfections

The initial imperfections of steel plates consist of initial out-of-plane deflections and residual stresses, which are usually induced in the process of construction and welding. Initial imperfections would considerably reduce the strength and stiffness of steel plates. The effects of initial geometric imperfections and residual stresses on the local buckling strength of steel plates in concrete-filled steel box columns under axial compression have been reported by Liang and Uy [14]. In the present study, the form of initial out-of-plane deflections was taken as the first local buckling mode which yields the minimum buckling load. The maximum magnitude of initial geometric imperfections at the plate centre was taken as $w_0 = 0.1t$ for steel plates in concrete-filled steel tubular beam-columns. A lateral pressure was applied to the plate to induce the initial out-of-plane deflection [14]. In a welded steel plate, tensile residual stresses develop in the region of the weld while compressive residual stresses are present in the remainder of the plate. It is noted that compressive residual stresses in the cross section of a welded plate are balanced by the tensile residual stresses that reach the yield strength of the steel plate. An idealized residual stress pattern in a concrete-filled welded steel tubular beam-column is schematically depicted in Fig. 2. In the present study, the compressive residual stress was taken as 25 percent of the yield strength of the steel plate. Residual stresses were incorporated in the finite element model by prestressing.

2.3 Stress-strain relationship for steel plates

Residual stresses have a considerable effect on the stress-strain curve of a welded steel plate as addressed by Liang and Uy [14]. A welded steel plate displays a rounded stress-strain form that differs from the tensile test behavior of a coupon without residual stresses. In the present study, the rounded stress-strain curve of steel plates with residual stresses was modeled using the Ramberg-Osgood formula [19], which is expressed by

$$\varepsilon = \frac{\sigma}{E}\left[1 + \frac{3}{7}\left(\frac{\sigma}{\sigma_{0.7}}\right)^{n}\right] \tag{1}$$

where σ and ε are the uniaxial stress and strain respectively, E is the Young's modulus, $\sigma_{0.7}$ is the stress corresponding to $E_{0.7} = 0.7E$, and n is the knee factor that defines the sharpness of the knee in the stress-strain curve. The knee factor n = 25 was used in Eq. (1) to account for the isotropic strain hardening of steel plates [14-17]. Since the proof stress and strain of structural steels are usually known, $\sigma_{0.7}$ can be determined by substituting them into Eq. (1). An ultimate strain of 0.2 was assumed for mild steels in the nonlinear analysis.

3. Steel plates under edge compression

The critical local and post-local buckling behavior of steel plates under linearly varying compressive stresses as shown in Fig. 3 is studied in this section. The stress gradient coefficient α is defined as the ratio of the minimum edge stress (σ_2) to the maximum edge stress (σ_1). Stress gradient coefficients ranging from 0.0 to 0.2, 0.4, 0.6, 0.8 and 1.0 were considered. Note that when the stress gradient coefficient is equal to 1.0, the plate is under uniform compression. Square steel plates (500 × 500 mm) with initial geometric imperfections and welding residual stresses were studied. The thickness of the steel plates was varied to give different *b/t* ratios ranging from 30 to 100. The yield strength of steel plates was 300 MPa and the Young's modulus was 200 GPa. The effects of initial geometric imperfections and residual stresses on both the critical local buckling strength and the ultimate strength of steel plates under uniform edge compression have been investigated previously [14] and were not studied here.

Load-lateral deflection curves for steel plates with various *b/t* ratios under a stress gradient of $\alpha = 0.8$ are presented in Fig. 4. The figure shows that all steel plates considered cannot attain the yield strength because of the effects of initial imperfections and stress gradients. The stiffness, critical local buckling strength and ultimate strength of steel plates generally decrease with an increase in the plate width-to-thickness ratios. However, steel plates with the *b/t* ratios of 30 and 40 can attain the same ultimate stress, for these stocky plates undergo yielding only. It appears from Fig. 4 that the ultimate strength of a steel plate with a *b/t* ratio of 100 is only 61.4 percent of its yield strength. Apparently, local buckling significantly reduces the ultimate strength of slender steel plates.

Fig. 5 demonstrates the effects of stress gradient coefficients on the load-deflection behavior of steel plates with a *b/t* ratio of 100. The figure shows that the lateral stiffness of the steel plate is reduced when increasing the stress gradient coefficient α . The reduction in the lateral stiffness of a steel plate would eventually lead to a lower critical local buckling strength of the plate with initial imperfections as depicted in Fig. 5. It is also seen that increasing the stress gradient coefficient α reduces the ultimate strength of the steel plate. When the stress gradient coefficient α increases from 0.0 to 0.2, 0.4, 0.6, 0.8 and 1.0, the ultimate stress of the steel plate decreases from $0.844 f_y$ to $0.825 f_y$, $0.751 f_y$, $0.652 f_y$, $0.614 f_y$ and $0.566 f_y$, respectively.

Fig. 6 provides the ultimate strengths of steel plates subjected to edge compression in concrete-filled steel box beam-columns. It can be seen from Fig. 6 that for steel plates under the same stress gradient the ultimate strength of steel plates decreases with an increase in the plate width-to-thickness ratio. As expected, increasing the stress gradient factor (α) would reduce the ultimate strength of a steel plate regardless of its width-to-thickness ratio. When the stress gradient coefficient α increases from 0.2 to 0.4, 0.6, 0.8 and 1.0, the ultimate stress of a steel plate with a *b/t* ratio of 60 decreases from $1.145 f_y$ to $1.04 f_y$, $0.935 f_y$, $0.845 f_y$ and $0.767 f_y$, respectively. For steel plates with small *b/t* ratios and small stress gradient coefficients, they can attain a higher ultimate strength than the yield strength due to strain hardening and stress gradients. Usami [5] also reported that the ultimate stress of steel plates with a stress gradient coefficient less than 1.0 exceeded its yield strength. It can be observed from Fig. 6 that the maximum increase in the ultimate strength due to strain hardening and stress gradients for steel plates is approximately18 percent of the yield strength.

4. Steel plates under in-plane bending

The critical local and post-local buckling strengths of steel plates under in-plane bending as depicted in Fig. 3 are investigated here. The geometry and material properties used in the analyses were the same as those presented in the preceding section. The stress gradient coefficient was varied from 0.2 to 0.4, 0.6, 0.8 and 1.0. Fig. 7 depicts the load-lateral deflection curves for steel plates with $\alpha = 0.2$. The figure indicates that increasing the width-to-thickness ratio remarkably reduces the lateral stiffness of the plates under the same stress gradient. When the *b/t* ratio increases from 60 to 100, the ultimate stress carried by the steel plate reduces slightly. The effect of stress gradient coefficients on the load-deflection behavior of the steel plate with a *b/t* ratio of 80 is schematically demonstrated in Fig. 8. It is seen that reducing the stress gradient coefficient increases the lateral deflection of the plate under the same loading level. The stress gradient coefficient only has a minor effect on the ultimate strength of the steel plate because local buckling unlikely occurs for plates with such a *b/t* ratio.

5. Critical local buckling strength

It can be seen from Fig. 4 that no bifurcation point on the load-lateral deflection curves for steel plates can be observed due to the presence of initial geometric imperfections. A simple method for determining the critical local buckling strength of steel plates with initial geometric imperfections and residual stresses has been developed by Liang and Uy [14]. In their method, the inflection point can be located by plotting the nondimensional central lateral deflection versus the ratio of the deflection to the applied load w/σ_1. The minimum value of w/σ_1 determined from the plot represents the inflection point where the critical local buckling occurs at the corresponding loading level.

Fig. 9 presents the load-deflection curves for determining the critical local buckling strengths of the steel plate with a *b/t* ratio of 100 under various stress gradients. The figure demonstrates that the ratio w/σ_1 decreases with a corresponding increase in the load and deflection in the first few loading increments. After reaching the minimum value, the ratio w/σ_1 increases with an increase in the load and deflection. Before the critical local buckling occurs, the lateral deflections of the plate have a small increase with the applied load. However, after the critical local buckling, lateral deflections increase rapidly even under a small load increment because local buckling has remarkably reduced the lateral stiffness of the steel plate. It is also seen from Fig. 9 that the ratio w/σ_1 increases by increasing the stress gradient coefficient α. The critical local buckling stresses of steel plates under edge compression are presented in Fig. 10. It appears from Fig. 10 that increasing the stress gradient coefficient α essentially reduces the critical local buckling stress of steel plates with a *b/t* ratio greater than 30.

6. Post-local buckling reserve strength

After critical local buckling, thin steel plates can still carry increased loads without failure. This behavior of thin steel plates is called post-local buckling. The post-local buckling reverse strength of a steel plate is the difference between the critical local buckling strength and the ultimate strength that the plate can carry, and can be expressed by

$$\sigma_{1p} = \sigma_{1u} - \sigma_{1c} \tag{2}$$

where σ_{1p} is the post-local buckling reserve strength, σ_{1u} is the ultimate strength and σ_{1c} is the critical local buckling stress of a plate with imperfections, which can be determined from the load-lateral deflection curves obtained from the nonlinear finite element analysis on the steel plate.

It can be seen from Figs. 6 and 10 that the post-local buckling reserve strength of a steel plate decreases with an increase in the stress gradient coefficient ($\alpha \geq 0.2$) when its width-to-thickness ratio is greater than 60. The post-local buckling reserve strength of slender steel plates is much higher than that of stocky ones when subjected to the same stress gradient. The post-local buckling reserve strength of steel plates with a *b/t* ratio of 100 and $\alpha = 1.0$ is 62.9 percent of the yield strength of steel material whilst it is only 27.3 percent of the yield strength for plates with a *b/t* ratio of 30.

7. Proposed design formulas

7.1 Design formulas for critical local buckling strength

As depicted in Fig. 10, the critical local buckling strengths of steel plates with prescribed geometric imperfections and residual stresses depend on the plate width-to-thickness ratio, the stress gradient coefficient α and the yield strength of the steel plates. Based on the results obtained from the nonlinear finite element analyses, design formulas for calculating the critical local buckling strengths of steel plates under linearly varying edge compression are proposed as

$$\frac{\sigma_{1c}}{f_y} = a_1 + a_2\left(\frac{b}{t}\right) + a_3\left(\frac{b}{t}\right)^2 + a_4\left(\frac{b}{t}\right)^3 \tag{3}$$

where b is the width of a steel plate, t is the thickness of the steel plate and a_1, a_2, a_3 and a_4 are constant coefficients which vary with the stress gradient coefficient α. The constant coefficients for determining the critical local buckling strengths of steel plates with various stress gradient coefficients are given in Table 1. The critical local buckling strengths of steel plates calculated using Eq. (3) are compared with those obtained from the finite element analyses in Fig. 11. It can be seen that the proposed design formulas fit very well the results of the finite element analysis.

7.2 Design formulas for ultimate strength

Thin steel plates in concrete-filled steel tubular beam-columns possess very high post-local buckling reverse strengths as discussed in the preceding section. Therefore, the post-local buckling reverse strengths of thin steel plates should be taken into account in the ultimate strength design. It can be seen from Fig. 6 that the ultimate strength of a steel plate with prescribed geometric imperfections and residual stresses is a function of the *b/t* ratio, stress gradient coefficient (α) and the yield strength (f_y). To quantify the ultimate strengths of steel plates under edge compression in concrete-filled steel tubular beam-columns, design formulas are proposed as

$$\frac{\sigma_{1u}}{f_y} = c_1 + c_2\left(\frac{b}{t}\right) + c_3\left(\frac{b}{t}\right)^2 + c_4\left(\frac{b}{t}\right)^3 \tag{4}$$

where σ_{1u} is the ultimate stress that corresponds to the maximum edge stress σ_1 at the ultimate state, and c_1, c_2, c_3 and c_4 are constant coefficients which vary with variation of the stress gradient coefficient. The constant coefficients for determining the ultimate strengths of steel plates with various stress gradient coefficients are given in Table 2. The ultimate strengths of steel plates calculated using Eq. (4) are compared with those obtained from the finite element analyses in Fig. 12. It can be seen that the proposed design formulas fit very well the results of the finite element analysis.

Since the ultimate strength of steel plates under stress gradients is a function of the stress gradient coefficient α. A single formula is proposed to approximately express the ultimate strength of steel plates with stress gradient coefficients greater than zero as follows:

$$\frac{\sigma_{1u}}{f_y} = (1 + 0.5\phi)\frac{\sigma_u}{f_y} \qquad (0 \le \phi < 1.0) \tag{5}$$

where $\phi = 1 - \alpha$ and σ_u is the ultimate stress of steel plates under uniform compression and can be calculated using Eq. (4) with the stress gradient coefficient of $\alpha = 1.0$.

7.3 Effective width formulas

The post-local buckling behavior of a thin steel plate under compression and in-plane bending is characterized by the stress redistribution within the buckled steel plate. The effective width concept, which is an ultimate strength criterion, is usually used to describe the post-local buckling behavior of a thin steel plate [14]. Fig. 13 schematically depicts the effective width of a thin steel plate in the post-local buckling regime under compression and in-plane bending. Usami [4,5] has proposed effective width formulas for predicting the ultimate strength of simply supported steel plates under compression and in-plane bending. Shanmugam et al. [7] also presented effective width formulas for simply supported steel plates in thin-walled steel box columns under biaxial loading. Effective width formulas can be incorporated in advanced analysis methods to account for local buckling effects on the strength and behavior of thin-walled steel tubular columns with and without concrete in-fill as presented by Shanmugam et al. [7] and Liang et al. [15]. However, no effective width formulas have been developed for clamped steel plates under compression and in-plane bending. Based on the results obtained from the nonlinear finite element analyses and the proposed ultimate strength formulas, effective width formulas for determining the ultimate strength of clamped steel plates under compression and in-plane bending in concrete-filled steel tubular beam-columns are proposed as

$$\frac{b_{e1}}{b} = 0.2777 + 0.01019\left(\frac{b}{t}\right) - 1.972\times10^{-4}\left(\frac{b}{t}\right)^2 + 9.605\times10^{-7}\left(\frac{b}{t}\right)^3 \quad \text{for } \alpha > 0.0 \tag{6a}$$

$$\frac{b_{e1}}{b} = 0.4186 - 0.002047\left(\frac{b}{t}\right) + 5.355\times10^{-5}\left(\frac{b}{t}\right)^2 - 4.685\times10^{-7}\left(\frac{b}{t}\right)^3 \quad \text{for } \alpha = 0.0 \tag{6b}$$

$$\frac{b_{e2}}{b} = (1+\phi)\frac{b_{e1}}{b} \tag{7}$$

where b_{e1} and b_{e2} are the effective widths as depicted in Fig. 13. Note that for $(b_{e1} + b_{e2}) \geq b$, the steel plate is fully effective in carrying loads and the ultimate strength of the steel plate can be determined using Eqs. (4) and (5).

8. Comparisons with existing formulas

8.1 Uniform compression

The proposed design formulas for determining the ultimate strength of steel plates in concrete-filled steel box columns under uniform compression are compared with existing formulas reported in the literature in this section. Effective width formulas presented by Liang and Uy [14] for steel plates in concrete-filled steel box columns are expressed by

$$\frac{b_e}{b} = 0.675\left(\frac{\sigma_{\mathrm{cr}}}{f_y}\right)^{1/3} \qquad \text{for } \sigma_{\mathrm{cr}} \le f_y \tag{8}$$

$$\frac{b_e}{b} = 0.915\left(\frac{\sigma_{\mathrm{cr}}}{\sigma_{\mathrm{cr}} + f_y}\right)^{1/3} \qquad \text{for } \sigma_{\mathrm{cr}} > f_y \tag{9}$$

where b_e is the total effective width of a steel plate, and σ_{cr} is the elastic critical local buckling stress of the steel plate without imperfections under uniform compression, and is written as [20]

$$\sigma_{\mathrm{cr}} = \frac{k\pi^2 E}{12\left(1-\nu^2\right)\left(b/t\right)^2} \tag{10}$$

where ν is the Poisson's ratio and k is the elastic local buckling coefficient, which is taken as 9.81 in Eq. (10) to calculate σ_{cr} used in Eqs. (8) and (9) for clamped plates as suggested by Liang [21]. The design formula proposed by Ge and Usami [8] for the ultimate strength predictions of steel plates in concrete-filled steel box columns in compression is expressed by

$$\frac{\sigma_u}{f_y} = \frac{1.2}{\beta} - \frac{0.3}{\beta^2} \tag{11}$$

where β is the width-to-thickness ratio parameter, which is given by

$$\beta = \frac{b}{t}\sqrt{\frac{12\left(1-\nu^2\right)}{\pi^2 k}\frac{f_y}{E}} \tag{12}$$

The elastic local buckling coefficient is taken as $k = 4.0$ in Eq. (12) when calculating β in Eq. (11). Nakai et al. [22] proposed an equation for the ultimate strength of steel plates clamped at four edges as

$$\frac{\sigma_u}{f_y} = 0.433(\beta - 0.5)^2 - 0.831(\beta - 0.5) + 1.0 \qquad (0.5 < \beta < 1.3) \tag{13}$$

in which β is calculated using the elastic local buckling coefficient of $k = 9.81$ for clamped plates.

Fig. 14 shows the comparison of the proposed design Eq. (4) with those given by Liang and Uy [14], Ge and Usami [8] and Natai et al. [22] for steel plates under uniform compression. It can be seen from Fig. 14 that the proposed design formula compares very well with those of Liang and Uy [14]. For steel plates with a *b/t* ratio greater than 60, the ultimate strength of the plates predicted by Eq. (4) is between those calculated using Eqs. (11) and (13). The proposed design formula yields conservative predictions of the ultimate strengths for steel plates with a *b/t* ratio less than 60 when compared with Eqs. (11) and (13).

8. 2 Non-uniform compression

The post-local buckling strength of clamped steel plates under compression and in-plane bending was rarely reported in the literature. Effective width formulas for simply supported steel plates under compression and bending proposed by Usami [5] are modified here for clamped steel plates using the buckling coefficients of clamped steel plates. The proposed effective width formulas for clamped steel plates under non-uniform compression are compared with formulas developed by Usami [5]. The effective width formulas at the ultimate state given by Usami [5] are expressed by

$$\frac{b_{e1}}{b} = \frac{\gamma}{4\beta_n}\left(\lambda - \sqrt{\lambda^2 - 4\beta_n}\right) \tag{14}$$

$$\frac{b_{e2}}{b} = (1 + 0.44\phi)\frac{b_{e1}}{b} \tag{15}$$

where

$$\lambda = 1 + C(\beta_n - \beta_0) + \beta_n \tag{16}$$

$$\beta_0 = A - B_0 \times \ln\left(\frac{w_0}{b}\right) \le 1.0 \tag{17}$$

$$A = -0.05 - 0.542 \times \exp\left(-11.9\frac{\sigma_r}{f_y}\right) \tag{18}$$

$$B_0 = 0.09 + 0.107 \times \exp\left(-12.4\frac{\sigma_r}{f_y}\right) \tag{19}$$

$$C = -157\left(\frac{w_0}{b}\right)\left(\frac{\sigma_r}{f_y}\right) + 43\left(\frac{w_0}{b}\right) + 1.2\left(\frac{\sigma_r}{f_y}\right) + 0.03 \tag{20}$$

$$\gamma = \frac{\sigma_r}{0.3 f_y}\left(1 + 45\frac{w_0}{b}\phi\right) + \left(1 - \frac{\sigma_r}{0.3 f_y}\right)\left(1 - \frac{\phi^2}{16}\right) \tag{21}$$

where σ_r is the compressive residual stress and the width-to-thickness ratio parameter β_n is calculated using Eq. (12) with the elastic local buckling coefficient k_n for clamped steel plates under non-uniform compression. Based on results obtained from the linear elastic buckling analyses, a formula for the elastic local buckling coefficient k_n of clamped steel plates under non-uniform compression is proposed as

$$k_n = 18.89 - 14.38\alpha + 5.3\alpha^2 \tag{22}$$

It is noted that when the stress gradient coefficient α is equal to 1.0, Eq. (22) yields a buckling coefficient of $k_n = 9.81$, which is the value for clamped steel plates under uniform compression.

Fig. 15 provides a comparison of the ultimate strengths predicted by the proposed effective width formulas Eqs. (6) and (7) and those given by Usami [5] with buckling coefficient k_n determined by Eq. (22) for clamped plates with stress gradient coefficients of 0.4, 0.6 and 0.8. The initial geometric imperfection was taken as $w_0 = 0.1t$ and the compressive residual stress was assumed to be $0.25 f_y$ for steel plates (500 × 500 mm). Fig. 15 shows that for steel plates with *b/t* ratios between 60 and 90, both the proposed effective width formulas and those given by Usami [5] yield almost the same ultimate strength. For steel plates with $b/t > 90$, the proposed formulas provide slightly higher ultimate stress for the plates than Eqs. (14) and (15). For steel plates with $b/t < 60$, the proposed formulas yield conservative strength predictions when compared with Eqs. (14) and (15). It should be noted that Eqs. (14) and (15) were developed based on simply supported steel plates. It can be concluded that the proposed effective width formulas yield accurate predictions of the ultimate loads of clamped steel plates under compression and in-plane bending in concrete-filled steel tubular beam-columns.

9. Conclusions

The critical local and post-local buckling behavior of steel plates in concrete-filled thin-walled steel tubular beam-columns has been investigated by undertaking the geometric and material nonlinear finite element analyses in this paper. Clamped square steel plates with various width-to-thickness ratios and geometric imperfections and residual stresses were studied. Two loading conditions including edge compression and in-plane bending encountered in concrete-filled steel tubular beam-columns were considered. The effects of stress gradient coefficients and width-to-thickness ratios on the critical local buckling strength, the post-local buckling reserve strength, the ultimate strength and load-deflection behavior of steel plates in concrete-filled steel box columns were investigated. Based on the results obtained from the nonlinear finite element analyses, a set of design formulas were proposed for determining the critical local buckling and ultimate strengths of steel plates under compression and in-plane bending. Effective width formulas were also developed for the ultimate strength design of clamped steel plates under edge compression in concrete-filled steel tubular beam-columns.

Numerical results indicate that increasing the width-to-thickness ratio of a steel plate under a predefined stress gradient reduces its lateral stiffness, critical local buckling stress and ultimate strength. It has also been shown that the lateral stiffness, critical local buckling stress and ultimate strength of steel plates under edge compression decreases with an increase in the stress gradient coefficient. The proposed design formulas for the ultimate strengths of steel plates under edge compression and in-plane bending were verified by comparisons with available solutions. These design formulas can be used directly in the design of steel plates in concrete-filled thin-walled steel tubular beam-columns and are suitable for inclusion in composite design codes. Moreover, they can be incorporated in the advanced analysis methods to account for local buckling effects on the strength and behavior of concrete-filled thin-walled steel tubular columns under axial load and biaxial bending.

Acknowledgements

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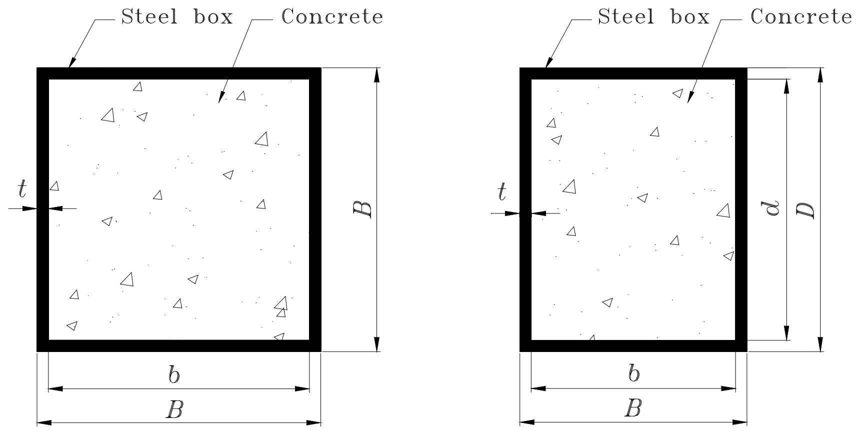

(a) Square section (b) Rectangular section

Fig. 1. Concrete-filled thin-walled steel tubular beam-columns.

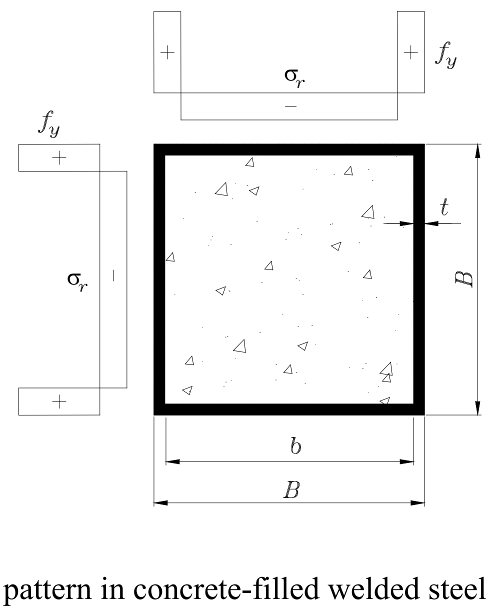

Fig. 2. Residual stress pattern in concrete-filled welded steel tubular beam-columns.

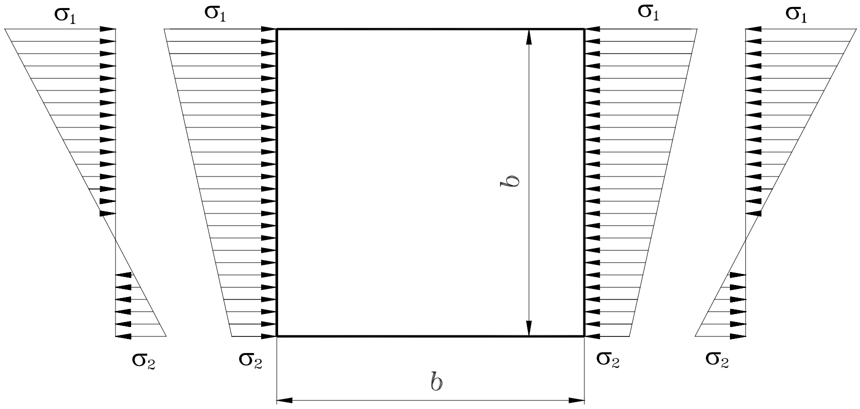

Fig. 3. Clamped steel plates under edge compression and in-plane bending.

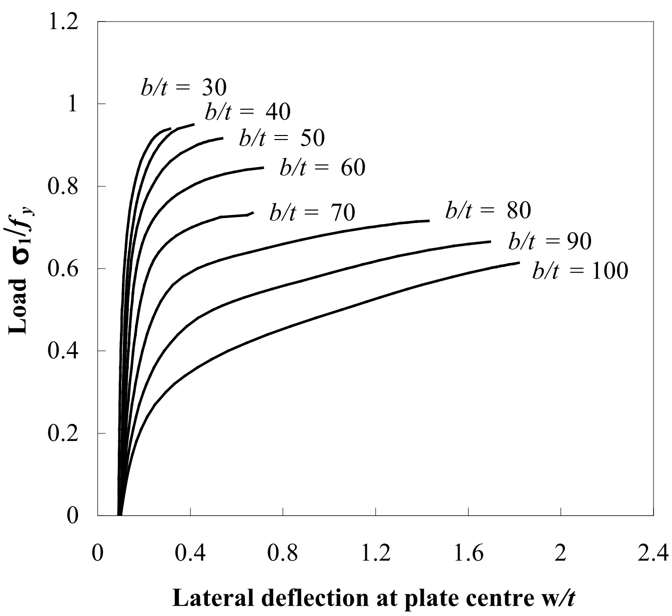

Fig. 4. Load-deflection curves for steel plates under edge compression $(\alpha = 0.8)$.

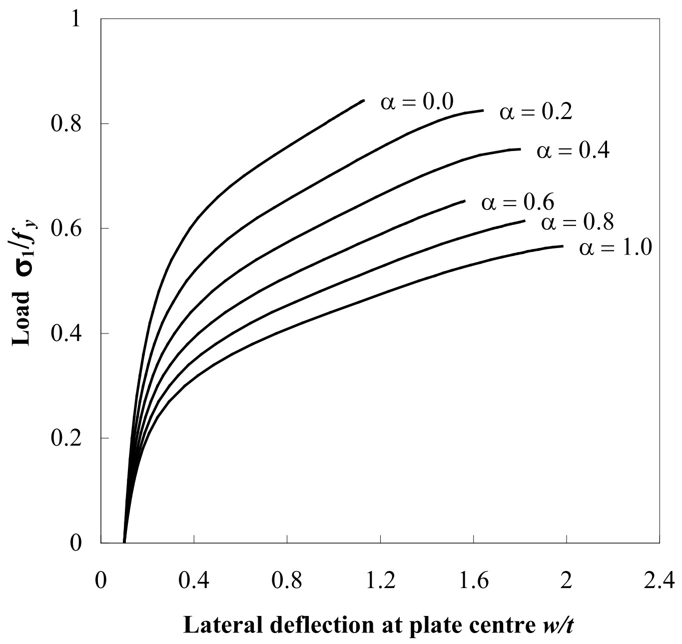

Fig. 5. Effects of stress gradients on the load-deflection curves for plates under compression $(b/t = 100)$.

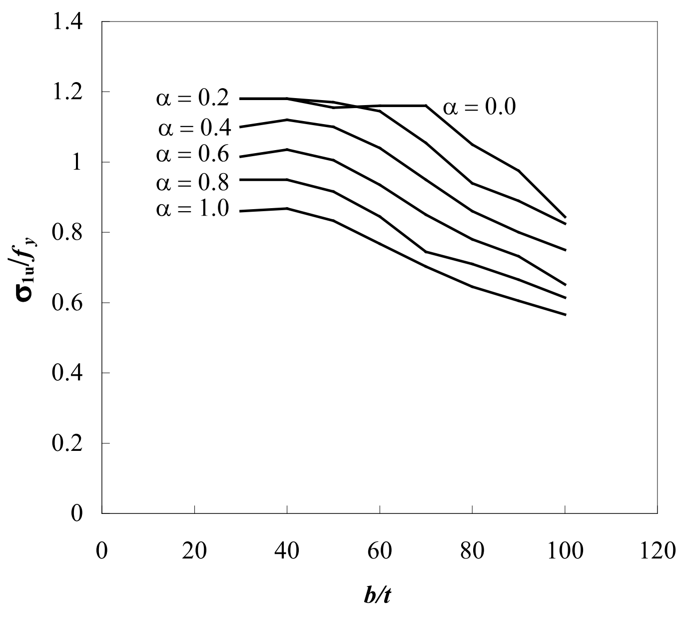

Fig. 6. Ultimate strengths of steel plates under edge compression.

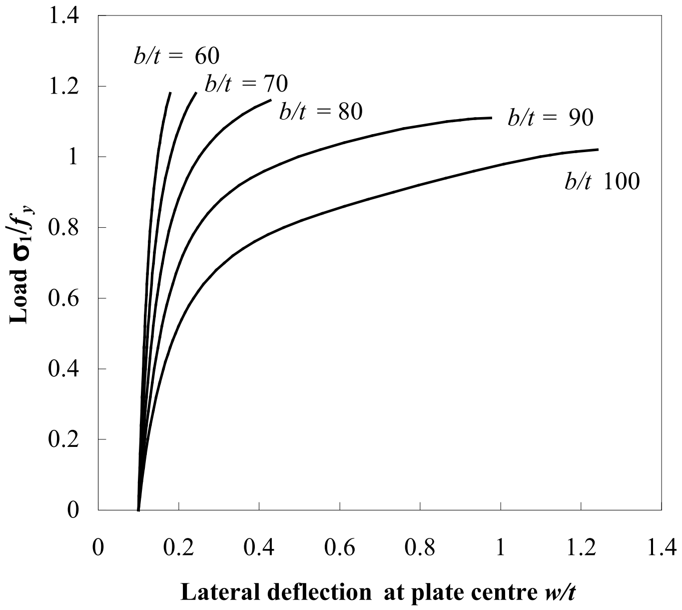

Fig. 7. Load-deflection curves for steel plates under in-plane bending $(\alpha = 0.2)$.

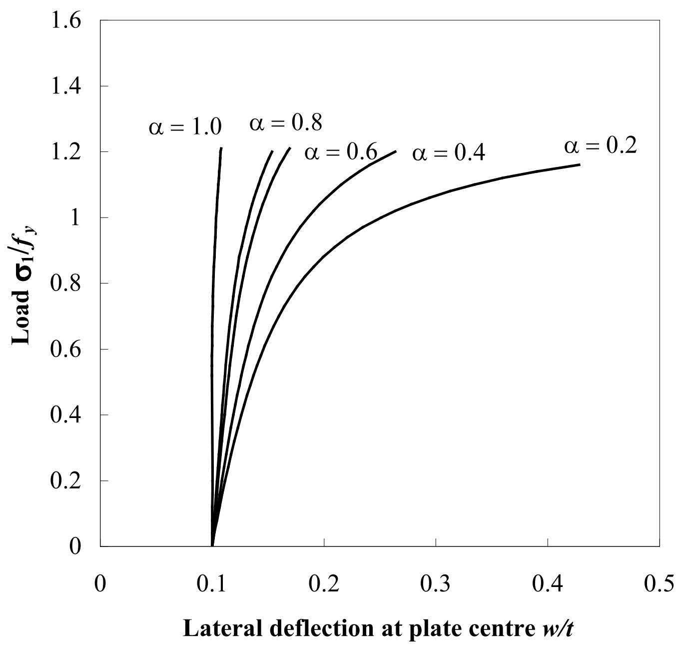

Fig. 8. Effect of stress gradients on the load-deflection curves for steel plates under in-plane bending $(b/t = 80)$.

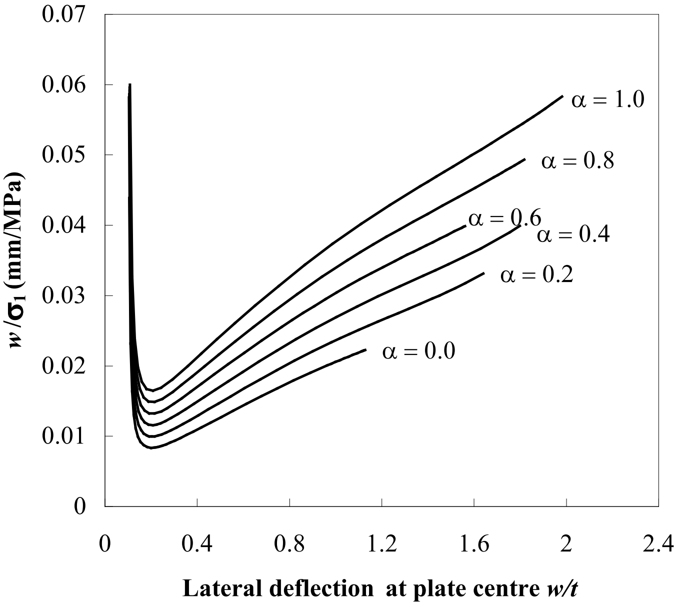

Fig. 9. Load-deflection curves for determining the critical local buckling strengths of plates $(b/t = 100)$.

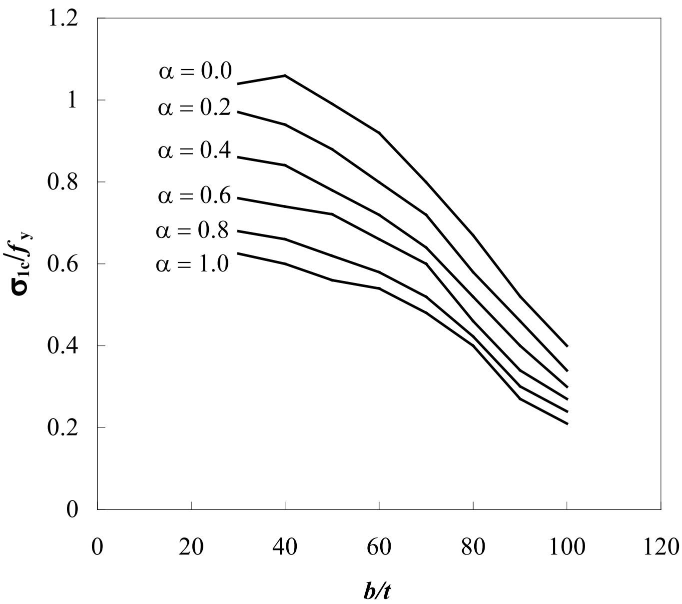

Fig. 10. Critical local buckling strengths of steel plates under compression.

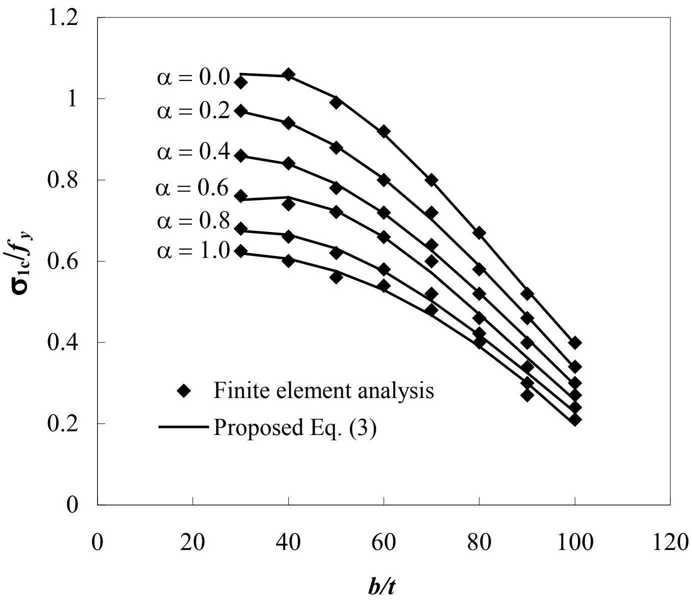

Fig. 11. Comparison of critical buckling strengths obtained by FEA and proposed formulas.

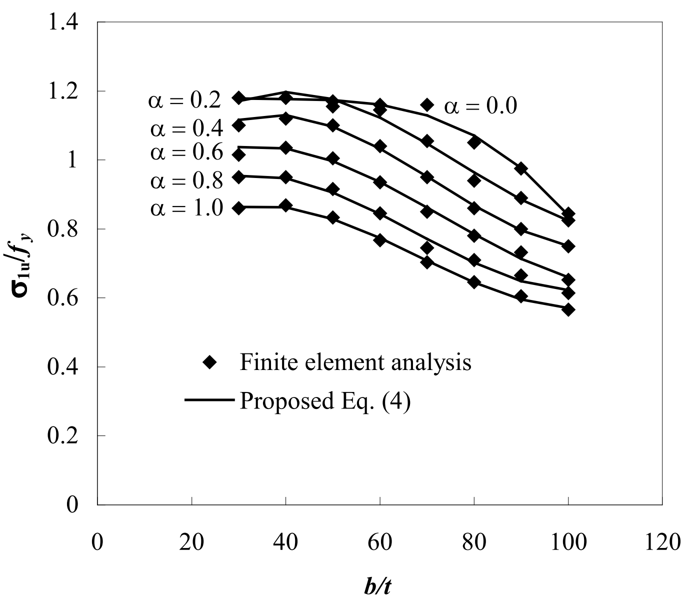

Fig. 12. Comparison of ultimate strengths obtained by FEA and proposed design formulas.

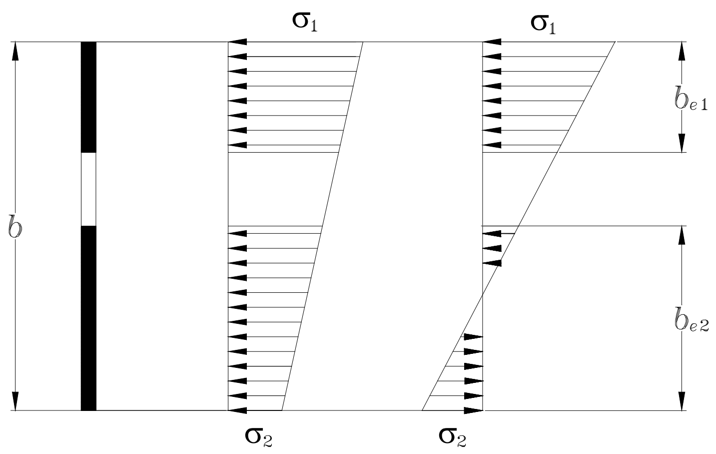

Fig. 13. Effective width of steel plate under compression and in-plane bending.

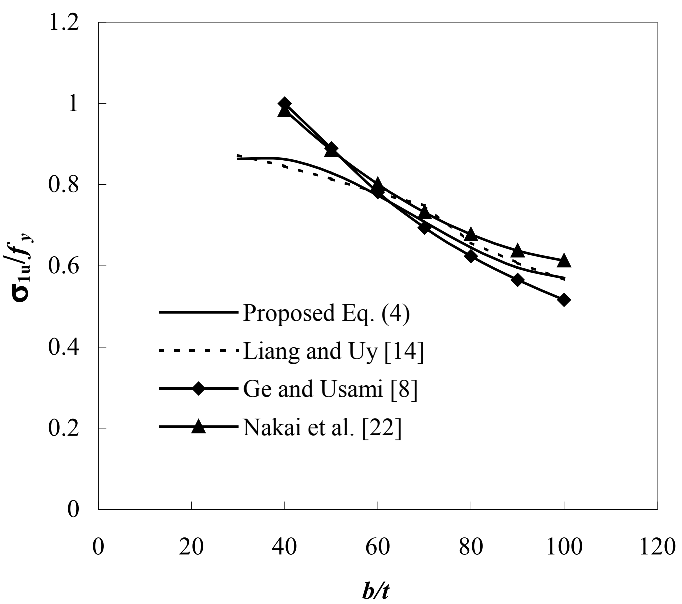

Fig. 14. Comparison of proposed strength design formulas with existing formulas.

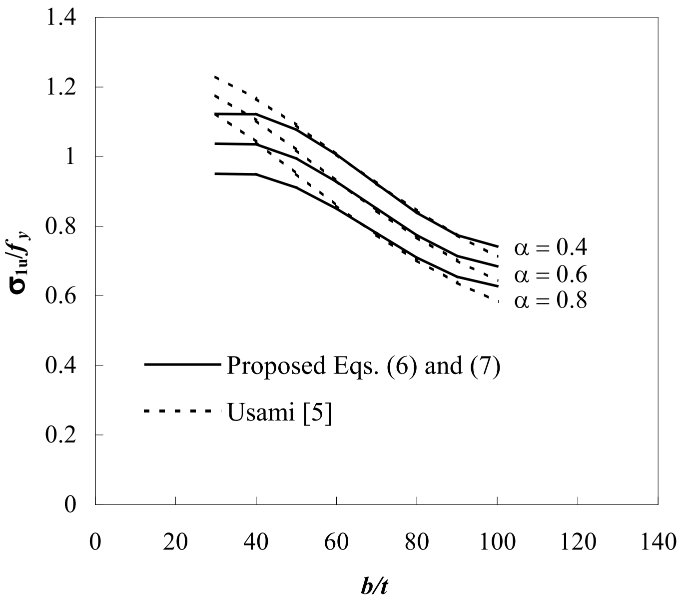

Fig. 15. Comparison of proposed effective width formulas with existing formulas for steel plates under non-uniform compression

Table 1. Constant coefficients for determining the critical local buckling strengths of plates.

α	a_1	a_2	a_3	a_4
0.0	0.6925	0.02394	-4.408×10^{-4}	1.718×10^{-6}
0.2	0.8293	0.01118	-2.427×10^{-4}	8.164×10^{-7}
0.4	0.6921	0.01223	-2.488×10^{-4}	8.676×10^{-7}
0.6	0.4028	0.02152	-3.742×10^{-4}	1.446×10^{-6}
0.8	0.5096	0.0112	-2.11×10^{-4}	7.092×10^{-7}
1.0	0.5507	0.005132	-9.869×10^{-5}	1.198×10^{-7}

Table 2. Constant coefficients for determining the ultimate strengths of steel plates.

α	c_1	c_2	c_3	c_4
0.0	1.257	-0.006184	1.608×10^{-4}	-1.407×10^{-6}
0.2	0.6855	0.02894	-4.89×10^{-4}	2.134×10^{-6}
0.4	0.6538	0.02888	-5.215×10^{-4}	2.424×10^{-6}
0.6	0.7468	0.01925	-3.689×10^{-4}	1.677×10^{-6}
0.8	0.6474	0.02088	-4.171×10^{-4}	2.058×10^{-6}
1.0	0.5554	0.02038	-3.944×10^{-4}	1.921×10^{-6}
-0.2	1.48	-0.01584	2.868×10^{-4}	-1.742×10^{-6}

Captions for Figures and Tables

Fig. 1. Concrete-filled thin-walled steel tubular beam-columns.

(a) Square section (b) Rectangular section

Fig. 2. Residual stress pattern in concrete-filled welded steel tubular beam-columns.

Fig. 3. Clamped steel plates under edge compression and in-plane bending.

Fig. 4. Load-deflection curves for steel plates under edge compression $(\alpha = 0.8)$.

Fig. 5. Effects of stress gradients on the load-deflection curves for plates under compression $(b/t = 100)$.

Fig. 6. Ultimate strengths of steel plates under edge compression.

Fig. 7. Load-deflection curves for steel plates under in-plane bending $(\alpha = 0.2)$.

Fig. 8. Effect of stress gradients on the load-deflection curves for steel plates under in-plane bending $(b/t = 80)$.

Fig. 9. Load-deflection curves for determining the critical local buckling strengths of plates $(b/t = 100)$.

Fig. 10. Critical local buckling strengths of steel plates under compression.

Fig. 11. Comparison of critical buckling strengths obtained by FEA and proposed formulas.

Fig. 12. Comparison of ultimate strengths obtained by FEA and proposed design formulas.

Fig. 13. Effective width of steel plate under compression and in-plane bending.

Fig. 14. Comparison of proposed strength design formulas with existing formulas.

Fig. 15. Comparison of proposed effective width formulas with existing formulas for steel plates under non-uniform compression

Table 1. Constant coefficients for determining the critical local buckling strengths of plates.

Table 2. Constant coefficients for determining the ultimate strengths of steel plates.