Ponder this

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As the focus of this issue of *ASMJ* is the teaching of calculus in the senior secondary and upper tertiary level, so is the focus of this problems section.

The purpose of the section is to provide teachers and students with a selection of calculus problems. In the time since the 17th century, basic ideas of calculus have undergone tremendous development. The works of many famous mathematicians exerted an enormous influence upon the development of calculus. One of them is Sir Isaac Newton whose name is connected with invention of the calculus. He used the method of fluxions for infinitesimals. Gottfried Wilhelm von Leibniz ranks with Newton as one of the inventors of the calculus. In particular, he worked out a convenient symbolism, an inverse method of tangents and the representation of transcendental lines by means of differential equations. Augustin-Louis Cauchy developed the concepts of limit and continuity. Cauchy was the first to prove Taylor's theorem rigorously, establishing his well-known form of the remainder. The calculus of variations is generally regarded as originating with the papers of Jean Bernoulli on the problem of the brachistochrone.

The calculus problems featured in this issue demonstrate diversity of calculus ideas, different concepts and topics. The first problem goes back in time as far as AD 1655. The Wallis product along with the Vieta formula

$$\frac{2}{\pi}=\sqrt{\frac{1}{2}}\cdot\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}\cdot\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}}\cdot\ldots$$

are the first examples of the infinite products in the history of calculus. Thank you to Michel Bataille for the proposed problem on a functional equation. The third problem is a particular case, though non-trivial one, of the Shapiro inequality that was proved in general form by Drinfeld nearly 20 years ago. The last two problems are from the editor's notes at the time of his study of calculus as an undergraduate student.

Problem set 2

From the work by John Wallis (1655)

1. Prove the Wallis product

$$\frac{\pi}{2} = \frac{2}{1}\cdot\frac{2}{3}\cdot\frac{4}{3}\cdot\frac{4}{5}\cdot\ldots\cdot\frac{2n}{2n-1}\cdot\frac{2n}{2n+1}$$

or
$$\frac{\pi}{2} = \lim_{n\to\infty}\frac{2\cdot 2\cdot 4\cdot 4\cdot\ldots\cdot 2n\cdot 2n}{1\cdot 3\cdot 3\cdot 5\cdot\ldots\cdot(2n-1)\cdot(2n+1)}$$

Proposed by Michel Bataille

2. Find all functions $f:\mathbb{R}\to\mathbb{R}$ such that

$$xy\,f(x+y) = y(x+3y)f(x) + x(y+3x)f(y)$$

for all real numbers x; y.

Particular case $n = 4$ of the Shapiro inequality

3. If $a, b, c, d > 0$ prove that

$$\frac{a}{b+c}+\frac{b}{c+d}+\frac{c}{d+a}+\frac{d}{a+b}\ge 2$$

4. Let for every $n\in\mathbb{N}$ a_n and b_n be integers from

$$\left(1+\sqrt{3}\right)^n = a_n + b_n\sqrt{3}$$

Evaluate
$$\lim_{n\to\infty}\frac{a_n}{b_n}$$

5. Prove that $x = 2$ is a unique solution on the set $\mathbb{R}$ to the equation

$$3^x + 4^x = 5^x$$

Solutions to this set or problems for publication should be submitted to:
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Solutions to this set will be made available on the AAMT website (www.aamt.edu.au) after 1 April 2009.