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A mathematical framework for regional hospital case mix planning and capacity appraisal

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ABSTRACT

This article considers current capacity issues in health care and the development of quantitative techniques to facilitate a high-level strategic assessment of hospital activity within a region. In providing that assessment, a variety of decision problems are foreseen, and we test the notion that it is useful to provide decision support for those. To achieve that support, several optimization models are developed and tested. In theory the presented models may help health care planners organise hospital resources and activity better, to treat more patients. The first model that we propose identifies a maximal caseload that meets the patient type proportions specified in a regional case mix imposed by a planner, executive or manager. The second model identifies how spatially distributed demand can best be met amongst the different hospitals, such that travel distance and unmet demand are minimised. The third model identifies how individual hospitals can jointly achieve their goals with the help of outsourcing. Each of the models has been implemented and tested on some demonstrative examples of a smaller nature, before a larger study is presented. Our case study demonstrates that appropriate data can be collected, and the proposed decision models can provide a rational appraisal of regional capacity and utilization.

1. Introduction

Increasing demand is an endemic issue in healthcare, exacerbated by the covid pandemic, and other social factors. With limited staff, equipment, facilities, and budget, hospitals are required to provide treatments and care for an increasing number of many patients with diverse illnesses and or disabilities. Without intervention and planning, however, a hospital's case mix (i.e., the composition of patient types treated in a cohort) is dictated by the training, skills and interests of staff, the referral patterns of patients, the productivity of the hospital, and prevalence of disease within catchment areas [1]. The problem of identifying a patient cohort (a.k.a., case mix) with a specific set of features deemed desirable or ideal is called case mix planning. Identifying the ideal composition and number of patients to be treated, however, is not straight-forward and is quite nuanced. A variety of challenges make this task challenging (Hof et. al. [2]). First, there are many different alternative case mixes that can be selected. Some case mixes are favourable for some patient type and unfavourable for others. Second, the term "ideal" is subjective and can mean different things in a practical setting. A case mix may be sought that is most equitable, for instance in the allocation and usage of hospital resources. A case mix may also be sought that is most economical or financially viable to treat. From a utilization and output-oriented perspective, a maximal cohort may also be sought. That cohort results in the greatest number of patients treated over time.

In most urban areas around the world, an assortment of public and privately owned hospitals is available to provide healthcare services. Each of these has a different focus and capabilities and is either motivated by professional interests or economic returns [3]. Otherwise, the overriding goal is to provide the public with equitable, accessible, high quality health care. Hospitals come in all shapes and sizes. Some are classified as specialist hospitals, local hospitals, major acute hospitals, elective centres, or clinics [4].

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Notation

Indices

g	Index for patient type
p	Index for patient subtype
w	Index for treatment area (i.e., wards and theatres)
h	Index for hospital
i	Index for region (sub)

Sets

G	Set of patient types
P_g	Set of subtypes of patient type g
P	Complete set of patient subtypes
$A_{g,p}$	A set of activities for patients of type g, subtype p
A	Complete set of all activities. $A = \bigcup_{(g,p)\in GP} A_{g,p}$

Parameters

$\phi^{\mathrm{HG}}_{h,g}$	Binary parameter – if patients of type g are treated at hospital h
$\phi^{\mathrm{HGP}}_{h,g,p}$	Binary parameter – if patients of type g, subtype p are treated at hospital h
$\mathbb{T}$	The period for the analysis. The unit is #weeks
$T_{h,w}$	Time availability of area $w \in W_h$ at hospital h during the analysis
$t_a, t_{a,h}$	Treatment time for activity a and specific to hospital h
$l_a, l_{h,a}$	Set of candidate locations for activity a generally and in hospital h
A_w	Activities that can be performed in area w. $A_w \subset A$
W_h	Set of treatment areas at hospital h
W	Total number of treatment areas
$r^{\mathrm{IG}}_{i,g}, r^{\mathrm{IGP}}_{i,g,p}$	Demographic information. The patients of each type requiring treatment in area i
$\widehat{N}^{\mathrm{G}}_g, \widehat{N}^{\mathrm{GP}}_{g,p}, \widehat{N}^{\mathrm{I}}_i$	Demands (a.k.a., targets)
$\overline{N}^{\mathrm{G}}_g, \overline{N}^{\mathrm{GP}}_{g,p}$	Upper bounds
$B, b_{h,w}$	Total number of treatment spaces (i.e., beds) and those in area $w \in W_h$
$\mu^{\mathrm{G}}_g, \mu^{\mathrm{GP}}_{g,p}$	Regional case mix and sub mix

Decision Variables

$\mathbb{N}$	Number of patients treated in the region.
N^{H}_h	Total number of patients treated in hospital h
N^{I}_i	Total number of patients treated in region i
$N^{\mathrm{G}}_g, N^{\mathrm{GP}}_{g,p}$	Total number of patients of type g treated, and of subtype p
$\breve{N}^{G}_g, \breve{N}^{\mathrm{GP}}_{g,p}, \breve{N}^{\mathrm{I}}_i$	Unmet demands
$n^{\mathrm{HG}}_{h,g}, n^{\mathrm{HGP}}_{h,g,p}$	Specific number of patients of type g treated in hospital h and of subtype p
$x^{\mathrm{IHG}}_{i,h,g}, x^{\mathrm{IHGP}}_{i,h,g,p}$	Patients of type g and subtype p originating in region i and treated in hospital h
$in^{\mathrm{HGP}}_{h,g,p}, in^{\mathrm{HG}}_{h,g}$	Number of patients insourced to hospital h
$out^{\mathrm{HGP}}_{h,g,p}, out^{\mathrm{HG}}_{h,g}$	Number of patients outsourced to hospital h
$\mu^{\mathrm{HG}}_{h,g}, \mu^{\mathrm{HGP}}_{h,g,p}$	Case mix and sub mix for individual hospitals
$U^{\mathrm{HW}}_{h,w}$	Usage of area w in hospital h

For better or worse, most hospitals operate independently in most of their activities, without any global managerial oversight, even if they are part of a multi-hospital system [5]. There are a variety of units that contribute to planning-related activities within each hospital, and these are tasked with organising patient admissions, surgeries, and other logistics [6]. These units are not typically equipped to perform strategic planning, or required to do so, and concentrate only on operational planning.

It is important for individual hospitals to treat patients quickly and efficiently and to use their resources wisely. By more efficiently utilising the existing hospital resources, which are contained therein, it may be possible to increase the number of patients that would otherwise be treated and reduce waiting lists and associated access delays [7]. It is also possible to reduce the number of resources that are required to meet future demands. However, using resources efficiently can only go so far. At some point in the future, expansions and other modernisations become warranted. Making ad-hoc decisions about health care facilities and their expansion, without proper consideration, analysis, and scientific support may result in oversights, excessive costs, inefficiencies, and unrealised performance potential.

Most hospitals are part of a larger health care system and working in a coordinated way with other hospitals may be advantageous. Furthermore, closer ties and collaboration would seem beneficial to keep up with future demands and the increasing health care needs of an aging population. However, that has yet to happen at scale. Decisions regarding the spatial organization of healthcare services, and resources, pose significant challenges to decision and policy makers [8].

As we will show in Section 2, there are various techniques in the literature to plan and assess individual hospitals. However, very few exist for multi-hospital scenarios [5]. Additional techniques, approaches, and strategies seem warranted. To test that assertion, this article considers whether a regional perspective is worthwhile and whether hospital case mix planning can be applied holistically across a region. Our investigations have resulted in the development of three mathematical models to help support strategic decision making. The attraction of these models is that they are transparent and conceptually easy to grasp. Once data has been extracted from data archives and data warehouses, solution time is instantaneous. Each model has a different focus, different data requirements, and provides a particular type of appraisal. The first model identifies the maximum output achievable in a region and which patient cohorts, individual hospitals should treat. The second model identifies if specified patient demands can be met and if so, how is it best distributed between different hospitals, given spatial demographic information. The third model considers the benefits of outsourcing patients between hospitals. The models that we have proposed may also be used to evaluate different hospital configurations and pinpoint (i.e., via an iterative approach or other model extensions), how the region's hospitals can be re-calibrated and reconfigured to achieve higher outputs. Our approach can in theory be used to judge whether a region's health care needs can be met.

It is worth noting that this article has a utilization and output-oriented perspective. As such, the concept of a "maximal" cohort is important. A maximal cohort is a measure of the region's capacity and is the greatest number of patients of each type that can be treated over time. To treat that cohort, many of the resources of the different hospitals must be continuously used (i.e., saturated). Hospital capacity is viewed as the maximum number of patients that can be treated in a specified period, rather than the actual number of hospital resources present, although this is a fair measure for making simplistic comparisons and assessments. Consequently, the capacity of a region is not the total number of operating theatres (a.k.a., rooms) and beds existing in current hospitals. Besides, theatres and beds are not equivalent, and they are not used to treat the same types of patients. Beds are positioned and staffed in different wards, and those wards are established for specific medical and surgical specialties. Similarly, operating theatres are often equipped with different equipment.

Questions pertaining to the capacity of multiple hospitals, is an emerging topic. Given recent developments in the literature analysing single hospitals [9, 10, 11], this topic is anticipated to be a frontier for new research in the coming years. To the best of our knowledge this topic has yet to be considered in detail. As such, there are many unknowns, and many questions. Determining how best to approach this topic is a significant research goal. In this article we propose a starting point, pose various questions, and seek answers for those, should they arise. For instance, how would hospitals best work together? How do multiple hospitals maximize the number of patients treated without exceeding the capacity of individual hospitals? Which types of patients should be treated in each hospital and what is the effect of the current status quo? Which hospitals should patients in different sub regions attend? Would a multi-hospital decision making framework be useful and to whom? How many hospitals are required and how should each be sized and configured? What is the impact of specific hospitals and how would the addition of a new hospital affect the capacity of a region? What is the effect of insourcing and outsourcing patients between hospitals? With answers to the above, one might then ask more wide-ranging questions like the following. For instance, should hospital services be centralized or decentralized? Should hospitals treat many patient types or specialise in treating a few? Which hospitals should treat which patient types? Is the capacity of a region better quantified by the number of specialists operating and their time availability for medical and surgical activities? Should more specialists be trained and in which hospitals(s) should they operate? Which hospitals should be expanded and how should they be expanded? Where and when should new hospitals be built and how should they be configured?

The format of this article is as follows. In Section 2 the current state of the art is examined, and an analysis is provided. In Section 3 the details of the quantitative framework are provided commencing with an outline of key technical details. In Section 4, a realistic case study is presented. Last, the conclusions, managerial insights and future research directions are detailed.

2. Literature review

The capacity of a single hospital system has been investigated quite frequently in past research. Specific questions like, i) how much capacity a hospital has, ii) how many patients can be treated in a specified period, and iii) how many beds are required, have been posed. The answer to these questions has been identified as “conditional” in nature, and reliant to a great extent upon the patient case mix chosen or imposed. There are different notions of case mix that may be adopted. A popular case mix description is the proportion of patients of each type within a cohort, relative to the total number treated. The case mix may also be viewed as the time apportioned to each patient type relative to the total time available. That definition, however, relates more to surgical patients and to the time allocated in the master surgical schedule.

A variety of quantitative assessments of hospital capacity (QAHC) and output have been performed. Many have occurred without observation of actual hospital operations. Assessments are performed parametrically (i.e., relative to key parameters) and geographic details (i.e., hospital layout and configuration). The papers of Burdett and colleagues take that approach. Their articles are noteworthy for the application of mathematical programming techniques. To determine the maximum number of patients that can be treated, Burdett et al. [11] found that it is sufficient to choose a patient case mix that saturates (i.e., fully utilizes) hospital resources. Their mathematical programming (optimization) model was developed to make that selection. In Burdett and Kozan [10] a multi-criteria approach that provides a sensitivity analysis of patient case mix was proposed. The approach generates a Pareto frontier of alternative case mix solutions and highlights patient cohorts with the highest outputs. Later a stochastic approach was developed in Burdett et al. (2022a) and a personal decision support system for QAHC was developed in Burdett et al. (2022b). McRae, Brunner, Bard [12] developed a non-linear mixed-integer programming model for case mix planning. Their model incorporates economies of scale and investigates the effect of changes in the efficiency of resource use on the optimal case mix. McRae and Brunner [9] also presented a framework for evaluating the effect of stochastic parameters on the case mix of a hospital. Hawkinson et al. [13] proposed a capacity allocation optimization methodology that reserves appointment slots for out-patients based on urgency in a complicated, integrated care environment where multiple specialties serve multiple types of patients. Their optimization reallocates and reserves network capacity to limit access delays.

There have been a variety of multi-objective mathematical programming models in healthcare pertaining to hospital case mix planning and capacity allocation. Abdelaziz and Masmoudi [14] proposed a multi-objective stochastic programming model for hospital bed planning. Their model identifies the optimal number of beds, physicians, and nurses in a region with numerous hospitals. The cost of creating new beds and the total number of employed nurses and physicians is minimised. A 157-hospital problem originating in Tunisia was solved. Shafaei and Mozdgir [15] considered the optimal allocation of theatre time amongst surgical groups. In their surrogate case mix planning problem, they considered three criterions as a means of calculating the value of patients. They formulated a linear programming model and applied the technique for order preference by similarity (TOPSIS) to rank solutions. Zhou et al. (2018) found that the joint optimization of revenue and equity is rare, with limited coverage in the literature. They proposed a multi-objective stochastic programming model to support capacity allocation of hospital wards, with revenue and equity as objectives. Revenue however is reported as challenging to model because payment mechanisms in healthcare are quite complex, and vary by disease, type of insurance cover and length of stay. Equity of access is modelled in terms of responsiveness and waiting time. Oliveira et al. [16] developed a distributed simulation-optimisation approach to identify the required number of hospital beds in each city. In the considered multicriteria location-allocation decision problem, they considered two objectives.

To perform planning within a multi-hospital system, the article by Mahar et al. [5] is noteworthy. In their article they developed a non-linear mixed integer optimization model to locate hospitals and specialized services within a region, subject to financial considerations, patient service levels, and diversions. They recognised that pooling specialized hospital procedures may leverage economies of scale to reduce cost and may also lead to an increase in actual or perceived quality of care. In their case study they considered five hospital locations in four cities. Bruno et al. [8] is also significant. They proposed a bi-objective mathematical programming model to redistribute beds among the existing hospitals of the network to maximize user’s accessibility. The two objectives are social cost and economic cost. The former is paid by users and is expressed by their spatial separation from the hospitals. The later cost is incurred by the hospitals when reconfiguring their capacities. For the selected case study, a spatial analysis was first performed to assess the user’s accessibility to the current network and to motivate the need for such a reorganization.

2.1. Final remarks

The article of Mahar et al. [5] and Bruno et al. [8], provides significant motivation for this article, for instance in suggesting and demonstrating the value of a regional assessment involving multiple hospitals. Although we have a similar focus, there are key differences. In this article we consider CMP which involves the optimal allocation of existing resources to different patient types, in specific hospitals across a whole region of hospitals. In the afore-said articles, completely different decision problems are considered. They instead identify where to position hospitals, services, and resources, subject to financial considerations and aspired service levels. It is worth noting that in our decision problem financial implications could also be included, but that is outside the scope of this article.

To reiterate, we consider utilization and output in a region with many hospitals. We do not consider the placement of hospitals, but what to do with existing hospitals. We assume that hospitals are already setup with the capacity to treat specialized patient types and the removal of wards and their beds from existing hospitals is not permitted, although this can be explored by users through a parametric analysis. In some research articles the optimal number of beds and staff is determined, however, this aspect is not concentrated upon in this article. We identify what to do with the beds that are already in place. Ravaghi et al. [17] provides a comprehensive review of approaches to identify the optimal number of beds. They did not however find an approach superior to all others.

3. Mathematical modelling

In this section three regional hospital capacity allocation models are proposed. The formal details and notation relevant to later modelling are, however, first described.

3.1. Formal specification and details

3.1.1. Hospital resources

There is a set of hospitals H. Each hospital $h \in H$ has limited operating theatres, wards, and beds. Operating theatres and wards consist of beds where treatments and care are provided. They are henceforth referred to as treatment areas. The set of treatment areas at hospital h is denoted by W_h and the number of treatment spaces in each area $w \in W_h$ is denoted $b_{h,w}$.

3.1.2. Patient types

It is assumed that there is a clearly defined set of patient types (a.k.a., groups) that are treatable across each hospital in the region. This is a necessary requirement. For each patient type $g \in G$, there is a set of patient subtypes denoted P_g. The full set of patient subtypes is hence denoted $GP = \{(g,p) | g \in G, p \in P_g\}$. For each patient sub type there is a set of activities to be performed of a medical or surgical nature, and we denote this as $A_{(g,p)}$. This set may be regarded as either a clinical pathway (CP) or as a resourcing profile (RP). The main distinction between the two viewpoints is how information is aggregated or not aggregated. In a RP, each record, pertains to the usage of a distinct resource type. In contrast, there may be multiple records for the same hospital resource type in a CP. The viewpoint chosen, however, has no bearing on the nature of the developed mathematical models.

For activity $a \in A_{(g,p)}$, the duration is denoted t_a and is measured in either hours or minutes. The set of treatment areas where the activity can be performed in each hospital is denoted $l_{h,a}$. The full set of treatment areas (i.e., candidate locations) is therefore, $L_a = \bigcup_{h \in H} l_{h,a}$. Some hospitals can only treat some patient types and subtypes and not others and this information is discernible via the definition of $l_{h,a}$. It is helpful to define the capabilities of hospitals upfront, more explicitly. For instance, let us define $\phi^{HG}_{h,g}$ as a binary parameter indicating whether patients of type g are treated at hospital h. Similarly, let us define $\phi^{HGP}_{h,g,p}$ as a binary parameter indicating whether patients of subtype (g,p) are treated at hospital h. Generally, $\phi^{HGP}_{h,g,p} \le \phi^{HG}_{h,g}\ \forall p \in P_g$. If for instance, $\phi^{HG}_{h,g} = 0$ then $\phi^{HGP}_{h,g,p} = 0\ \forall p \in P_g$. However, if $\phi^{HGP}_{h,g,p} = 0$ then $\phi^{HG}_{h,g}$ may be either zero or one.

3.1.3. Hospital caseload

The regional caseload is to be determined. This is the number of patients of each type and subtype to be treated in each hospital. These are respectively denoted $n^{HG}_{h,g}$ and $n^{HGP}_{h,g,p}$ (a.k.a., $n^{HGP}_{h,gp}$). Several ancillary decision variables are also defined, namely, $\mathbb{N}$ the number of patients to be treated in the region, N^{H}_{h} the total number of patients treated in hospital h, and N^{G}_{g} the number of each type treated. It is worth noting the following inherent dependencies:

$$\mathbb{N} = \sum_{h \in H} N^{H}_{h} \tag{1}$$

$$N^{G}_{g} = \sum_{h \in H} n^{HG}_{h,g} \qquad \forall g \in G \tag{2}$$

$$N^{H}_{h} = \sum_{g \in G} n^{HG}_{h,g} \qquad \forall h \in H \tag{3}$$

$$N^{GP}_{g,p} = \sum_{h \in H} n^{HGP}_{h,g,p} \qquad \forall g \in G, p \in P_g \tag{4}$$

$$n^{HG}_{h,g} = \sum_{p \in P_g} \left(n^{HGP}_{h,g,p} \right) \qquad \forall g \in G, h \in H \tag{5}$$

It is also worth noting that, $n^{HG}_{h,g} \le \phi^{HG}_{h,g} M$ and $n^{HGP}_{h,g,p} \le \phi^{HGP}_{h,g,p} M$ where M is a placeholder for either a large value or an upper bound. Within each hospital treatment area, the treatment spaces are available for a specified number of hours per period. The time availability of spaces within treatment area $w \in W_h$ at hospital h is denoted $T_{h,w}$. The number of activities of type a to be performed in area w of hospital h is a major decision and is called the allocation. This is denoted $\alpha_{a,h,w}$. It is worth noting that $\alpha_{a,h,w} = 0$, if $\phi^{HG}_{h,g} = 0$ or $\phi^{HGP}_{h,g,p} = 0$. Similarly, $\alpha_{a,h,w} = 0\ \forall w \in W_h\ l_{h,a}$.

3.1.4. Case mix

The mix of patients to be treated in a hospital is selectable, and changes throughout the year. There are various financial and social implications associated with each case mix. The notion of a case mix is a well-established [10, 11], to regulate the competition for shared resources. If a case mix is not imposed when performing CMP, a multi-criteria optimization problem eventuates. In a single hospital scenario, the case mix and sub mix are specific to that hospital. In a regional based approach, however, it is hypothesised that the case mix should be defined for the region instead, and the case mix and sub mix at each hospital to become a decision. This is because the case mix for a region is obtainable from historical events and is routinely published in government documents. Consequently, we define a general region-based case mix as μ^{G}_{g} and sub mix as $\mu^{GP}_{g,p}$ where $\sum_{g \in G} \mu^{G}_{g} = 1$ and $\sum_{p} \mu^{GP}_{g,p} = 1$. A local hospital case mix is denoted by the same variables, albeit with an additional subscript for the hospital, i.e., $\mu^{HG}_{h,g}, \mu^{HGP}_{h,g,p}$.

3.2. Capacity allocation within a region: determining a maximal caseload

The first capacity allocation model has been created to appraise the output of a region of hospitals. It determines the greatest caseload that can be treated amongst the different hospitals present over a designated period (a.k.a., planning horizon) of $\mathbb{T}$ weeks. The caseload should match the proportions defined for the region, namely the regional case mix. The variables defined in (1)-(5) have a dual meaning in this model. They describe rates of output over the planning horizon. As such, $\mathbb{N}$ may also be viewed as a rate of output. A key feature of this model is that it identifies how best to distribute the patient types amongst the hospitals present. This model may also highlight situations where some hospitals are not required to treat as many patient types as they are capable of treating. The exact details of the model are as follows:

Maximize $\mathbb{N}$

Subject To:

$$0 \le n^{HG}_{h,g} \le \phi^{HG}_{h,g} M \qquad \forall h \in H, g \in G \tag{6}$$

$$0 \le n^{HGP}_{h,g,p} \le \phi^{HGP}_{h,g,p} M \qquad \forall h \in H, g \in G, p \in P_g \tag{7}$$

$$0 \le \alpha_{a,h,w} \le \phi^{HGP}_{h,g,p} M \qquad \forall h \in H, g \in G, p \in P_g, a \in A_{g,p}, w \in W_h \tag{8}$$

$$n_{h,g,p}^{\mathrm{HGP}} = \sum_{w \in I_{a,h}} \alpha_{a,h,w} \quad \forall h \in H, g \in G, p \in P_g, a \in A_{g,p} \tag{9}$$

$$U_{h,w}^{\mathrm{HW}} = \sum_{a \in A} \alpha_{a,h,w} t_{a,h} \quad \forall h \in H, w \in W_h \tag{10}$$

$$U_{h,w}^{\mathrm{HW}} \leq b_{h,w} T_{h,w} \quad \forall h \in H, w \in W_h \text{ where } T_{h,w} = \mathbb{T} \times \#hrs \Big/ week \tag{11}$$

$$N_g^{\mathrm{G}} \geq \mu_g^{\mathrm{G}} \mathbb{N} \quad \forall g \in G \tag{12}$$

$$N_{g,p}^{\mathrm{GP}} \geq \mu_{g,p}^{\mathrm{GP}} N_g^{\mathrm{G}} \quad \forall g \in G, p \in P_g \tag{13}$$

In this model we utilize the relationships summarised previously in equation (1) – (5). These are substituted into relevant places. The main decision variables have restricted ranges as shown in (6) – (8). Resource allocations and patient treatments are aligned in constraint (9). Resource usage is defined by constraint (10) and restricted by constraint (11). Both constraints, however, can be merged. Case mix and sub mix are enforced by (12) and (13). After the model is solved, case mix and sub mix for individual hospitals can be evaluated by equation (14).

$$\mu_{h,g}^{\mathrm{HG}} = n_{h,g}^{\mathrm{HG}} \Big/ N_h^{\mathrm{H}} \text{ and } \mu_{h,g,p}^{\mathrm{HGP}} = n_{h,g,p}^{\mathrm{HGP}} \Big/ n_{h,g}^{\mathrm{HG}} \quad \forall h \in H, g \in G, p \in P_g \tag{14}$$

3.2.1. Additional remarks

This model can be further extended and modified in many useful ways. Those details are provided in Appendix B. A critical element of the model that should be discussed, concerns the parameter $A_{(g,p)}$, and the identification of locations where activities are performed. To initialize these parameters, it is necessary to define where each medical and surgical activity can be performed. In a multi-hospital scenario, this requires considerable effort. However, in many circumstances, automating that task is possible.

3.2.2. Demonstrative example

Let us consider a fictional regional area with two hospitals to demonstrate the solution of the proposed mathematical model and the nuances of this type of assessment. In this region, each hospital has different resources and layout. The details of the two hospitals are shown in Table 1, where OT and ICU are abbreviations for operating theatre and intensive care unit, respectively. It is assumed that each ward (i.e., W1 – W5) and intensive care bed is available 24 hrs per day per week (i.e., 168 hours). Theatres are deemed available eight hours per day, five days a week (i.e., for 40 hours). The capacity of this regional area is appraised over a period of four weeks.

There are many different patient types, but for clarity we restrict that number to five in this example. As shown in Table 2, each patient type has a unique resourcing profile. Table 3 summarises the focus of each ward.

In each profile the patient needs to spend a specific amount of time in an operating theatre, intensive care bed or ward bed. We generated this information to be indicative of those occurring in local hospitals using a pseudo-random process. Hospital H2 has fewer beds overall but is staffed/nursed differently. Their wards are more generic and can look after more patient types. Hospital H1 is less generic, and their wards are only for specific types. From a practical perspective, we could expect the quality of care to be better in H1.

For the purposes of this investigation, let us assume the case mix is [0.328, 0.295, 0.164, 0.082, 0.131]. The model provides the solutions shown in Tables 4 and 5. In Table 4, each hospital is considered as a separate entity, and must treat a cohort with the proportions specified in the regional case mix. In Table 5, the region is considered as a whole, and each hospital can treat a different cohort, if jointly that cohort meets the proportions specified for the whole region. Clearly, more patients can be treated if an integrated approach is applied. Here, the increase was 20%. Different ward utilisations occur, and some wards are very lightly used depending on how patient types are distributed between the two hospitals. In Table 5 we can see that some specialization is implied. In H1, type two is not treated at all, and in H2, type one and four are not treated. Consequently, it may be worth considering whether some wards should be reassigned to other patient types. For instance, ward H1-W2 in H1, and H2-W1 and H2-W3 in H2. If both hospitals are generic, it may not make sense to treat none of some types. Adding some minimum treatment numbers, however, is an easy thing to do.

In both tables, ward utilization levels are low, and the operating theatres are clearly bottlenecks restricting further outputs. This is a quirk of this toy scenario. This is unlikely to be the case in real-life hospitals and one would expect bed numbers to be more closely aligned with operating theatre outputs. Also, there are many medical inpatients, who would use the wards for minor treatments and care, and not the theatres and ICU. Those have not been included, however, in this example.

Table 1
Hospital resources and beds in the region.

Hosp.	#OT	#ICU	#BEDS	#W1	#W2	#W3	#W4	#W5
H1	10	15	78	25	15	10	15	13
H2	6	5	45	15	20	10	-	-

Table 2
Resourcing details and case mix.

Type	Sub Type	Mix (%)	OT (hrs)	ICU (hrs)	Ward (hrs)	Ward Options
T1	T1-1	70	0.25	0	2.16	H1-W1, H2-W1
	T1-2	25	1.25	0	3.65	H1-W1, H2-W1
	T1-3	5	0	0	0.25	H1-W1, H2-W1
T2	T2-1	90	2.4	0	5.22	H1-W2, H2-W1
	T2-2	10	0	0	0.5	H1-W2, H2-W1
T3	T3-1	25	6.5	2.6	22.94	H1-W3, H2-W2
	T3-2	40	4.56	1.5	18.39	H1-W3, H2-W2
	T3-3	28	7.6	14.8	55.54	H1-W3, H2-W2
	T3-4	7	0	0	1	H1-W3, H2-W2
T4	T4-1	50	3.4	0	10.43	H1-W4, H2-W3
	T4-2	30	5.7	12	48.12	H1-W4, H2-W3
	T4-3	20	0	0	0.33	H1-W4, H2-W3
T5	T5-1	85	4.1	8.67	32.81	H1-W5, H2-W3
	T5-2	15	0	0	2	H1-W5, H2-W3

Table 3
Patient types treatable in each ward.

Ward	Treats
H1-W1	T1-1, T1-2, T1-3
H1-W2	T2-1, T2-2
H1-W3	T3-1, T3-2, T3-3, T3-4
H1-W4	T4-1, T4-2, T4-3
H1-W5	T5-1, T5-2
H2-W1	T1-1, T1-2, T1-3, T2-1, T2-2
H2-W2	T3-1, T3-2, T3-3, T3-4
H2-W3	T4-1, T4-2, T4-3, T5-1, T5-2

Table 4
Caseload and utilization for separate hospitals each operating with the same case mix.

Hosp.	Type 1	2	3	4	5	Tot	
H1	214.39	192.82	107.195	53.6	85.63	653.63	
H2	415.14	373.37	207.57	103.78	165.8	1265	
Region	629.53	566.19	314.765	157.38	251.43	1918.63	
Hosp.	ICU	OT	W1	W2	W3	W4	W5
H1	13.91	100	3.11	9.16	45.69	10.48	27.63
H2	80.81	76.84	27.62	44.34	100	-	-

Table 5
Caseload and utilization for integrated regional approach.

Hosp.	Type 1	2	3	4	5	Tot	
H1	754.65	0	74.68	188.66	54.13	1072.12	
H2	0	678.73	302.64	0	247.27	1228.64	
Region	754.65	678.73	377.32	188.66	301.4	2300.76	
Hosp.	ICU	OT	W1	W2	W3	W4	W5
H1	18.48	100	10.95	0.26	39.89	36.9	20.33
H2	91.45	100	31.97	60.47	100	-	-

3.3. *Capacity allocation incorporating spatially distributed patients*

In the previous section a strategic hospital capacity allocation model was proposed to appraise regional capacity. It was assumed that hospitals present in the region were close, and distance travelled was not significantly different, or else was not important to be modelled. However, when assigning specific numbers of patients of each type to be treated it is worth considering where patients originate. In most scenarios, the location of patients is unlikely to be uniformly distributed across a region, and this motivates the development of a second capacity allocation model.

The novel model presented in this section includes spatial demographics and proximity information. The intention of this model is to assign specified demands originating in different sub regions to the hospitals within the region. The objective is to cover as much demand as possible, but also to minimize the total incurred travel distance. This decision problem is inherently multi-objective and suggests different trade-offs may be made. It is unlikely that both criterions can be optimised simultaneously.

This model requires population demographics of sub regions to be collected. A typical example is shown in Fig. 1.

There are six sub regions, and two hospitals are positioned in two of the sub regions. Patients who live in sub regions $i = 2, 3, 5, 6$ may need to travel further than those positioned in region one and four. The proximity between sub regions is vital information. We denote $d_{i,i'}$ as the average distance between region i and region i', $\forall i \neq i'$. The hospitals present in sub region $i \in I$ is denoted by set $H_i \subset H$. The specific region that hospital h resides in is designated $\mathscr{L}_h \in I$. If $h \in H_i$ then $\mathscr{L}_h = i$.

As indicated, the demand varies spatially but is assumed fixed within the region. For instance, in each sub region, the number of patients of each type are known. Let us define $r^{\mathrm{IG}}_{i,g}$ and $r^{\mathrm{IGP}}_{i,g,p}$ as demographic information describing the number of patients of each type requiring treatment in area i where $r^{\mathrm{IG}}_{i,g} = \sum_{p \in P_g} r^{\mathrm{IGP}}_{i,g,p}$. Let us then define aggregated

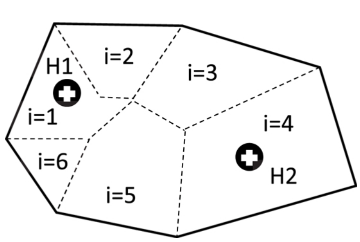

Fig. 1. Position of two hospitals in a region and demarcation of sub regions.

demand (a.k.a., goals) for the region by $\widehat{N}^{\mathrm{GP}}_{g,p}$ and $\widehat{N}^{\mathrm{I}}_i$ where $\widehat{N}^{G}_g = \sum_i r^{\mathrm{IG}}_{i,g}$, $\widehat{N}^{\mathrm{GP}}_{g,p} = \sum_i r^{\mathrm{IGP}}_{i,g,p}$ and $\widehat{N}^{\mathrm{I}}_i = \sum_{g \in G} r^{\mathrm{IG}}_{i,g}$. Let also us define $\breve{N}^{\mathrm{G}}_g$, $\breve{N}^{\mathrm{GP}}_{g,p}$ and $\breve{N}^{\mathrm{I}}_i$ as the unmet demands. We can now quantify the following relationships:

$$\ddot{N}^{\mathrm{G}}_g = \max\left(\widehat{N}^{\mathrm{G}}_g - N^{\mathrm{G}}_g, 0\right) \qquad \forall h \in H, g \in G \tag{15}$$

$$\ddot{N}^{\mathrm{GP}}_{g,p} = \max\left(\widehat{N}^{\mathrm{GP}}_{g,p} - N^{\mathrm{GP}}_{g,p}, 0\right) \qquad \forall h \in H, g \in G, p \in P_g \tag{16}$$

$$\ddot{N}^{\mathrm{I}}_i = \max\left(\widehat{N}^{\mathrm{I}}_i - N^{\mathrm{I}}_i, 0\right) \qquad \forall i \in I \tag{17}$$

The main decision is how many patients should be treated and where. Let us define $x^{\mathrm{IHGP}}_{i,h,g,p}$ as the number of patients of type (g,p) originating in sub region i treated in hospital h. Let us also define $x^{\mathrm{IHG}}_{i,h,g}$ as the total number of patients of type g treated and impose that $x^{\mathrm{IHG}}_{i,h,g} = \sum_{p \in P_g} x^{\mathrm{IHGP}}_{i,h,g,p}$. Consequently, we can write that:

$$n^{\mathrm{HGP}}_{h,g,p} = \sum_{i \in I} x^{\mathrm{IHGP}}_{i,h,g,p} \qquad \forall h \in H, g \in G, p \in P_g \tag{18}$$

$$n^{\mathrm{HG}}_{h,g} = \sum_{i \in I} x^{\mathrm{IHG}}_{i,h,g} \qquad \forall h \in H, g \in G \tag{19}$$

$$N^{\mathrm{I}}_i = \sum_{h \in H} \sum_{g \in G} x^{\mathrm{IHG}}_{i,h,g} \qquad \forall i \in I \tag{20}$$

$$\sum_{h \in H} x^{\mathrm{IHGP}}_{i,h,g,p} \leq r^{\mathrm{IGP}}_{i,g,p} \qquad \forall i \in I, g \in G, p \in P_g \tag{21}$$

$$\sum_{h \in H} x^{\mathrm{IHG}}_{i,h,g} \leq r^{\mathrm{IG}}_{i,g} \qquad \forall i \in I, g \in G \tag{22}$$

$$n^{\mathrm{IH}}_{i,h} = \sum_{g \in G} x^{\mathrm{IHG}}_{i,h,g}, \; n^{\mathrm{IG}}_{i,g} = \sum_{h \in H} x^{\mathrm{IHG}}_{i,h,g} \qquad \forall h \in H, i \in I, g \in G \tag{23}$$

Constraints (18) and (19) aggregate the allocations over the different sub regions. Constraints (21) and (22) ensures that the number of patients assigned to hospitals from a sub region does not exceed the number available in the sub region. Equation (23) provides additional auxiliary variables to describe more information about the allocations that have been made. A bi-objective capacity allocation can now be posed. The full model is as follows:

$$\text{Minimize } Z_1 = \sum_{i \in I} \sum_{\forall h \in H} d_{i,\mathscr{L}_h} x^{\mathrm{IH}}_{i,h} \tag{24}$$

$$\text{Minimize}\, Z_2 = \sum_{g\in G} \breve{N}_g^{G},\ \sum_{g\in G}\sum_{p\in Pg} \breve{N}_{g,p}^{\mathrm{GP}}\ \text{or} \sum_{i\in I} \breve{N}_i^{I} \tag{25}$$

Subject to:

$$x_{i,h,g}^{\mathrm{IHG}} = \sum_{p\in Pg} x_{i,h,g,p}^{\mathrm{IHGP}} \qquad \forall i \in I, h \in H, g \in G \tag{26}$$

$$\dddot{N}_g^{\mathrm{G}} \geq \widehat{N}_g^{\mathrm{G}} - N_g^{\mathrm{G}};\quad \dddot{N}_g^{\mathrm{G}} \geq 0 \qquad \forall g \in G \tag{27}$$

$$\dddot{N}_g^{\mathrm{G}} \leq \left(\widehat{N}_g^{\mathrm{G}} - N_g^{\mathrm{G}}\right) + \left(1 - \beta_g^{\mathrm{G}}\right) M \text{ and } \dddot{N}_g^{\mathrm{G}} \leq \beta_g^{\mathrm{G}} M \qquad \forall g \in G \tag{28}$$

$$\dddot{N}_{g,p}^{\mathrm{GP}} \geq \widehat{N}_{g,p}^{\mathrm{GP}} - N_{g,p}^{\mathrm{GP}};\quad \dddot{N}_{g,p}^{\mathrm{GP}} \geq 0 \qquad \forall g \in G, p \in P_g \tag{29}$$

$$\dddot{N}_{g,p}^{\mathrm{GP}} \leq \left(\widehat{N}_{g,p}^{\mathrm{GP}} - N_{g,p}^{\mathrm{GP}}\right) + \left(1 - \beta_{g,p}^{\mathrm{GP}}\right) M \text{ and } \dddot{N}_{g,p}^{\mathrm{GP}} \leq \beta_{g,p}^{\mathrm{GP}} M \qquad \forall g \in G, p \in P_g \tag{30}$$

$$\dddot{N}_i^{\mathrm{I}} - N_i^{\mathrm{I}} \leq \dddot{N}_i^{\mathrm{I}} \leq \left(\dddot{N}_i^{\mathrm{I}} - N_i^{\mathrm{I}}\right) + \left(1 - \beta_i^{\mathrm{I}}\right) M \qquad \forall i \in I \tag{31}$$

$$0 \leq \dddot{N}_i^{\mathrm{I}} \leq \beta_i^{\mathrm{I}} M;\ N_i^{\mathrm{I}} = \sum_{h\in H}\sum_{g\in G} x_{i,h,g}^{\mathrm{IHG}} \qquad \forall i \in I \tag{32}$$

$$\beta_g^{\mathrm{G}}, \beta_{g,p}^{\mathrm{GP}}, \beta_i^{\mathrm{I}} \in \{0,1\} \qquad \forall g \in G, p \in P_g, i \in I \tag{33}$$

$$N_g^{\mathrm{G}} = \sum_{p\in Pg} N_{g,p}^{\mathrm{GP}};\ N_{g,p}^{\mathrm{GP}} = \sum_{h\in H}\sum_{i\in I} x_{i,h,g,p}^{\mathrm{IHGP}} \qquad \forall g \in G, p \in P_g \tag{34}$$

+ Constraints (1) – (6), (10), (11), (18), (19), (21), (22)

Constraints (27)-(32) facilitate a linearisation of (15)-(17). Constraints (27) and (28) are a required substitute for (15). Similarly, constraint (29) and (30) are a required substitute for (16), and constraints (31) and (32) are a required substitute for (17). Constraints (28), (30) and (32) are needed because $\breve{N}_g^{\mathrm{G}}$, $\breve{N}_{g,p}^{\mathrm{GP}}$, $\breve{N}_i^{\mathrm{I}}$ may not always be in the objective. As such, the model can select invalid values. To force exact equality, the binary decision variables $\beta_g^{\mathrm{G}}, \beta_{g,p}^{\mathrm{GP}}, \beta_i^{\mathrm{I}}$ are necessary.

3.3.1. Additional remarks

If the designated demands are too high, then the model may not solve. This is an indication to the decision maker that the infrastructure and constraints of the system are not well aligned with the case mix and demand volume.

This model can be adjusted to identify the time horizon required to meet all demand if demands cannot be met within one period. As such the objective is to minimize $\mathbb{T}$ subject to $Z_2 = 0$. Further adjustments can be made to restrict the distance travelled. This however may inflate $\mathbb{T}$. Assessments of this nature are particularly useful for strategic considerations like estimating the minimum time to work through a large waiting list or backlog of patients. Operational or tactical considerations, concerning the time to treat a selected caseload, is also a potential use-case. Explicit scheduling of a caseload is a competing approach, but that decision problem is NP-hard [7].

3.3.2. Demonstrative example

Let us consider a slight variation of example from the preceding section. The demographic shown in Table 6 is now specified; it provides $r_{i,g,p}^2$ values. The theatres, wards, and intensive care beds of the two hospitals are given in Table 7.

For the purposes of this example, the distance between region centroids is used as an acceptable proxy for the average distance travelled. This is an acceptable and pragmatic option when there is no information about individual patients or when regions are sized appropriately. This approach has been applied in many fields where objects and materials are transported. Earthworks is one example [18]. The centroid of the region's six sub regions is respectively (265,150), (290,192), (326, 173), (355,93), (296,92), and (260, 93). Given that meeting unmet demand is the objective, a solution like the one shown in Fig. 2 can be obtained.

In the presented solution all demands are met. Both hospitals have high OT and ICU utilisations, however, there is still some capacity left in several wards. Significant travel is required in the displayed solution. The total distance required is 143,360 patient-km, which, is 77km per person on average. There are many alternative optimal solutions, and each has different travel distances. If the distance travelled is the objective, then no patients are chosen, and no distance is travelled. If we force unmet demand to be zero, however, then a solution with 116,521 km can be obtained, which is significantly better than the first solution obtained. The following decision matrix best summarises that solution:

A multi-objective assessment can be performed. The well-known epsilon-constraint method (ECM) is sufficient for this task and easily implemented, for instance in IBM's ILOG CPLEX software using scripting or using the C++ Concert Technology. The ECM method involves the repeated application of the CMP model with an additional constraint restricting the value of one of the objectives. The trade-off between distance travelled and number of patients can be built up as shown in Fig. 3. In this chart all solutions above the lower line and below the top line are achievable. The lower line represents the Pareto frontier. All solutions below this are not achievable. Fig. 3 hence shows how much flexibility there is in the system to assign patients to other locations. In each chart, the star denotes the fictional ideal solution. This figure demonstrates that when the number of patients treated is low, then patients can be sent to their nearest hospital and thus incur the least travel distance. However, as more patients are treated, less capacity is free, and patients are more often needed to travel further afield, or risk not being treated at all. Within the range $\mathbb{N} \in [500, 1300]$ the distance increases linearly. As utilisation approaches the regions capacity, however, the distance required to be travelled increases most steeply.

From a pragmatic perspective it is worth considering what this Pareto frontier highlights and whether it suggests regional hospital executives, managers, or planners to do anything. Ideally, any regional

Table 7
Hospital configuration.

Hosp.	#OT	#ICU	#WARD	W1	W2	W3	W4	W5
H1	10	5	74	22	15	10	14	13
H2	6	2	42	15	20	7	-	-

Table 6
The spatial demographic information for the example.

Region	g=1, $p = 1$	g=1, $p = 2$	g=2, $p = 1$	g=3, $p = 1$	g=3, $p = 2$	g=3, $p = 3$	g=4, $p = 1$	g=4, $p = 2$	g=5, $p = 1$	
R1	34	22	72	1	29	14	19	21	17	229
R2	92	36	146	35	4	21	28	13	56	431
R3	162	45	95	17	53	25	4	0	33	434
R4	43	13	10	4	10	11	8	21	92	212
R5	7	39	147	19	21	14	8	4	27	286
R6	90	28	80	0	5	22	9	17	19	270
	428	183	550	76	122	107	76	76	244	1862
	611		550	305			152		244	

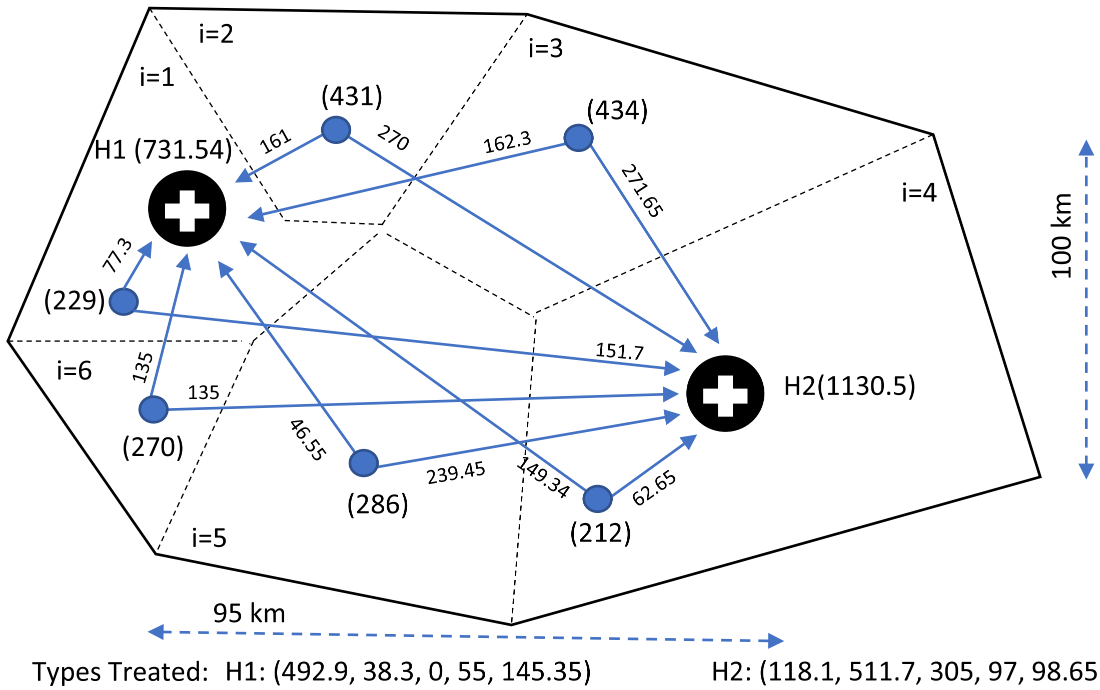

Fig. 2. Caseload distribution that minimises unmet demand.

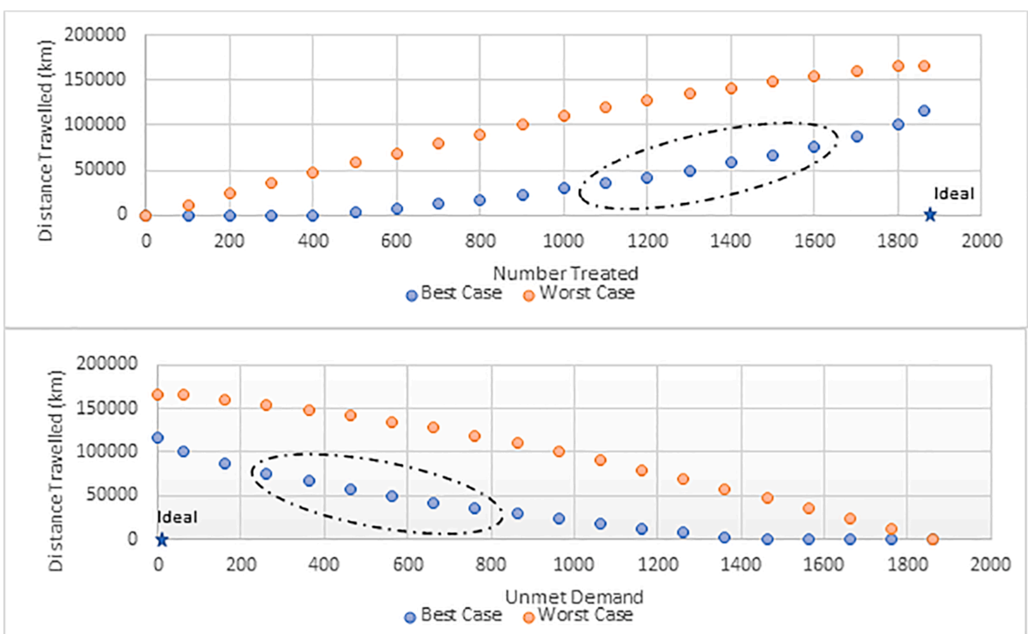

Fig. 3. Pareto frontier of distance travelled versus a) unmet demand and b) number treated.

plan should sit somewhere on the Pareto front. If it did not, then perhaps some replanning should be initiated. In the public sector, it is preferable to meet as much demand, as possible. Yet, this is not necessarily true of the private. If all demand is to be met, then the minimum total distance to be incurred by patients within the region should be identified and an appropriate distribution of patients to hospitals should be chosen. The required distance to be travelled could be quite excessive though and it may not be desirable to enforce a regional plan like that. As shown, this requirement could be reduced in some circumstance, for instance by treating fewer overall patients. So, that is a course of action worth considering under certain circumstances.

If meeting demand is high in priority, but distance travelled is also a high priority, then it is possible to reduce the total distance travelled by treating fewer patients. The exact trade-off for that consideration is shown in Fig. 4. In that chart, the proximity to the ideal solution has been computed. The function shown is skewed to the right and suggests that it is better to treat more patients, rather than less, but not to saturate the two hospitals. The average distance to be travelled was also computed. In this situation it increases as more patients are treated and does not decrease.

The best solution is to treat 1300 patients with a total distance of $Z_1 = 49601$ patient-km to be travelled (i.e., 38.16 km on average). That

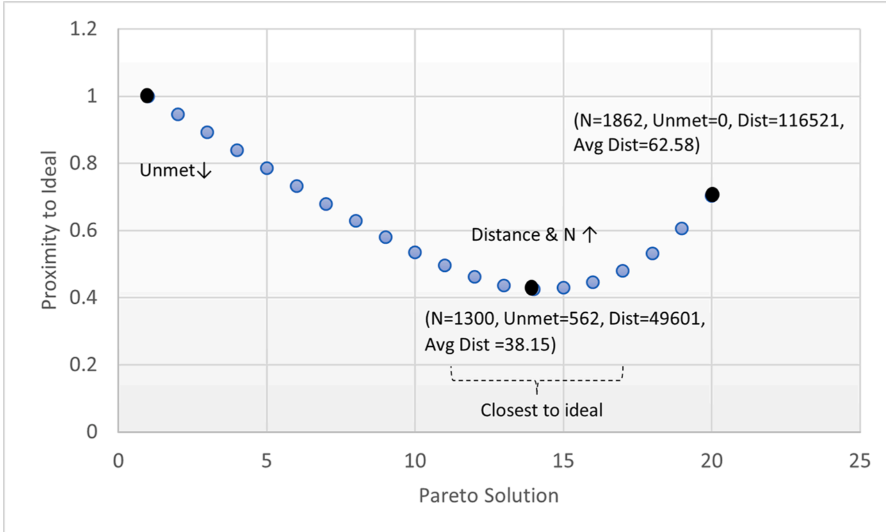

Fig. 4. Proximity to the ideal solution.

is a reduction of over 50% in the distance travelled for a reduction of 30% of the cohort. This also means that 562 patients will not be treated during the four-week period over which this analysis has been performed. This leads us to our next consideration, which is, which 562 patients are not treated? Looking at the solution provided by the model for $\mathbb{N} = 1300$ we find the following:

- Treated (by hospital): $N^{\mathrm{H}} = (835.94, 464.06)$
- Treated (by type): $N^{\mathrm{G}} = (611, 390.65, 112.35, 77, 109)$

  Unmet demand: $\breve{N}^{\mathrm{G}} = (0, 159.35, 192.65, 75, 135)$

- Treated (by sub region): $N^{\mathrm{I}} = (218.35, 274, 207, 212, 255, 133.65)$

  Unmet demand: $\breve{N}^{\mathrm{I}} = (10.65, 157, 227, 0, 31, 136.35)$

- Treated (by subtype): $N^{\mathrm{GP}} = ((428,183),(390.65),(24,60,28.35),(35,42),(109))$

  Unmet demand: $\breve{N}^{\mathrm{GP}} = ((0,0),(159.35),(52,62,78.65),(41,34),(135))$

The demand has not been met equitably. The demands present in regions R2, R3 and R6 for instance have been met least of all, while R1, R4 and R5 have been prioritised. Amongst the different patient types, the inequity varies. Only patient type one is entirely treated. To make the solution more equitable, we need to adjust the number of patients of each type treated, so that the relative number of unmet patients should be the same. The following model alterations are required:

$$\begin{gathered}\text{Minimize}\,\lambda\\ \mathbb{N} = 1300\,\text{and}\,Z_1 = 49601\\ N_g^G \geq \widehat{N}_g^G - \lambda\widehat{N}_g^G \quad \forall g \in G\,\text{or}\,N_i^{\mathrm{I}} \geq \widehat{N}_i^{\mathrm{I}} - \lambda\widehat{N}_i^{\mathrm{I}} \quad \forall\, i \in I\end{gathered} \tag{35}$$

To obtain constraint (35) we define $\frac{\widehat{N}_g^{\mathrm{G}} - N_g^{\mathrm{G}}}{\widehat{N}_g^{\mathrm{G}}} \leq \lambda\ \forall g \in G$ or $\frac{\widehat{N}_i^{\mathrm{I}} - N_i^{\mathrm{I}}}{\widehat{N}_i^{\mathrm{I}}} \leq \lambda\ \forall i \in I$. Numerical investigations demonstrate that if the distance is fixed, there is no way to equitably treat the patients in each region, or the patients of each type. However, removal of the condition $Z_1 = 49601$, does permit a more equitable caseload to be treated. We can obtain the following solutions:

Solution 1: $Z_1 = 85022$ & $N^{\mathrm{H}} = (712.78, 587.22)$. $N^{\mathrm{G}} = (426.58, 384, 212.94, 106.12, 170.35)$

$\breve{N}^{\mathrm{G}} = (184.42, 166, 92.06, 45.88, 73.65)$

Solution 2: $Z_1 = 97305$ & $N^{\mathrm{H}} = (647.28, 652.72)$. $N^{\mathrm{I}} = (159.88, 300.91, 303, 148, 199.68, 188.51)$

$\breve{N}^{\mathrm{I}} = (69.12, 130.09, 130.99, 63.99, 86.32, 81.49)$

The first provides more equity by patient type, and the second more equity by region. It is a harder proposition by region, as some regions are located further away from the hospitals present.

3.4. Case mix planning with outsourcing

In this section we consider a regional scenario where each hospital is independently operated and has their own agenda. Additionally, let us also assume that each hospital has a target caseload that they would like to treat. The primary goal of case mix planning is to identify a caseload for each hospital, which satisfies as many targets as possible. Each hospital may, however, outsource their patients to other hospitals. Hospitals that treat patients from other hospitals are said to insource those patients. Based on the specifics of the situation and the price, a hospital may prefer to treat a patient type they had not originally planned to treat as opposed to one they had planned.

Let us define the caseload of each hospital by decision variables $n_{h,g}^{\mathrm{HG}}$ and $n_{h,g,p}^{\mathrm{HGP}}$. Both hospitals have patient type treatment targets denoted $\widehat{n}_{h,g}^{\mathrm{HG}}$ and $\widehat{n}_{h,g,p}^{\mathrm{HGP}}$, where $\widehat{n}_{h,g}^{\mathrm{HG}} = \sum_{p \in P_g} \widehat{n}_{h,g,p}^{\mathrm{HGP}}$. These may also be used to designate general bounds. We assume that exceeding targets is not permitted, and it makes no sense to permit greater output. If a larger value is permitted, then the target should just be increased. If there is no target, then a large value should be chosen upfront for these parameters. If zero is chosen, then no patients of that type will be treated, and no insourcing will be permitted.

For patient type g let us define $⫛_{h,h',g}^{1}$ as the amount of demand outsourced from hospital h to hospital h'. For patient subtype $gp = (g,p)$, let us also define $⫛_{h,h',gp}^{2}$ as the demand outsourced from h to h'. Let us also denote unmet demand by variables $\breve{n}_{h,g}^{\mathrm{HG}}$ and $\breve{n}_{h,gp}^{\mathrm{HGP}}$. The cost to outsource demand is designated $C_{h,h',g,p}$. This cost may, however, may be specific to the location where demand is transferred from, and in that circumstance, we can define the cost parameter $\acute{C}_{h,g,p}$ and set $C_{h,h',g,p} = \acute{C}_{h,g,p}$. The hospital where demand is transferred to may also earn income from insourcing, and we designate this by $\acute{I}_{h,g,p}$. Hence, we may define $C_{h,h',g,p} = \acute{C}_{h,g,p} - \acute{I}_{h',g,p}$.The model for outsourcing and capacity allocation is as follows:

$$\text{Maximize} Z_1 = \sum_{h \in H} \sum_{g \in G} \omega_{h,g}^{\text{HG}} n_{h,g}^{\text{HG}} \tag{36}$$

$$\begin{aligned} &\text{Minimize } Z_2 = OS = \sum_{\forall h,h' \in H / h \neq h'} \sum_{\forall gp \in GP} \left(\pitchfork_{h,h',gp}^{2} \right) \text{OR} \\ &\text{Minimize } Z_2 = OSC = \sum_{\forall h,h' \in H / h \neq h'} \sum_{g} \sum_{p} \left(C_{h,h',g,p} \pitchfork_{h,h',gp}^{2} \right) \end{aligned} \tag{37}$$

$$\begin{aligned} \text{Minimize } Z_3 = U^{\text{HG}} &= \sum_{h \in H} \sum_{g \in G} \left(\omega_{h,g}^{\text{HG}} \breve{n}_{h,g}^{\text{HG}} \right) \text{ or } Z_3 = U^{\text{HGP}} \\ &= \sum_{h \in H} \sum_{gp \in GP} \omega_{h,gp}^{\text{HGP}} \breve{n}_{h,gp}^{\text{HGP}} \end{aligned} \tag{38}$$

Subject to:
Constraint (1) – (6), (10), (11), (18), (19)

$$\pitchfork_{h,h',g}^{1} = \sum_{p \in P_g} \pitchfork_{h,h',g,p}^{2} \qquad \forall h, h' \in H | h \neq h', g \in G \tag{39}$$

$$\pitchfork_{h,h',gp}^{2} \leq \kappa_{h,h',gp} M \qquad \forall (h,h') \in H^2, g \in G, p \in P_g \tag{40}$$

$$\pitchfork_{h',h,gp}^{2} \leq \left(1 - \kappa_{h,h',gp}\right) M \qquad \forall (h,h') \in H^2, g \in G, p \in P_g \tag{41}$$

$$\pitchfork_{h,h',g}^{1} \geq 0 \qquad \forall h, h' \in H | h \neq h', \; g \in G \tag{42}$$

$$\pitchfork_{h,h',gp}^{2} \geq 0 \qquad \forall h, h' \in H | h \neq h', \; g \in G, p \in P_g \tag{43}$$

$$\kappa_{h,h',gp} \in \{0, 1\} \qquad \forall (h,h') \in H^2, g \in G, p \in P_g \tag{44}$$

$$in_{h,gp}^{\text{HGP}} = \sum_{h' \in H / h' \neq h} \pitchfork_{h',h,gp}^{2}; \; in_{h,g}^{\text{HG}} = \sum_{p \in P_g} in_{h,g,p}^{\text{HGP}} \qquad \forall h \in H, g \in G, p \in P_g \tag{45}$$

$$out_{h,gp}^{\text{HGP}} = \sum_{h' \in H | h' \neq h} \pitchfork_{h,h',gp}^{2}; \; out_{h,g}^{\text{HG}} = \sum_{p \in Pg} out_{h,g,p}^{\text{HGP}} \qquad \forall h \in H, g \in G, p \in P_g \tag{46}$$

$$n_{h,g}^{\text{HG}} \geq in_{h,g}^{\text{HG}}, \quad n_{h,g,p}^{\text{HGP}} \geq in_{h,g,p}^{\text{HGP}} \qquad \forall h \in H, g \in G, p \in P_g \tag{47}$$

$$out_{h,g}^{\text{HG}} \leq \widehat{n}_{h,g}^{\text{HG}}; \quad out_{h,g,p}^{\text{HGP}} \leq \widehat{n}_{h,g,p}^{\text{HGP}} \qquad \forall h \in H, g \in G, p \in P_g \tag{48}$$

$$\dddot{n}_{h,g}^{\text{HG}} = \widehat{n}_{h,g}^{\text{HG}} - met_{h,g}^{\text{HG}}; \quad met_{h,g}^{\text{HG}} = n_{h,g}^{\text{HG}} - in_{h,g}^{\text{HG}} + out_{h,g}^{\text{HG}} \qquad \forall h \in H, \forall g \in G \tag{49}$$

$$\dddot{n}_{h,gp}^{\text{HGP}} = \widehat{n}_{h,gp}^{\text{HGP}} - met_{h,gp}^{\text{HGP}}; \; met_{h,gp}^{\text{HGP}} = n_{h,gp}^{\text{HGP}} - in_{h,gp}^{\text{HGP}} + out_{h,gp}^{\text{HGP}} \qquad \forall h \in H, g \in G, p \in P_g \tag{50}$$

$$\dddot{n}_{h,g}^{\text{HG}} \geq 0, \quad n_{h,g}^{\text{HG}} \geq 0 \qquad \forall h \in H, \forall g \in G \tag{51}$$

$$\dddot{n}_{h,gp}^{\text{HGP}} \geq 0, \; n_{h,gp}^{\text{HGP}} \geq 0 \qquad \forall h \in H, g \in G, p \in P_g \tag{52}$$

$$in_{h,g}^{\text{HG}} \leq \sum_{\forall h' \in H | h' \neq h} \widehat{n}_{h',g}^{\text{HG}}; in_{h,g,p}^{\text{HGP}} \leq \sum_{\forall h' \in H | h' \neq h} \widehat{n}_{h',gp}^{\text{HGP}} \qquad \forall h \in H, g \in G, p \in P_g \tag{53}$$

In this model, equation (39) aggregates the outsourcing across all patient subtypes. Constraints (40) and (41) are required to restrict redundant outsourcing. It is not permitted to outsource and simultaneously insource the same patient type. As such we must enforce that $\pitchfork_{h',h,g,p}^{2} \pitchfork_{h,h',g,p}^{2} = 0 \; \forall (h,h') \in H^2, g \in G, p \in P_g$ where $H^2 = \{(h,h') | h,h' \in H, h < h'\}$. To linearise that condition it is necessary to define a binary decision variable $\kappa_{h,h',g,p}$. If $\kappa_{h,h',g,p} = 1$ then outsourcing is permitted from h to h'. Otherwise, if $\kappa_{h,h',g,p} = 0$ then insourcing is permitted from h' to h, i.e., outsourcing is permitted from h' to h. Constraints (42) and (43) ensure positive outsourcing, while (44) defines the bounds for the binary variable $\kappa_{h,h',gp}$. Equation (45) computes the number of insourced patients added to the hospital's caseload while equation (46) computes the number of outsourced patients that are treated elsewhere. Constraint (47) forces the number of patients treated in a hospital to exceed the obligation to treat patients from other hospitals. Constraint (48) ensures that outsourcing is performed for a reason and the level cannot exceed the original target that has been designated. Constraints (49) and (50) compute the met and unmet demand. Constraints (51) and (52) force unmet demand to be positive. Constraint (53) provides a general bound on the total amount of insourcing that could be achieved.

There are three objectives of interest. Naturally, the first, denoted Z_1, is to maximize the number of weighted patients treated across the region. If $\omega_{h,g}^{\text{HG}} = 1 \; \forall h \in H, \; g \in G$ then $Z_1 = \mathbb{N}$. The second objective, denoted Z_2, is to minimize the outsourcing required and/or the outsourcing costs. The third objective, denoted Z_3, is to meet the treatment targets as best possible, by minimizing either of the unmet demands, namely $\check{n}_{h,g}^{\text{HG}}$ and $\check{n}_{h,gp}^{\text{HGP}}$. Minimizing Z_2, however, does not make sense on its own. The model may just zero the patients treated, and no outsourcing will be performed. Or else, the model will just choose the maximum number of patients each hospital has capacity to treat. The targets specified will be ignored completely. If Z_3 is minimised, then inefficient outsourcing may be chosen. If it is known that a solution with $Z_3 = 0$ can be obtained (i.e., all target demands can be met) then we can minimize the outsourcing in a second step. The model will identify if outsourcing is required. In some circumstances it may not be necessary but in others it may be vital. If there are no specific outsourcing costs, then minimising unmet demand has a higher importance. We may set $c_{h,h',g,p} = 1$ and minimize $Z_3 + 1E^{-5} Z_2$. That strategy minimizes both Z_2 and Z_3 quite well in limited numerical testing.

3.4.1. *Final remarks*

The described outsourcing model does not equitably meet the targets that are defined, and some patient types will inevitably be favoured. The use of priority weightings $\omega_{h,g}^{\text{HG}}$ and $\omega_{h,g,p}^{\text{HGP}}$ may however overcome the inequity. How those values should be defined, however, is a practical consideration. Additional restrictions may be imposed to restrict the outsourcing of certain patient types and subtypes. It is unlikely that there would be flexibility to consider unrestricted outsourcing and it is anticipated that most real-world scenarios would consider only a few. It is a simple matter to add the necessary restrictions to the model. The model could be used to make "high-level" strategic decisions around outsourcing. For that analysis we need only assume one patient type per hospital, with a length of stay and treatment time defined as the average across all patient types treated.

3.4.2. *Demonstrative example*

A two-hospital case mix planning scenario is considered to demonstrate the model. The patient types and subtypes from Table 2 have again been adopted. Table 8 summarises the resources present in each hospital and Table 10 introduces a set of target demands for each patient type, subtype, and hospital. The demands are noticeably greater in the first hospital than the second. We take into consideration outsourcing possibilities but introduce no outsourcing costs (Table 9).

The solution of the model was first investigated without any re-

Table 8
Solution with minimal distance.

Hospital	Region 1	2	3	4	5	6
H1	143.09	197	207.11	2.15	41.66	154
H2	85.91	234	226.89	209.85	244.34	116

Table 9
Hospital configuration.

Hosp.	#OT	#ICU	#WARD	W1	W2	W3	W4	W5
H1	10	5	74	22	15	10	14	13
H2	6	2	42	15	20	7	-	-
Total	16	7	116	37	35	17	14	13

striction on outsourcing. This means that our objective was $Z_3 = U^{HG}$ (i.e., total unmet demand). Targets were designated for individual patient types, and those values are the aggregate of the different subtype targets. The results are shown in Table 11. Associated with this solution we have, $\mathbb{N} = 4749.9$, $U^{HG} = 3068.1, OS = 779$. Table 11 shows that many targets cannot be met and there is also a lot of outsourcing to H2.

The solution of the model with objective $Z_3 = U^{HGP}$ was also investigated. The results are summarised in Table 12, and match those shown in Table 11. To check whether a superior solution with less outsourcing is achievable, the model was resolved with OS minimization as the objective. The condition $U^{HGP} = 3068.1$ was however enforced. An OS of 750.37 is possible and is achieved by some subtle changes to the solution shown in Table 12. It is worth noting that with no insourcing and outsourcing permitted, we have $\mathbb{N} = 4278.8$, $U^{HGP} = 3539.2$. In contrast, fewer patients (i.e., 471.1) are treated overall), and the unmet demand is higher. The benefits of collaboration between the hospitals are clearly demonstrated here.

3.4.3. *Pareto analysis – without outsourcing*

Solution of the model demonstrates that $\mathbb{N} \in [0, 4278.84]$ and $U^{HGP} \in [3539.16, 7818]$ and the ideal solution is $\mathbb{N} = 7818$ with $U^{HGP} = 0$. Hence, at best, 54.73% of the targets can be met. There is a linear relationship between the total number of patients treated and the total unmet targets, such that $U^{HGP}(\mathbb{N}) = 7818 - \mathbb{N}$. As more patients are treated, more of the targets are met.

3.4.4. *Pareto analysis – with outsourcing*

When outsourcing is included, a third dimension is added to the objective space. For each feasible $(\mathbb{N}, U^{HGP})$ pairing there may be as we have shown, alternative solutions involving outsourcing. Solution of the model demonstrates that $\mathbb{N} \in [0, 4749.932]$, $U^{HGP} \in [3068.07, 7818]$ and $OS \in [0, 1156.51]$. To fully understand the objective space, a multi-criteria analysis can be performed. For the current example, the set of Pareto optimal solutions are shown in Fig. 5.

To get those solutions, we discretised the U^{HGP} and OS domain into $\mho = 50$ points and created a 2D grid. We then solved the model with epsilon-constraints $U^{HGP} \le \epsilon_i^U$ and $OS \le \epsilon_j^{OS}$ where $\epsilon_i^U = 3068.068 + i\Delta^U$, $\epsilon_j^{OS} = j\Delta^{OS}$, $\Delta^U = \frac{7818-3068.068}{\mho-1}$ and $\Delta^{OS} = \frac{1156.511}{\mho-1}$.

Fig. 5 shows that as the outsourcing is increased, more targets are met, and a higher number of patients are treatable. The frontier tapers off once a certain level of outsourcing is reached, and the extra benefit that occurs is marginal. This might occur if outsourcing encroaches upon a hospitals capacity to service their own catchment. The ideal solution is ($\mathbb{N} = 4749.932$, $U^{HGP} = 3068.07$, $OS = 0$) and the closest solutions are (4663.57, 3154.43, 424.84), (4646.29, 3171.71, 401.24), (4680.84, 3137.16, 448.44). The closest solutions to the ideal are in dark blue, and the ones furthest away are red. The Pareto frontier is not a surface but a line. The rest of the objective space is shown in Fig. 6. Those solutions are coloured by their proximity to the ideal.

4. Regional case study

To demonstrate the mathematical models and the associated methodologies, we considered the Brisbane (BN) and Gold Coast (GC) regions in Queensland, Australia. To solve the mathematical models, we have applied the IBM ILOG CPLEX software. It is worth noting upfront that no runtimes are shown as the models solved instantly. The BN and GC regions are adjacent and have populations of approximately 2.4 and 0.7 million people, respectively. The Gold Coast is directly south of Brisbane. In these regions, there are 20-30 hospitals. A summary of the largest hospitals and their facilities (a.k.a., assets) is shown in Table 13. Fourteen are in BN and five are in the GC. In addition, ten are public and nine are private.

4.1. *Caveats*

An attempt was made to identify the main facilities of each of the hospitals in Table 13, and their function. Due to a lack of information, the GRN, MA, and MP hospitals were fully omitted from further consideration. As the QCH is a children's hospital, and does not treat adults, it was also omitted. As the only children's hospital in the region providing paediatric care, it may be analysed independently. The WES hospital has been included, but only partial information has been collected.

The main facilities of each of those hospitals was modelled. Some other specialist facilities/clinics, units, and services, however, are not included. For instance, we do not include emergency departments,

Table 11
Allocation solution for objective $Z_3 = U^{HG}$.

Variable	h	g=1	2	3	4	5	TOT
Target ($\hat{n}_{h,g}^{HG}$)	H1	2000	1500	1000	500	800	5800
# Treated ($n_{h,g}^{HG}$)	H1	1770	578.13	0	0	429	2777.1
# Insourced ($in_{h,g}^{HG}$)	H1	0	0	0	0	29	29
# Outsourced ($out_{h,g}^{HG}$)	H1	230	200	120	200	0	750
Unmet Targets ($\tilde{n}_{h,g}^{HG}$)	H1	0	721.88	880	300	400	2301.9
Met Targets ($met_{h,g}^{HG}$)	H1	2000	778.13	120	200	400	3498.1
Target ($\hat{n}_{h,g}^{HG}$)	H2	271	322	160	1176	89	2018
# Treated ($n_{h,g}^{HG}$)	H2	501	522	266.39	683.41	0	1972.81
# Insourced ($in_{h,g}^{HG}$)	H2	230	200	120	200	0	750
# Outsourced ($out_{h,g}^{HG}$)	H2	0	0	0	0	29	29
Unmet Targets ($\breve{n}_{h,g}^{HG}$)	H2	0	0	13.61	692.59	60	766.2
Met Targets ($met_{h,g}^{HG}$)	H2	271	322	146.39	483.41	29	1251.81

Table 10
Hospital targets.

Hosp.	Type 1			Type 2		Type 3				Type 4			Type 5		Total
Subtype	1	2	3	1	2	1	2	3	4	1	2	3	1	2	
H1	1000	1000	0	1000	500	250	650	100	0	200	300	0	400	400	5800
H2	200	30	41	200	122	40	40	40	40	258	678	240	60	29	2018
Total	1200	1030	41	1200	622	290	690	140	40	458	978	240	460	429	7818

Table 12
Allocation solution when $Z_3 = U^{\text{HGP}}$.

Variable		g=1			g=2		g=3				g=4			g=5		TOT
	$p = 1$	2	3	1	2	1	2	3	4	1	2	3	1	2		
$\widehat{n}^{HG}_{h,g}$	H1	1000	1000	0	1000	500	250	650	100	0	200	300	0	400	400	5800
$n^{HG}_{h,g}$	H1	800	970	0	78.13	500	0	0	0	0	0	0	0	0	429	2777.13
$in^{HG}_{h,g}$	H1	0	0	0	0	0	0	0	0	0	0	0	0	0	29	29
$out^{HG}_{h,g}$	H1	200	30	0	200	0	40	40	40	0	200	0	0	0	0	750
$\breve{n}^{HG}_{h,g}$	H1	0	0	0	721.88	0	210	610	60	0	0	300	0	400	0	2301.88
$met^{HG}_{h,g}$	H1	1000	1000	0	278.13	500	40	40	40	0	200	0	0	0	400	3498.13
$\widehat{n}^{HG}_{h,g}$	H2	200	30	41	200	122	40	40	40	40	258	678	240	60	29	2018
$n^{HG}_{h,g}$	H2	400	60	41	400	122	80	80	66.39	40	443.4	0	240	0	0	1972.81
$in^{HG}_{h,g}$	H2	200	30	0	200	0	40	40	40	0	200	0	0	0	0	750
$out^{HG}_{h,g}$	H2	0	0	0	0	0	0	0	0	0	0	0	0	0	29	29
$\breve{n}^{HG}_{h,g}$	H2	0	0	0	0	0	0	0	13.61	0	14.59	678	0	60	0	766.19
$met^{HG}_{h,g}$	H2	200	30	41	200	122	40	40	26.39	40	243.4	0	240	0	29	1251.81

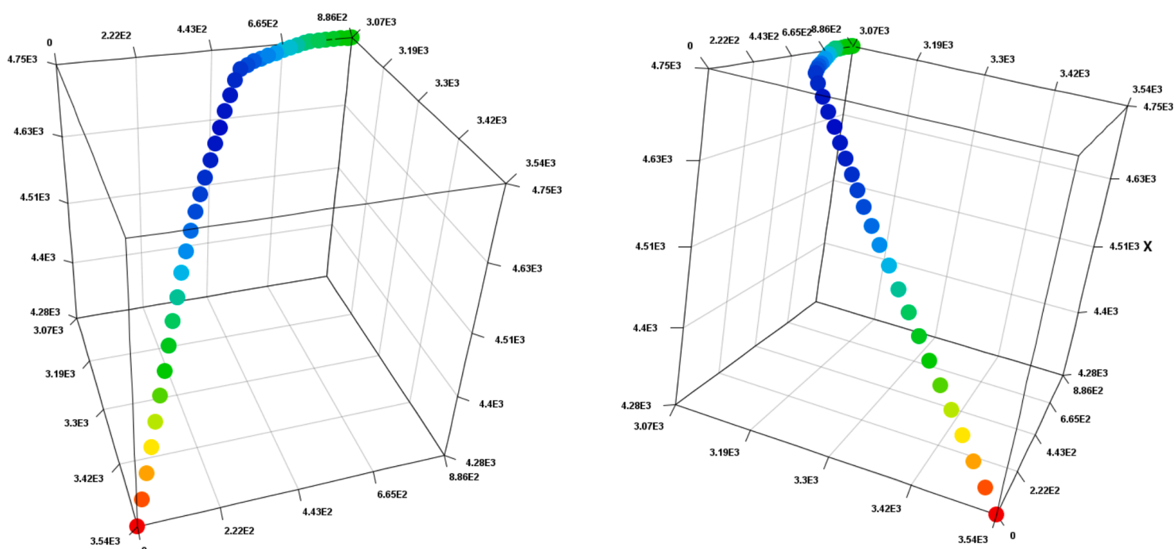

Fig. 5. Pareto optimal solutions coloured by proximity to the ideal.

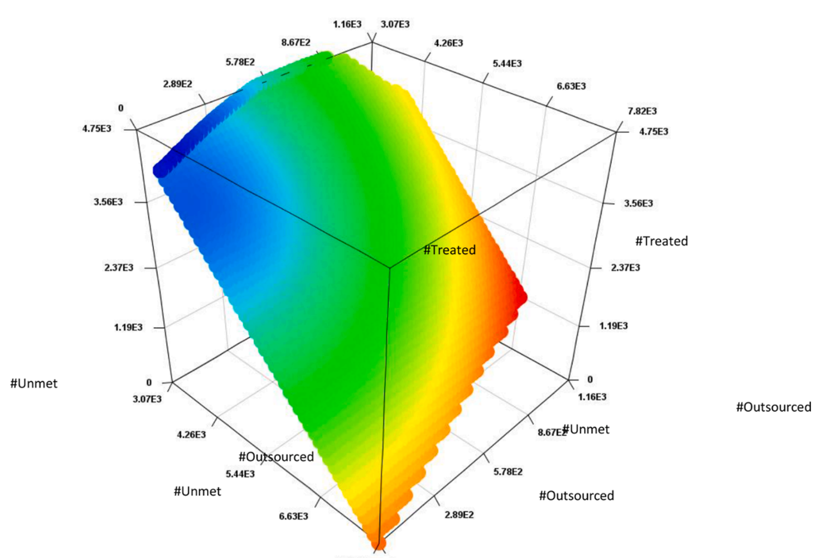

Fig. 6. Objective space coloured by proximity to the ideal.

Table 13
Hospital information.

Hospital	Abbrev.	Region	#Beds (Quoted Online)	Identified			
				#OT	# ICU Beds	# Ward Beds	#Wards
Brisbane (Private)	BPH	BN	181	4	5	181	7
Gold Coast University (Public| Tertiary)	GCUH	GC	750	20	21	630	21
Gold Coast (Private)	GCPR	GC	314	21	6	287	9
Greenslopes (Private| Tertiary)	GRN	BN	631	23	18	462	16
John Flynn (Private)	JFN	GC	354	12	5	330	11
Logan (Public| Major Centre)	LOG	BN	485	10	8	461	15
Mater Adult (Public| Tertiary)	MA	BN	343	20	30	343	-
Mater (Private)	MP	BN	323	10	16	323	-
North-West (Private)	NW	BN	150	7	6	150	5
Princess Alexandra (Public| Tertiary)	PA	BN	1058	19	25	786	29
Pindarra (Private)	PIN	GC	348	19	9	320	11
Prince Charles (Public| Tertiary)	PRCH	BN	657	20	27	546	17
Qld Childrens (Public)	QCH	BN	359	14	36	361	21
Queen Elizabeth 2 Jubilee (Public| Medium)	QE2	BN	239	10	5	244	10
Royal Brisbane Womens (Public| Tertiary)	RBWH	BN	966	22	36	660	22
Redland (Public| Major Centre)	RED	BN	183	5	0	180	6
Robina (Public)	ROB	GC	403	10	10	272	9
St Andrews (Private)	STAN	BN	250	15	15	234	9
Wesley (Private| Tertiary)	WES	BN	535	19	28	532	19
			8676	280	306	6374	227

medical assessment and planning units, rehabilitation units, and palliative care. It is worth noting, that many of these hospitals are currently being expanded and will have additional beds added in the months and years ahead. Our data is representative of reality now and is sufficiently accurate for the purposes of demonstrating the proposed methods. The details of the considered hospitals are freely accessible online, for instance in hospital websites, online reports, media releases, Facebook, and LinkedIn. Some of this information, however, is unsubstantiated and ambiguous, and some of it is inexact or just erroneousness. The biggest uncertainty in our data is the exact number of beds in each specialty ward. Typically, these range from 20 - 40 beds. This information is rarely provided. As a compromise we assume 30 beds are present in any ward that we have no exact information. The second biggest uncertainty is the specialty of the ward, and the type of patients that can be placed therein. Some wards are described as general medical or general surgical, and this is open to interpretation, varying across hospitals.

4.2. Determining a maximal caseload

The hospitals in the BN and GC region were first assessed using Model 1. To apply that model, it was necessary to assemble a list of patient types and associated medical and surgical activities needed in their care. That information was provided by one of the largest public hospitals in the region. This hospital has many specialists, and so captures a degree of variation between different specialists performing similar tasks. It was not possible or practical to collect confidential data from each hospital in the region. Also, there is insufficient evidence to suggest that expected surgery times and lengths of stay will vary across different hospitals.

Of the many available alternatives, it was decided that a patient characterisation based upon specialty was most reasonable. The specialty treating the patient is often the most dominant describer of the patient's condition and care pathway. Within each group there are various patient sub types. We looked at two patient sub type characterisations. The first being surgical or medical. The second was by Diagnostic Resource Group (DRG). In Table 14 the number of DRG within each specialty and the associated sub mix is provided.

For our assessment, the regional case mix and surgical to medical sub mix shown in Table 15 was defined. These percentages were selected according to information found reported on a government web site. The "UB" column describes the maximum number of patients that can be treated across all hospitals, if specialties do not compete for resources.

For the first patient sub type characterisation, we computed an average surgical and medical inpatient. Each DRG code has a "sur" or "med" descriptor, and this made it easy to partition the data into the two subtypes. We then proceeded to average the length of stay and treatment time data to complete our resource consumption profiles. All patient subtypes spend time in operating theatres, intensive care units, and medical/surgical wards, and treatment numbers are restricted by the number of assets present. Outpatients of both medical and surgical persuasions were not defined as insufficient data was available. Those patients are rarely restricted by the facilities, but the specialist. They are most often seen/treated in consultation rooms, and do not use the main facilities of the hospital.

Model 1 was then solved for a 12-week period (i.e., a single quarter) and the results are summarised in Tables 16 and 18. The maximum number of patients that are treatable is 53762 and Table 16 shows the optimal way to assign those patient types to individual hospitals. It is worth noting that there are no hospitals that treat every patient type.

Following the afore-said assessment, the full set of DRG sub types were instated and the DRG sub mix shown in Table 14 was instated. The model was then reapplied. The main results are shown in Table 17-20. The caseload partitioned by DRG is shown specifically in Table 20. In summary, the model was able to find a caseload obeying the case mix and sub mix constraints totalling 72053 patients, an increase of 18291 patients. More patient types are also treated in each hospital. Evidently, the averaging of activity durations to create fewer patient sub types is an inferior approach, but in practice warranted if insufficient fine-grained information is available. There is clear evidence of how different the resource consumption profiles are to the profile of an average sub patient type. Many profiles have no surgery, or no icu stay, and minimal ward time. By creating an average patient type, time in all three areas is forced. That means, the model can not make use of the hospital resources as well, and that is why less output was originally obtained for the sur-med sub type characterisation.

Table 14
DRG sub types and sub mix within each specialty group, partitioned by surgical and medical.

GROUP	#DRG	DRG MIX (%)
CARD	21	[0.19,0.06,0.51,0.39,0.74,0.72,0.52,0.37,0.6,0.09,0.17,0.01,0.63, 4.54,15.02,5.21,15.59,8.62,23.33,19.23,3.46]
ENDO	16	[8.91,9,4.22,11.09,3.2,3.66,3.67,11.69,6.54,5.78, 7.9,7.11,7.56,3.93,1.82,3.92]
ENT	15	[18.3,3.02,21.24,13.07,15.41,6.15,5.25,12.56, 0.31,0.04,0.96,0.87,1.7,0.81,0.31]
FMAX	2	[95, 5]
GAST	19	[2.26,1.58,0.62,1.87,0.96,2.03,1.88,1.55,2.26, 3.64,1.94,6.68,13.46,14.48,3.3,11.78,8.59,5.55,15.57]
GYN	10	[0.34,0.84,0.25,0.37,0.8,1.12,1.28, 51.37,10.36,33.27]
HEPA	11	[5.82,5.03,14.61,15.58,7.27,1.69,0.47,0.01,18.4,10.28,20.84,
IMMU	7	[5, 21.1,4.81,18.99,10.23,15.76,24.11]
NEPH	20	[1.7,0.99,0.18,0.85,0.25,0.26,0.73,0.04, 12.2,11.04,11.05,0.36,14.22,2.4,6.28,4.13,2.28,12.27,15.65,3.12]
NEUR	26	[6.12,4.06,5.85,3.73,5.39,1.39,0.46,3.8, 3.59,2.99,1.2,4.18,6.24,3.72,5.64,4.18,6.57,0.6,6.04,6.11,3.52,1.33,0.4,6.31,2.86,3.72]
OBS	2	[20, 80],
ONC	8	[1.44,1.8,1.42,0.34, 4.87,51.39,30.38,8.36]
OPHT	16	[9.34,11.65,8.71,6.68,12.96,1.35,6.28,9.62,8.58,7.1,7.6,5.13, 1.41,1.25,0.7,1.64]
ORTH	49	[4.75,0,0.73,0.59,2.15,3.9,4.36,2.82,4.9,0.21,0.7,4.71,4.59,2,3.86,4.13,2.3,4.99,3.81,2.97,4.5,4.33,3.55,3.71,2.14, 4.94,3.59,5.2,4.57, 0.04,0.19,0.34,0.4,0.16,0.36,0.49,0.22,0.26,0.36,0.19,0.22,0.01,0.06,0.31,0.25,0.28,0.25,0.32,0.29]
PLAS	18	[7.09,3.92,13.59,11.54,4.47,10.08,12,13.59,14.32,4.4, 0.46,1.2,0.12,1.31,0.19,1.29,0.02,0.41]
PSY	10	[0, 4.55,3.58,8.7,15.42,15.21,14.58,5.24,0.93,20.18,11.61]
RESP	20	[4.31,0.69, 4.24,6.74,8.06,0.39,8.21,1.93,3.58,8.44,2.98,7.2,1.98,6.24,6.1,8.31,6.17,4.03,5.41,4.98]
TRANS	1	[100, 0]
UROL	11	[9.58,30.57,17.47,5.06,4.82,27.5, 1.36,0.59,1.1,0.97,0.98]
VASC	17	[25.4,15.72,11.1,4.98,7.43,30.36, 0.3,0.77,0.85,0.19,0.86,0.48,0.31,0.08,0.26,0.62,0.29]
	301	

Table 15
Regional case mix, sub mix and upper bounds.

TYPE	CASE MIX (%)	SUB MIX (%)		UB	TYPE	CASE MIX (%)	SUB MIX (%)		UB
SUR	MED	SUR	MED						
Cardiology (CARD)	6.95	5	95	8984	**Obstetrics (OBS)**	5.17	20	80	186270
Endocrinology (END)	1.2	62	38	16134	**Oncology (ONC)**	6.38	5	95	13131
Ear, Nose, Throat (ENT)	2.65	95	5	31151	**Ophthalmology (OPHT)**	4.94	95	5	29841
Facio-Maxillary (FMAX)	0.01	95	5	10283	**Orthopaedics (ORTH)**	8.5	95	5	10291
Gastroenterology (GAST)	16.73	15	85	17173	**Plastic (PLAS)**	3.1	95	5	9543
Gynaecology (GYN)	4.3	5	95	40707	**Mental Health (PSY)**	5.25	0	100	7858
Hepatology (HEP)	0.01	50	50	16157	**Respiratory (RESP)**	5.28	5	95	14107
Immunology (IMMU)	0.77	5	95	5338	**Transplants (TRANS)**	0.02	100	0	159.386
Nephrology	17.68	5	95	35780	**Urology (UROL)**	4.96	95	5	20399
Neurology	5.07	27	73	14036	**Vascular (VASC)**	1.03	95	5	4824

4.3. Incorporating spatially distributed patients

In this section, spatial demographics and proximity information are incorporated. A fictional “demand” scenario was created for demonstrative purposes as information about individual patients and their location is not identifiable without significant help from government bodies. We have however used real region information and real patient treatment data. As such our case study can be viewed as pseudo-real. Model 2 was then applied to see if specified demands can be met, and if so, how. The model assigns demand originating in different sub regions to the hospitals within the region. The trade-offs between patients treated and distance travelled is also explored.

The sub regions in our case study were extracted from the Australian Bureau of Statistics (ABS) at https://dbr.abs.gov.au/. We chose to use Statistical Area Level 3 (SA3) regions. These geographical areas have between 30 and 130 thousand people and have a distinct identity and similar social and economic characteristics. They often represent the area serviced by a major transport and commercial hub, and closely align with large urban local government areas. In total there are 48 SA3 sub regions within the BN and GC region. The longitude and latitude of the centroid of each SA3 sub region was also identified and the distances between all pairs of centroids was computed (i.e., using the Haversine formula).

The patient subtype demand in each sub region was chosen randomly using the solution from Section 4.1 as a starting point. Hence all demands sum to 53762.1 patients. As there are 48 SA3 regions and 38 patient subtypes, a total of 1824 demand values were selected. As the demands are the outputs from the first model, we can be assured that the second model is feasible and will produce a solution. The demands in each region are summarised in Fig. 7.

For our analysis, the second model was repeatedly solved as in Section 3.3 and the trade-off between distance and number of patients treated was obtained. The results are shown in Fig. 8.

That chart shows significant difference between the best and worst cases. For any patient cohort, there are many alternative ways to allocate patients to hospitals. As more regional capacity is utilized, the distance required to be travelled increases moderately to start with, and more steeply later. The average distance required per patient also increases. At full capacity utilization, the average is at best, about 15km per patient. At worst, however, it is about 60km. The exact difference between the best and worst solution, is quite transparent. The wort solution is obtained by treating BN based patients in the GC and vice versa.

Table 21 reports for each region, the hospitals assigned to treat the patients originating in that region. A similar table could also be shown, that describes the list of regions that each hospital treats. In Table 21, a single star (*) highlights a required travel distance of 30km and more, whereas a double star (**) distances of 50km or more. Nine of the 48 regions could be serviced (i.e., fully) by local hospitals. These are shown in bold text. All other regions needed to send some of their patients further afield. Table 22 and Table 23 describes the full extent of the travel needed. Table 22 describes the specific number of patients that are treated in each hospital, and Table 23 the total distance to be travelled. Upon closer scrutiny, these tables show patients are predominantly treated locally. A much smaller number are sent further afield. The

Table 16
The number of patients treated by hospital, specialty, and subtype [SUR-MED SUB TYPES].

HOSPITAL	# PATIENTS	# TYPES TREATED		SPECIALTY	# PATIENTS	#SUR	#MED
BPH	1662.34	5		CARD	3736.46	186.82	3549.64
GCUH	7651.73	10		ENDO	645.14	399.99	245.16
GCPR	1791.69	3		ENT	1424.69	1353.46	71.23
JFN	3035.17	6		FMAX	5.38	5.11	0.27
LOG	2517.13	4		GAST	8994.40	1349.16	7645.24
NW	1232.90	4		GYN	2311.77	115.59	2196.18
PA	7230.64	16		HEPA	5.38	2.69	2.69
PIN	3078.57	6		IMMU	413.97	20.70	393.27
PRCH	2926.51	4		NEPH	9505.13	475.26	9029.88
QE2	2095.74	4		NEUR	2725.74	735.95	1989.79
RBWH	14010.06	11		OBS	2779.50	555.90	2223.60
RED	132.79	1		ONC	3430.02	171.50	3258.52
ROB	3538.55	4		OPHT	2655.85	2523.05	132.79
STAN	2556.89	2		ORTH	4569.78	4341.29	228.49
WES	301.38	1		PLAS	1666.62	1583.29	83.33
	53762.08			PSY	2822.51	2822.51	
				RESP	2838.64	141.93	2696.71
				TRANS	10.75	10.75	
				UROL	2666.60	2533.27	133.33
				VASC	553.75	526.06	27.69
					53762.08	19854.28	33907.79

Table 17
The number of patients treated by hospital, specialty, and subtype [DRG SUB TYPES].

HOSPITAL	# PATIENTS	# TYPES TREATED		SPECIALTY	# PATIENTS	#SUR	#MED
BPH	2192.97	9		CARD	5007.66	250.39	4757.27
GCUH	7338.72	14		ENDO	864.63	535.92	328.75
GCPR	3844.52	13		ENT	1909.39	1813.93	95.47
JFN	5346.92	10		FMAX	7.21	6.85	0.36
LOG	5984.01	13		GAST	12054.41	1809.36	10245.04
NW	4821.11	9		GYN	3098.26	154.92	2943.35
PA	6253.68	15		HEPA	7.21	3.59	3.60
PIN	5877.78	14		IMMU	554.81	27.74	527.07
PRCH	6890.14	12		NEPH	12738.91	636.95	12101.96
QE2	6248.41	0		NEUR	3653.07	986.32	2666.76
RBWH	10010.43	9		OBS	3725.12	745.02	2980.10
RED	0.00	15		ONC	4596.96	229.86	4367.12
ROB	2971.99	0		OPHT	3559.40	3381.43	177.97
STAN	3836.51	11		ORTH	6124.47	5818.23	306.22
WES	435.43	8		PLAS	2233.63	2121.95	111.67
	72052.62			PSY	3782.76	0.00	3782.77
				RESP	3804.38	190.22	3614.18
				TRANS	14.41	14.41	0.00
				UROL	3573.81	3409.52	178.69
				VASC	742.14	704.95	37.17
					72052.64	22841.56	49225.52

patients originating in areas, BEAU, CAB, BRIB, JIM, IPS and NAR must travel the most to obtain care. This makes sense as these areas are all on the boundary of the Brisbane region.

4.3.1. Last remarks

It is likely that patients have a preference and/or requirement for care in a particular type of hospital. For instance, in a public or private

Table 18
The number of patients treated in each hospital of each type [SUR-MED SUB TYPES].

	CARD	END	ENT	FMAX	GAST	GYN	HEPA	INFD	NEPH	NEUR	OBS	ONC	OPHT	ORTH	PLAS	PSY	RESP	TRANS	UROL	VASC	
BPH	0	0	423	0	0	0	0	0	0	182	0	0	745	227	0	85.3	0	0	0	0	1662.34
GCUH	40.4	0	0	0	1789	2240	0	20.7	1194	491	0	351	0	0	0	0	444	0	844	238	7651.74
GCPR	0	0	0	0	596	0	0	0	0	0	0	351	0	0	0	0	0	0	844	0	1791.69
GRN	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
JFN	0	314	0	0	1186	0	0	0	0	0	0	724	0	0	493	0	169	0	0	149	3035.16
LOG	0	0	0	0	0	0	0	0	0	0	0	335	0	1.22	768	1413	0	0	0	0	2517.13
NW	0	0	0	0	596	0	0	0	0	0	0	172	0	143	322	0	0	0	0	0	1232.9
PA	0	0	71.2	5.11	3660	71.9	5.38	393	592	301	0	351	0	516	83.3	234	370	10.8	401	166	7230.64
PIN	0	0	0	0	651	0	0	0	0	241	556	679	0	810	0	0	142	0	0	0	3078.57
PRCH	187	0	0	0	0	0	0	0	0	0	0	0	0	552	0	857	1331	0	0	0	2926.51
QE2	0	0	0	0	517	0	0	0	0	477	0	0	0	718	0	0	384	0	0	0	2095.74
RBWH	1910	245	930	0.27	0	0	0	0	4776	733	2224	468	1778	368	0	0	0	0	577	0	14010.1
RED	0	0	0	0	0	0	0	0	0	0	0	0	133	0	0	0	0	0	0	0	132.79
ROB	0	85.7	0	0	0	0	0	0	2943	0	0	0	0	276	0	234	0	0	0	0	3538.56
STAN	1599	0	0	0	0	0	0	0	0	0	0	0	0	957	0	0	0	0	0	0	2556.89
WES	0	0	0	0	0	0	0	0	0	301	0	0	0	0	0	0	0	0	0	0	301.38
	3736	645	1425	5.38	8994	2312	5.38	414	9505	2726	2780	3430	2656	4570	1667	2823	2839	10.8	2667	554	53762.1

Table 19
The number of patients treated in each hospital of each type [DRG SUB TYPES].

	CARD	ENDO	ENT	FMAX	GAST	GYN	HEPA	IMMU	NEPH	NEUR	OBS	ONC	OPHT	ORTH	PLAS	PSY	RESP	TRANS	UROL	VASC	
BPH	0	158	100	6.85	0	321	0	0	0	757	0	0	0	177	268	228	0	0	177	0	2193
GCUH	963	36.5	0	0	1193	0	0.42	0	2046	538	745	15.6	50.2	490	177	0	469	0	611	2.23	7338.8
GCPR	534	49.7	32.5	0	1420	0	0.52	57.2	31.8	81.4	0	0	48.1	264	0	0	371	0	950	3.56	3844.5
GRN	412	0	0	0	1340	0	0	0	306	50.8	0	2362	268	253	29.3	0	153	0	172	0	5346.9
JFN	245	0	0	0	1036	11.5	0	134	1431	408	0	0	24.9	956	7.96	782	312	0	624	12.6	5984.1
LOG	0	31.6	0	0	0	32.5	1.24	0	894	0	2980	66.2	0	453	344	0	0	0	0	18.8	4821.1
NW	463	95.9	350	0	300	10.5	1.9	0	0	103	0	0	685	889	371	1620	1100	14.4	60.4	191	6253.7
PA	829	50	73.9	0	1391	1031	0	0	324	221	0	0	305	651	98.3	63.1	435	0	181	225	5877.8
PIN	1545	0	123	0	2122	0	1.33	143	1562	272	0	0	58.4	0	487	48.1	347	0	0	180	6890.2
PRCH	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
QE2	15.9	0	513	0	460	60.7	0	0	3400	0	0	0	1114	306	2.68	376	0	0	0	0	6248.4
RBWH	0	350	16.6	0	2606	0	1.79	133	2126	769	0	608	673	600	129	552	541	0	797	107	10010
RED	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ROB	0	33.9	0	0.36	187	0	0	87.4	397	0	0	1462	0	293	320	114	75.3	0	0	1.93	2972
STAN	0	59.4	700	0	0	1631	0	0	222	16.8	0	82.7	332	792	0	0	0	0	0	0	3836.5
WES	0	0	0	0	0	0	0	0	0	435	0	0	0	0	0	0	0	0	0	0	435.43
	5008	865	1909	7.21	#####	3098	7.21	555	#####	3653	3725	4597	3559	6124	2234	3783	3804	14.4	3574	742	72053

hospital. Patients also see differences between hospitals of the same type, even if both hospitals provide the same services and treat the same types of conditions. The current model ignores those requirements and sees no differences. For the purposes of a strategic capacity planning approach that seems reasonable.

4.4. Discussion, insights and regional framework

In this section important considerations and insights are discussed in further detail constituting a complete framework for enabling regional assessments. *The main insights are italicized.*

4.4.1. Case study

The case study gives an indication of the type of assessment that can be performed. It demonstrates the complexity of real life, and the necessary compromises needed when modelling. Because of data unavailability, the case study only provides an indication of the capacity achievable in the BN and GC region, and any further generalisations should be avoided. To gain the most insights, it would be necessary to collect data from each hospital. That is a mammoth undertaking.

The case study shows various things that seem self-evident to those living in the BN and GC region, and common sense. Depending upon the demands present in local areas, and the capacity of local hospitals, *patients may have to travel further to receive prompt treatment.* Otherwise, they may need to stay on waiting lists longer, and be treated in later time

Table 20
Caseload broken up by patient type and DRG sub type.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
CARD	9.51	3	25.5	19.5	37.1	36.1	26	18.5	30.1	4.51	8.51	0.5	31.6	227	752	261	781	432	1168	963	173					
ENDO	77	77.8	36.5	95.9	27.7	31.7	31.7	101	56.6	50	68.3	61.5	65.4	34	15.7	33.9										
ENT	349	57.7	406	250	294	117	100	240	5.92	0.76	18.3	16.6	32.5	15.5	5.92											
FMAX	6.85	0.36																								
GAST	272	190	74.7	225	116	245	227	187	272	439	234	805	1623	1745	398	1420	1035	669	1877							
GYN	10.5	26	7.75	11.5	24.8	34.7	39.7	1592	321	1031																
HEPA	0.42	0.36	1.05	1.12	0.52	0.12	0.03	0	1.33	0.74	1.5															
IMMU	27.7	117	26.7	105	56.8	87.4	134																			
NEPH	217	126	22.9	108	31.9	33.1	93	5.1	1554	1406	1408	45.9	1811	306	800	526	290	1563	1994	397						
NEUR	224	148	214	136	197	50.8	16.8	139	131	109	43.8	153	228	136	206	153	240	21.9	221	223	129	48.6	14.6	231	104	136
OBS	745	2980																								
ONC	66.2	82.8	65.3	15.6	224	2362	1397	384																		
OPHT	332	415	310	238	461	48.1	224	342	305	253	271	183	50.2	44.5	24.9	58.4										
ORTH	291	0	44.7	36.1	132	239	267	173	300	12.9	42.9	288	281	122	236	253	141	306	233	182	276	265	217	227	131	303
	220	318	280	2.45	11.6	20.8	24.5	9.8	22.1	30	13.5	15.9	22.1	11.6	13.5	0.61	3.67	19	15.3	17.2	15.3	19.6	17.8			
PLAS	158	87.6	304	258	99.8	225	268	304	320	98.3	10.3	26.8	2.68	29.3	4.24	28.8	0.45	9.16								
PSY	172	135	329	583	575	552	198	35.2	763	439																
RESP	164	26.3	161	256	307	14.8	312	73.4	136	321	113	274	75.3	237	232	316	235	153	206	189						
TRANS	14.4																									
UROL	342	1093	624	181	172	983	48.6	21.1	39.3	34.7	35															
VASC	189	117	82.4	37	55.1	225	2.23	5.71	6.31	1.41	6.38	3.56	2.3	0.59	1.93	4.6	2.15									

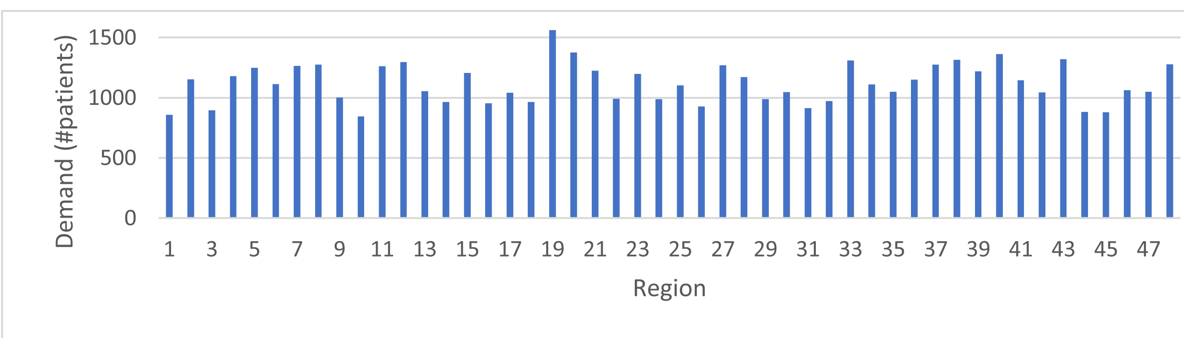

Fig. 7. Summary of regional demand.

periods. It also shows which regions do not have sufficient facilities nearby, and hence where new facilities could be built, or existing facilities expanded. *It also highlights the importance of reconfiguring hospitals to meet the demands within their local area, and the need to forecast accurately, those demands.*

4.4.2. Solution visualizations, reporting and display

Problem instances are generally inconsequential to solve. However, the information and outputs provided by the models, is extensive; certainly, too lengthy to be viewed in one glance. It is possible to delve down and look at each facility, ward, patient type and subtype. *A dashboard for summarising and navigating the results would be beneficial for health care managers and executives when implemented in real-world healthcare settings.*

4.4.3. Practical relevance

Mathematics aside, there are various repercussions to be considered regarding the proposed models. It is intended that the models will be useful and motivate further discussion and research on this important topic. It remains to be seen how applicable mathematical approaches are, and how easy it is for health care professionals, managers, and executives to adopt them. More generally, whom these techniques are exactly targeted, and who has the power to act upon the provided solutions. The context upon which they would be applied, and how often they should be applied is also important and in need of discussion. We pose some thoughts and responses to these inquiries below.

Who should use the mathematical models proposed in this article? Our mathematical approaches are developed for application by those with expertise in data analysis, mathematical modelling, and operational research. *A regional health-care master planning study, a process which typically involves government and a range of consultants, is also an opportunity to apply this article's models.*

Would a multi-hospital master planning framework be useful to hospital executives and managers? Conceptually, the idea of strategic regional planning is sound, *but the mathematical models are nuanced, requiring access to appropriate information, and other software support.* Much of the administrative aspects, however, can be automated with forethought.

4.4.4. Regional appraisals and optimization

In this article the necessity to perform a regional analysis of hospital capacity was demonstrated and several planning tasks that could be performed were motivated. We pose some thoughts regarding how a region with a multitude of hospitals can be appraised and optimized.

How do multiple hospitals maximize the number of patients treated over time? Section 3.2. demonstrated that more patients can be treated *if*

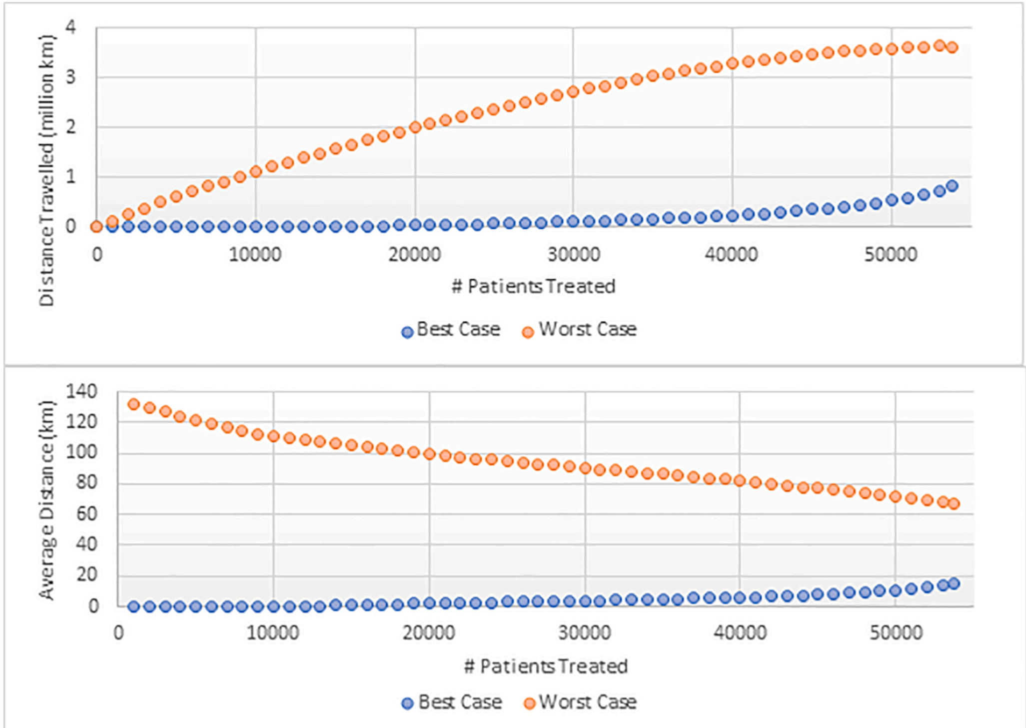

Fig. 8. Trade-off between patients treated and necessary travel distance.

the case mix of each hospital is selected, and an integrated approach is taken. The first model can identify how to treat the largest number of patients, given resource availability information. The exact number of patients depends on the setup of each hospital, for instance, in the wards and beds present, and their associated functions. Worth mentioning, is the need to follow through with treating planned numbers of patients. *This can be enforced most easily by changing the master surgical schedule of each hospital*. Otherwise, patient admissions across a region would have to be monitored, on an ongoing basis.

How would hospitals best work together and who are the decision makers? Coordinating function and caseload across multiple hospitals would be challenging under a distributed horizontally integrated management structure. A centralised planning and decision-making body (e.g., a regional strategic planning unit (RSPU)) would be best placed to apply modelling and issue directives to optimise the allocation of regional health services. The role of this body could be to perform strategic regional planning, optimisation, and analysis, and to report plans, actions, and requirements, to individual hospitals. Monitoring outcomes and collecting data is expected to be an ongoing task. Liaisons with individual hospitals are not expected to be necessary daily unless their authority extends to operational matters. It may be necessary for each hospital to have a “point of call” to relay information to other staff and to coordinate any activities that may be necessary. That staff member would be part of an existing planning unit and would not require the creation of a new one.

If the regions hospitals are from the same organisation, then the RSPU could be given access to electronic platforms, and to real-time information. This would reduce the need for excessive liaisons and permit faster access to data. This is also possible even if hospitals are from different organisations.

The analysis performed by the regional planning body is of a strategic nature and it is envisaged that changes to the operations and configuration of existing hospitals may sometimes be desirable to improve overall throughput or efficiency across the region. Changing how hospitals operate, however, has financial and logistical implications, and there are costs involved in making changes. *Those costs may need to be incorporated into strategic models, in the future.*

How often should regional strategic planning be performed? The application of strategic regional models is expected to be a regular task. In Australia, there are many regions. In the state of Queensland (Australia) for instance, there are fourteen, with about 123 public hospitals [19].

The selected regional case mix has a great impact on outputs. Any change or introspection in that would require a model resolve. The treatment times and lengths of stay are critical pieces of information too. These uncertain values affect all the models and reflect current trends in quality of care, technological innovation, staff fatigue and staff competency. It would be reasonable to expect that data could be updated and model scenario analysis conducted every six months (or annually) to check regional capacity and consider any changes that may be needed to meet future demand. Changes in the demographic of a region and the demand for healthcare, along with changes in patterns of patient flow, would trigger the need for both the first and second model to be reapplied. The construction of new hospital(s), the closure of existing one(s), or the introduction of a new region would also be the trigger point for model resolves.

Which hospitals should treat which patient types? Our mathematical models determine the number of patients of each type that can be feasibly treated in each hospital subject to deterministic treatment durations and length of stay. Those decisions, however, are dictated by the configuration of each hospital and the specific objective defined for decision making. As shown, our models provide a single solution, but *there are many alternatively optimal ways to distribute a regional caseload across individual hospitals*. The ramifications of this are that in each of those alternatives, each hospital could be treating a different mix of patients. As there is no overriding pattern or policy, this may be hard for hospital executives and managers to tolerate. We would expect that they would rather the same case mix be treated year in year out. Further research on this quirk is evidently important. It has not been considered before in the literature to the best of our knowledge. It is also worth noting that each hospital caseload has different staffing requirements and needs different medical supplies. However, across the region, the

Table 21
The hospitals servicing each region in the solution.

REG	HOSPITALS SERVICING REGION	REG	HOSPITALS SERVICING REGION
CAP	BPH,GCUH*,GCPR*,LOG, PA,QE2,RBWH,RED,STAN,	**SPFD**	BPH,GCUH**,JFN**,LOG*, PA,QE2,RBWH,ROB**, STAN,
CLEV	GCUH*,GCPR*,LOG,PA, QE2,RBWH,RED,STAN,	**BEAU**	GCUH*,GCPR*,JFN*,PA**, RBWH**,RED**,
WYN	BPH,GCPR**,LOG,PA, PRCH,RBWH,STAN,	**BEEN**	GCUH,GCPR,LOG,PA,PIN*, QE2,RBWH*,RED,
BALD	***BPH,NW,PA,PRCH, RBWH,***	**BRN**	GCUH*,JFN**,LOG,PA,QE2, RBWH,RED,ROB**,
CHERM	***PA,PRCH,RBWH,***	**JIMB**	GCUH*,GCPR*,JFN**,LOG, PA*,QE2*,RBWH*,RED*, ROB*,
NUN	BPH,GCUH**,LOG*,PA, PRCH,RBWH,STAN,	**LOG**	GCUH*,LOG,PA,PIN*,QE2, RBWH,RED,
SAND	BPH,GCUH**,LOG*,NW, PA,PRCH,RBWH,	**SWOOD**	GCUH*,LOG,PA,PIN**,QE2, RBWH,RED,
CAR	BPH,GCPR**,LOG,PA, PRCH,RBWH,STAN,	**BRIB**	BPH*,GCPR**,NW*,PA*, PRCH*,RBWH*,STAN*,
HOLL	***BPH,PA,RBWH,***	**CAB**	NW*,PA**,PRCH*,RBWH*, WES*,
MGR	GCUH*,LOG,PA,PIN**, QE2,RBWH,STAN,	**NAR**	NW,PA*,PRCH,RBWH*, WES*,
NATH	LOG,PA,PIN**,QE2, RBWH,	**RCLIFF**	BPH,GCUH**,GCPR**,NW, PA*,PRCH,RBWH,STAN,
ROCK	GCUH*,LOG,PA,PIN**, QE2,RBWH,	**HILL**	NW,PA*,PRCH,RBWH,STAN, WES,
SUNY	GCUH*,LOG,PA,PIN**, QE2,RBWH,	**NLAKE**	LOG**,NW,PA,PRCH,RBWH, STAN,
CENT	BPH,GCUH**,PA,PIN**, PRCH,QE2,RBWH,STAN,	**SPINE**	***BPH,NW,PA,PRCH,RBWH,***
KEN	BPH,JFN**,LOG*,PA, PIN**,PRCH,RBWH,STAN, WES,	**BROAD**	GCUH,JFN,PA**,PIN, RBWH**,RED**,ROB,
SHER	***BPH,PA,PRCH,RBWH, STAN,***	**COOL**	GCUH,JFN,PA**,PIN, RBWH**,RED**,
GAP	***BPH,PA,PRCH,RBWH, STAN,WES,***	**GCN**	GCUH,GCPR,JFN,PA**, RBWH**,RED*,
BRIN	***BPH,PA,PRCH,RBWH, STAN,***	**GCHIN**	GCUH,JFN,PA**,PIN, RBWH**,RED**,ROB,
BRINE	BPH,GCPR**,LOG,PA, PRCH,RBWH,STAN,	**MUD**	GCUH,JFN,PA**,PIN, RBWH**,RED**,
BRINN	***BPH,PA,PRCH,RBWH, STAN,***	**NER**	GCUH,JFN,PA**,PIN, RBWH**,RED*,ROB,
BRINW	***BPH,PA,PRCH,RBWH, STAN,WES,***	**ORM**	GCUH,GCPR,JFN*,PA*,PIN, RBWH*,RED,
FOR	JFN**,PA,QE2,RBWH, ROB**,STAN,	**ROB**	GCUH,JFN,PA**,PIN, RBWH**,RED**,ROB,
IPSW	BPH*,GCUH**,JFN**,PA*, PRCH**,QE2*,RBWH*, ROB**,STAN*,WES*,	**SPORT**	GCUH,JFN,PA**,PIN, RBWH**,RED*,
IPSWI	JFN**,PA,PRCH*,QE2, RBWH,ROB**,STAN,	**SURF**	GCUH,JFN,PA**,PIN, RBWH**,RED*,

same resources are potentially needed.

Should hospitals treat many patient types or specialise in treating a few? This question pertains to the provision of different forms of care within hospitals. We cannot say at this point, whether hospitals should be permanently reconfigured as we do not know what future demands will be, and what expectations will be required of the health care system. We do not know how technological progress and innovations, will affect the nature of healthcare in the future.

As mentioned in Flegel [20], what aspects of medical care are or are not included in a hospital depends upon those providing the care. There are however funding implications for the academic institutions that have the label "tertiary care," and prestige and billing benefits for the practitioners. Large hospitals that treat every type of patient seem advantageous because they have a full complement of health care resources on-site. As such they can treat complex cases. Breen et al. [21] commented that many hospitals often struggle to meet complex needs because they are designed for episodic care, provided by health care professionals who are highly specialised to single organ systems and are accustomed to working in silos defined by their medical specialties, institutions, and geographical areas. But in larger hospitals, quality of care is reduced as wards become full, and the placement of patients in wards of other specialties occurs.

How many hospitals are required in a region and how should each be sized and configured? This question is open-ended, and the answer depends on many things. There are pros and cons to different hospital types and to different configurations, and further investigation is warranted. The hospitals that are built should be sufficient to meet demand but should also be economically viable. Hospitals should be staffed appropriately, and their revenue should be sufficient to meet salaries. Resources should be well maintained, and funds should be available to repair or replace them in the event of breakdown or update them as more modern versions arise.

As posed, this article's mathematical models are not designed to provide direct recommendations in relation to how many hospitals are needed or how each should be configured. The models can, however, provide useful quantitative insights informing the merit of any proposed scenario that is being investigated. *The models can inform the staging of future expansion of hospitals (i.e., adding additional beds, wards, and theatres), and identify the triggers for expansion based on forecast growth in demand.*

Is the capacity of a region better quantified by the number of specialists operating and their time availability for medical and surgical activities? This requires further investigation, but we believe, this viewpoint is not sufficient, for the simple reason that operating theatres, ward beds, surgical care areas, and medical equipment, which are limiting factors, are not inherently included. Otherwise, the idea is sound. Regarding medical and surgical procedures, a specialist allocates their time, and if they are not operating, then patients are not being treated. *Alteration of this time is key to achieving higher or lower outputs.*

5. Conclusions

This article contributes to healthcare planning by developing and testing mathematical models to support various hospital capacity appraisals and capacity allocation tasks necessary in case mix planning in a regional healthcare setting. This contrasts with previous research in which only a single hospital is considered. The academic literature seems sparse and to the best of our knowledge there are no comparable approaches. From a practical perspective, there is a strong need for the proposed methods. Determining how hospitals can be used collectively, in a collaborative manner (i.e., as we have done), is not straightforward nor is it a task that falls within the usual expertise of politicians, health care managers, planners, and consultants. This task can benefit greatly from an optimisation approach.

The mathematical models that we have developed have many benefits. They provide a robust and agile approach to appraise a region's hospitals. They instantaneously determine how hospitals can best be used as they are currently configured and a means to quickly evaluate alternative configurations. The results are reproducible and defendable. The data requirements needed to facilitate such an endeavour, however, become more prohibitive as the number of considered hospitals increases.

Regarding the developed models, our first mathematical model provides an original approach to appraise the capacity of a region of hospitals. It identifies the global maximum output of the hospitals within a region, subject to relevant technical constraints, like a regional case mix. Our second model considers how to treat a spatially distributed cohort of patients. Specifically, it identifies how to assign that caseload to individual hospitals within a region. This is a multicriteria decision problem as it considers the trade-offs between meeting demand and the collective distance to be travelled by patients. The third model is constructed on the premise that each hospital may have its own target caseload, and that caseload can be met by other hospitals via

Table 22
Patients treated in each hospital from each region.

	BPH	GCUH	GCPR	GRN	JFN	LOG	NW	PA	PIN	PRCH	QE2	RBWH	RED	ROB	STAN	WES	
CAP	15.1	115.63	24.37	0	0	290.2	0	263.98	0	0	77.22	53.65	4.4	0	12.65	0	857.2
CLEV	0	1.37	124.52	0	0	711.5	0	73.2	0	0	69.63	9.07	2.48	0	161.97	0	1153.74
WYN	189.24	0	110.24	0	0	34.56	0	249.55	0	22.66	0	254.15	0	0	34.5	0	894.9
BALD	56.58	0	0	0	0	0	433.61	18.3	0	273.07	0	397.01	0	0	0	0	1178.57
CHERM	0	0	0	0	0	0	0	101.1	0	857.7	0	289.26	0	0	0	0	1248.06
NUN	5	36.08	0	0	0	49.56	0	36.86	0	491.34	0	481.87	0	0	11.71	0	1112.42
SAN	3.43	57.67	0	0	0	95.13	7.47	3.94	0	715.67	0	381.97	0	0	0	0	1265.28
CAR	6.44	0	102.03	0	0	17.12	0	861.09	0	78.68	0	88.71	0	0	121.84	0	1275.91
HOLL	8.01	0	0	0	0	0	0	850.79	0	0	0	143.05	0	0	0	0	1001.85
MGR	0	2.15	0	0	0	122.7	0	397.57	19.65	0	164.98	53.3	0	0	84.38	0	844.73
NATH	0	0	0	0	0	35.17	0	101.84	96.75	0	963.32	64.53	0	0	0	0	1261.61
ROC	0	114.33	0	0	0	91.82	0	41.1	31.02	0	995.96	21.87	0	0	0	0	1296.1
SUNY	0	2.03	0	0	0	306.89	0	36.53	57.94	0	568.28	82.35	0	0	0	0	1054.02
CEN	1.09	14.18	0	0	0	0	0	534.68	153.65	0.21	127.64	51.98	0	0	81.27	0	964.7
KEN	52.54	0	0	0	44.03	69.62	0	333.97	59.7	35.17	0	198.63	0	0	350.25	62.29	1206.2
SHER	31.43	0	0	0	0	0	0	717	0	5.67	0	143.59	0	0	55.79	0	953.48
GAP	167.75	0	0	0	0	0	0	217.32	0	102.86	0	436.21	0	0	45.6	72.23	1041.97
BRIN	139.99	0	0	0	0	0	0	18.51	0	35.94	0	576.77	0	0	192.57	0	963.78
BRINE	21.73	0	26.21	0	0	107.19	0	779.7	0	104.31	0	269.63	0	0	253.68	0	1562.45
BRINN	169.69	0	0	0	0	0	0	131.98	0	193.74	0	794.85	0	0	85.3	0	1375.56
BRINW	137.93	0	0	0	0	0	0	204.7	0	52.57	0	700.57	0	0	91.89	36.46	1224.12
FOR	0	0	0	0	72.04	0	0	246.8	0	0	526.02	88.34	0	23.68	33.88	0	990.76
IPS	126.59	21.51	0	0	97.75	0	0	295.87	0	108.88	35.13	223.87	0	23.93	255.02	10.87	1199.42
IPSI	0	0	0	0	11.56	0	0	474.54	0	85.57	61.7	198.59	0	30.77	125.65	0	988.38
SPFD	22.73	3.52	0	0	84.47	39.55	0	311.92	0	0	434.28	14.52	0	67.13	123.57	0	1101.69
BEAU	0	643	233.04	0	32.93	0	0	4.73	0	0	0	8.35	4.48	0	0	0	926.53
BEEN	0	400.81	26.1	0	0	680.99	0	10.64	60.41	0	87.25	0.08	4.04	0	0	0	1270.32
BRN	0	52.69	0	0	29.35	837.87	0	27.93	0	0	175.87	1.14	2.4	45.65	0	0	1172.9
JIM	0	395.87	55.72	0	90.06	299.09	0	19.21	0	0	82.32	2.7	1.6	42.22	0	0	988.79
LOG	0	28.83	0	0	0	763.05	0	18.88	97.51	0	124.26	9.19	3.96	0	0	0	1045.68
SWOOD	0	1.74	0	0	0	722.72	0	31.98	67.17	0	84.08	1.14	5	0	0	0	913.83
BRIB	3.71	0	105.18	0	0	0	60.76	27.53	0	433.76	0	181.74	0	0	159.25	0	971.93
CAB	0	0	0	0	0	0	557.76	0.26	0	482.36	0	254.14	0	0	0	15.46	1309.98
NAR	0	0	0	0	0	0	671.08	0.32	0	178.62	0	208.76	0	0	0	51.73	1110.51
RCLIFF	59.85	79.45	74.06	0	0	0	19.79	14.43	0	593.95	0	203.06	0	0	5.46	0	1050.05
HILL	0	0	0	0	0	0	20.22	46.91	0	557.8	0	447.27	0	0	26.47	52.34	1151.01
NLAKE	0	0	0	0	0	8.34	205.16	11.87	0	582.46	0	369.61	0	0	98.27	0	1275.71
SPINE	10.87	0	0	0	0	0	499.69	0.12	0	227.31	0	575.93	0	0	0	0	1313.92
BROAD	0	131.46	0	0	57.46	0	0	3.43	89.36	0	0	2.28	3.6	932.67	0	0	1220.26
COOL	0	30.21	0	0	1272.16	0	0	1.23	53.67	0	0	3.58	2.32	0	0	0	1363.17
GCN	0	604.82	530.83	0	1.67	0	0	4.66	0	0	0	0.8	1.96	0	0	0	1144.74
GCHIN	0	146.16	0	0	198.69	0	0	5.44	193.43	0	0	0.51	4.88	496.53	0	0	1045.64
MUD	0	45.32	0	0	1208.18	0	0	4.35	51.38	0	0	8.77	2.92	0	0	0	1320.92
NER	0	188.06	0	0	83.68	0	0	4.78	556.99	0	0	6.58	4.08	37.68	0	0	881.85
ORM	0	405.11	456.31	0	1.67	0	0	6.81	3.45	0	0	4.05	1.6	0	0	0	879
ROB	0	40.14	0	0	30.27	0	0	7.08	247.73	0	0	8.6	2.16	725.63	0	0	1061.61
SPORT	0	285.56	0	0	1.8	0	0	8.02	743.07	0	0	10.2	1.28	0	0	0	1049.93
SURF	0	244.65	0	0	2.28	0	0	7.73	1014.59	0	0	5.9	1.88	0	0	0	1277.03
	1229.7	4092.35	1868.61	0	3320.05	5283.07	2475.54	7571.18	3597.47	6220.3	4577.94	8332.72	55.04	2425.89	2410.97	301.38	

outsourcing agreements. Outsourcing is a significant innovation of this article's modelling. This model has similarities to the second model, but spatial demographic information is not incorporated. It also requires the definition of multiple target caseloads, one for each hospital, in contrast to one. That problem also has multiple criterions and is a multicriteria decision problem. In summary, it is worth noting that the models provide an appraisal. They do not recommend direct actions. The data handling and manipulation is extensive to facilitate the models.

Table 23
Distances to be travelled.

	BPH	GCUH	GCPR	GRN	JFN	LOG	NW	PA	PIN	PRCH	QE2	RBWH	RED	ROB	STAN	WES	
CAP	259.50	5694.34	1200.13	0.00	0.00	5180.56	0.00	4008.76	0.00	0.00	1199.64	922.00	44.12	0.00	217.40	0.00	18726.44
CLEV	0.00	54.09	4916.36	0.00	0.00	7957.79	0.00	1643.40	0.00	0.00	1471.79	231.35	0.00	0.00	4131.36	0.00	20406.14
WYN	2340.79	0.00	6262.00	0.00	0.00	847.06	0.00	3066.42	0.00	319.18	0.00	3143.68	0.00	0.00	426.74	0.00	16405.88
BALD	831.10	0.00	0.00	0.00	0.00	0.00	0.00	343.92	0.00	1492.30	0.00	5831.61	0.00	0.00	0.00	0.00	8498.93
CHERM	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1371.60	0.00	0.00	0.00	2777.12	0.00	0.00	0.00	0.00	4148.71
NUN	60.25	2379.13	0.00	0.00	0.00	1644.05	0.00	545.09	0.00	3320.79	0.00	5806.65	0.00	0.00	141.11	0.00	13897.06
SAND	58.09	4266.25	0.00	0.00	0.00	3902.88	39.84	81.39	0.00	5400.96	0.00	6468.52	0.00	0.00	0.00	0.00	20217.93
CAR	51.74	0.00	5567.69	0.00	0.00	362.99	0.00	5335.02	0.00	1126.85	0.00	712.68	0.00	0.00	978.83	0.00	14135.80
HOLL	33.51	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	598.49	0.00	0.00	0.00	0.00	632.00
MGR	0.00	103.90	0.00	0.00	0.00	1815.76	0.00	3597.66	1128.54	0.00	1204.91	680.71	0.00	0.00	1077.60	0.00	9609.09
NATH	0.00	0.00	0.00	0.00	0.00	717.03	0.00	407.52	6033.91	0.00	0.00	520.70	0.00	0.00	0.00	0.00	7679.16
ROCK	0.00	5637.91	0.00	0.00	0.00	1628.52	0.00	467.56	1784.61	0.00	7425.20	332.02	0.00	0.00	0.00	0.00	17275.82
SUNY	0.00	97.87	0.00	0.00	0.00	4700.21	0.00	344.14	3293.54	0.00	3186.73	1119.09	0.00	0.00	0.00	0.00	12741.58
CEN	12.98	859.72	0.00	0.00	0.00	0.00	0.00	5935.73	10559.54	4.15	1332.24	618.87	0.00	0.00	967.59	0.00	20290.82
KEN	718.35	0.00	0.00	0.00	3977.07	2408.48	0.00	4879.24	4456.90	666.35	0.00	2715.74	0.00	0.00	4788.74	689.02	25299.88
SHER	181.41	0.00	0.00	0.00	0.00	0.00	0.00	3829.33	0.00	82.17	0.00	828.79	0.00	0.00	322.02	0.00	5243.73
GAP	1496.10	0.00	0.00	0.00	0.00	0.00	0.00	2648.34	0.00	1075.49	0.00	3890.48	0.00	0.00	406.70	362.53	9879.64
BRIN	0.00	0.00	0.00	0.00	0.00	0.00	0.00	77.44	0.00	345.06	0.00	0.00	0.00	0.00	0.00	0.00	422.50
BRINE	91.77	0.00	1553.01	0.00	0.00	2764.62	0.00	4175.32	0.00	1011.54	0.00	1138.65	0.00	0.00	1071.29	0.00	11806.20
BRINN	863.36	0.00	0.00	0.00	0.00	0.00	0.00	1168.96	0.00	921.52	0.00	4044.08	0.00	0.00	433.98	0.00	7431.90
BRINW	538.83	0.00	0.00	0.00	0.00	0.00	0.00	1536.47	0.00	461.35	0.00	2736.80	0.00	0.00	358.97	0.00	5632.42
FOR	0.00	0.00	0.00	0.00	5699.98	0.00	0.00	3090.55	0.00	0.00	5266.78	1316.20	0.00	1608.31	504.79	0.00	17486.60
IPS	5904.77	1825.18	0.00	0.00	10113.54	0.00	0.00	13837.65	0.00	5571.97	1612.23	10442.20	0.00	2225.29	11895.06	479.07	63906.96
IPSI	0.00	0.00	0.00	0.00	1044.93	0.00	0.00	13722.40	0.00	3031.73	1710.44	5829.81	0.00	2444.83	3688.54	0.00	31472.67
SPFD	523.82	207.39	0.00	0.00	6815.10	1229.50	0.00	6630.26	0.00	0.00	8230.72	334.65	0.00	4672.76	2848.02	0.00	31492.22
BEAU	0.00	25084.35	9091.09	0.00	1621.30	0.00	0.00	252.61	0.00	0.00	0.00	477.58	230.24	0.00	0.00	0.00	36757.18
BEEN	0.00	11469.97	746.85	0.00	0.00	4239.95	0.00	302.87	2254.13	0.00	2187.32	2.61	67.93	0.00	0.00	0.00	21271.62
BRN	0.00	2221.38	0.00	0.00	1909.60	12990.08	0.00	584.63	0.00	0.00	2978.51	28.34	58.75	2459.12	0.00	0.00	23230.42
JIM	0.00	14345.52	2019.18	0.00	4980.30	7015.51	0.00	678.39	0.00	0.00	2578.30	105.86	55.20	1876.75	0.00	0.00	33655.00
LOG	0.00	966.88	0.00	0.00	0.00	0.00	0.00	441.82	4158.06	0.00	2533.47	251.82	44.29	0.00	0.00	0.00	8396.35
SWOOD	0.00	71.79	0.00	0.00	0.00	5777.61	0.00	500.28	3367.30	0.00	1045.39	22.51	64.41	0.00	0.00	0.00	10849.29
BRIB	170.76	0.00	10354.76	0.00	0.00	0.00	1979.35	1365.87	0.00	15880.95	0.00	8364.93	0.00	0.00	7329.79	0.00	45446.41
CAB	0.00	0.00	0.00	0.00	0.00	0.00	18817.94	13.66	0.00	18814.78	0.00	12307.45	0.00	0.00	0.00	716.56	50670.38
NAR	0.00	0.00	0.00	0.00	0.00	0.00	13737.85	12.56	0.00	4611.11	0.00	7336.67	0.00	0.00	0.00	1708.26	27406.44
RCLIFF	1684.74	6534.73	6091.40	0.00	0.00	0.00	307.43	457.18	0.00	11215.13	0.00	5715.89	0.00	0.00	153.69	0.00	32160.18
HILL	0.00	0.00	0.00	0.00	0.00	0.00	327.28	1443.63	0.00	11459.58	0.00	11981.78	0.00	0.00	709.15	1219.85	27141.27
NLAKE	0.00	0.00	0.00	0.00	0.00	420.18	2238.40	348.46	0.00	9198.80	0.00	9377.77	0.00	0.00	2493.31	0.00	24076.91
SPINE	227.76	0.00	0.00	0.00	0.00	0.00	3151.34	3.01	0.00	2667.25	0.00	12067.58	0.00	0.00	0.00	0.00	18116.93
BROAD	0.00	2245.02	0.00	0.00	526.67	0.00	0.00	250.58	661.09	0.00	0.00	175.96	203.51	3289.25	0.00	0.00	7352.07
COOL	0.00	791.98	0.00	0.00	0.00	0.00	0.00	101.00	888.92	0.00	0.00	308.62	152.39	0.00	0.00	0.00	2242.91
GCN	0.00	0.00	0.00	0.00	43.78	0.00	0.00	265.07	0.00	0.00	0.00	48.74	77.39	0.00	0.00	0.00	434.98
GCHIN	0.00	3832.55	0.00	0.00	4909.72	0.00	0.00	371.51	4091.78	0.00	0.00	36.95	281.67	8945.93	0.00	0.00	22470.11
MUD	0.00	1219.09	0.00	0.00	8601.97	0.00	0.00	350.01	887.99	0.00	0.00	742.25	191.66	0.00	0.00	0.00	11992.98
NER	0.00	2051.32	0.00	0.00	1756.53	0.00	0.00	294.12	4274.28	0.00	0.00	432.27	191.50	370.31	0.00	0.00	9370.33
ORM	0.00	6104.60	6876.16	0.00	68.04	0.00	0.00	284.74	83.58	0.00	0.00	185.74	40.77	0.00	0.00	0.00	13643.63
ROB	0.00	647.43	0.00	0.00	339.40	0.00	0.00	502.30	1614.05	0.00	0.00	645.78	119.18	0.00	0.00	0.00	3868.14
SPORT	0.00	1850.97	0.00	0.00	36.21	0.00	0.00	498.85	2659.85	0.00	0.00	676.30	58.43	0.00	0.00	0.00	5780.62
SURF	0.00	2391.04	0.00	0.00	37.76	0.00	0.00	508.41	0.00	0.00	0.00	412.27	92.41	0.00	0.00	0.00	3441.90
	16049.62	102954.41	54678.65	0.00	52481.90	65602.77	40599.41	92575.71	52198.08	98679.00	43963.65	134746.58	1973.84	27892.53	44944.68	5175.29	834516.12

In Section 4.1, an optimal solution was shown. There are others, however. It would be an interesting exercise to identify alternatively optimal solutions to gain further insights. The development of an inter-regional or national approach was outside the scope of this article but may be considered at a future time. The current models are suitable, and all that is required is further testing, via the creation of additional real-life case studies. Stochastic versions of the proposed models should also be developed and tested and would be hugely beneficial.

Economy of scale is an important concept. In a health care setting, which specialises its services, there is a high chance to reduce costs when many patients of the same type are treated daily or weekly. It may even be possible to reduce treatment times and improve quality of care. Modelling economy of scale (EOS) and integrating that aspect into this article's models may be worth considering, in a financial focused appraisal of hospital capacity.

CRediT authorship contribution statement

Robert L Burdett: Conceptualization, Data curation, Formal analysis, Methodology, Software, Validation, Writing – original draft. **Paul Corry:** Writing – original draft, Writing – review & editing. **Prasad Yarlagadda:** Funding acquisition, Project administration, Supervision. **David Cook:** Investigation, Writing – review & editing. **Sean Birgan:** Investigation, Writing – review & editing. **Steven M McPhail:** Investigation, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Appendix A

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Appendix B. Model Extensions

Extension 1. Let us assume that there is an enforceable case mix and sub mix within each hospital, respectively denoted $\mu_{h,g}^{\mathrm{HG}}$ and $\mu_{h,g,p}^{\mathrm{HGP}}$, where $\sum_{g\in G}\mu_{h,g}^{\mathrm{HG}}=1$ and $\sum_{p\in P_g}\mu_{h,g,p}^{\mathrm{HGP}}=1$. The following constraints are therefore necessary:

$$n_{h,g}^{\mathrm{HG}} \geq \mu_{h,g}^{\mathrm{HG}} N_h^{\mathrm{H}} \quad \forall g\in G, h\in H \text{ (Case mix adherence)}$$

$$n_{h,g,p}^{\mathrm{HGP}} \geq \mu_{h,g,p}^{\mathrm{HGP}} n_{h,g}^{\mathrm{HG}} \quad \forall g\in G, p\in P_g, h\in H \text{ (Subtype mix adherence)}$$

Another possibility worth considering is to remove designated case mix and sub mix constraints in each hospital, and to minimize deviations and disparities. To do that it is necessary to explicitly quantify the differences as follows:

$$v_{h,g}^{\mathrm{HG}} = \left|\mu_{h,g}^{\mathrm{HG}} N_h^{\mathrm{H}} - n_{h,g}^{\mathrm{HG}}\right| \Rightarrow v_{h,g}^{\mathrm{HG}} \geq \mu_{h,g}^{\mathrm{HG}} N_h^{\mathrm{H}} - n_{h,g}^{\mathrm{HG}} \text{ and } v_{h,g}^{\mathrm{HG}} \geq n_{h,g}^{\mathrm{HG}} - \mu_{h,g}^{\mathrm{HG}} N_h^{\mathrm{H}}$$

$$v_{h,g,p}^{\mathrm{HGP}} = \left|\mu_{h,g,p}^{\mathrm{HGP}} n_{h,g}^{\mathrm{HG}} - n_{h,g,p}^{\mathrm{HGP}}\right| \Rightarrow v_{h,g,p}^{\mathrm{HGP}} \geq \mu_{h,g,p}^{\mathrm{HGP}} n_{h,g}^{\mathrm{HG}} - n_{h,g,p}^{\mathrm{HGP}} \text{ and } v_{h,g,p}^{\mathrm{HGP}} \geq n_{h,g,p}^{\mathrm{HGP}} - \mu_{h,g,p}^{\mathrm{HGP}} n_{h,g}^{\mathrm{HG}}$$

where $n_{h,g}^{\mathrm{HG}} = \sum_{p\in P_g} n_{h,g,p}^{\mathrm{HGP}}$

Extension 2. Let us assume there is no case mix and sub mix, but there are specific requirements for each patient type and subtype, denoted respectively $\widehat{N}_g^{\mathrm{G}}$ and $\widehat{N}_{g,p}^{\mathrm{GP}}$. To force these demands are met, the following constraints are required:

$$N_g^{\mathrm{G}} \geq \widehat{N}_g^{\mathrm{G}} \forall\ g\in G \text{ (Demands – patient type)}$$

$$N_{g,p}^{\mathrm{GP}} \geq \widehat{N}_{g,p}^{\mathrm{GP}} \forall\ g\in G, p\in P_g \text{ (Demands – Patient subtype)}$$

There is however no guarantee that specified demands can be met, and the model will not solve if they cannot. In that event, unmet demand may be minimized to determine a best fit caseload. To find the best fit caseload given the targets $\widehat{N}_g^{\mathrm{G}}$ and $\widehat{N}_{g,p}^{\mathrm{GP}}$, the following approaches could be taken:

i) Set $N_g^{\mathrm{G}} \leq \widehat{N}_g^{\mathrm{G}}$ and/or $N_{g,p}^{\mathrm{GP}} \leq \widehat{N}_{g,p}^{\mathrm{GP}}$ and Minimize $\sum_g(\widehat{N}_g^{\mathrm{G}} - N_g^{\mathrm{G}}) + \sum_g\sum_{p\in P_g}(\widehat{N}_{g,p}^{\mathrm{GP}} - N_{g,p}^{\mathrm{GP}})$ or Minimize $\sum_g(\widehat{N}_g^{\mathrm{G}} - N_g^{\mathrm{G}})^2 + \sum_g\sum_{p\in P_g}(\widehat{N}_{g,p}^{\mathrm{GP}} - N_{g,p}^{\mathrm{GP}})^2$

ii) Set $N_g^{\mathrm{G}} \geq \widehat{N}_g^{\mathrm{G}} - \Omega\overline{N}_g^{\mathrm{G}}$ (i.e., $\frac{\widehat{N}_g^{\mathrm{G}} - N_g^{\mathrm{G}}}{\overline{N}_g^{\mathrm{G}}} \leq \Omega$) and Minimize Ω where $\overline{N}_g^{\mathrm{G}}$ is the upper bound

iii) Set $N_{g,p}^{\mathrm{GP}} \geq \widehat{N}_{g,p}^{\mathrm{GP}} - \Omega\overline{N}_{g,p}^{\mathrm{GP}}$ (i.e., $\frac{\widehat{N}_{g,p}^{\mathrm{GP}} - N_{g,p}^{\mathrm{GP}}}{\overline{N}_{g,p}^{\mathrm{GP}}} \leq \Omega$) and Minimize Ω where $\overline{N}_{g,p}^{\mathrm{GP}}$ is the upper bound

In the second and third options, the smallest value of Ω is chosen such that the relative difference between the actual value and the target is as small as possible across all patient types. This produces an equitable best fit case mix.

Extension 3. When targets cannot be met, a reconfiguration of the hospital's wards may be required. The parameter $b_{h,w}$ may instead be regarded as a decision variable. The model may then be solved as a satisfaction problem without any objective and additional constraints $N_g^{\mathrm{G}} \geq \widehat{N}_g^{\mathrm{G}}$ and $N_{g,p}^{\mathrm{GP}} \geq \widehat{N}_{g,p}^{\mathrm{GP}}$. Additional technical constraints governing and restricting how each area is expanded can easily be added.

Reconfiguring to maximize output may also be considered. Without bounds, however, the model will increase $b_{h,w}$ without limit to achieve increased outputs. One way is to have a budget C or restrictions on the number of extra spaces, denoted $\overline{b}_{h,w}$. If $c_{h,w}$ is the cost of a new treatment space in area w of hospital h, then the following constraints may be included:

$$U_{h,w} \leq (b_{h,w} + \Delta b_{h,w})T_{h,w} \quad \forall h \in H, w \in W_h \text{ (Restricted time availability)}$$

$$\sum_{h\in H}\sum_{w\in W_h} c_{h,w}\Delta b_{h,w} \leq C \text{ (Restricted changes)}$$

$$0 \leq \Delta b_{h,w} \leq \overline{b}_{h,w} \quad \forall h \in H, w \in W_h \text{ (Bounds)}$$

Extension 4. To treat more patients of specific types, the beds in some wards could be repurposed or relocated. For a particular $h \in H$ the following balance constraint could be included.

$$\sum_{w\in W_h} (\Delta b_{h,w}) = 0 \text{ where } -b_{h,w} \leq \Delta b_{h,w} \leq \overline{b}_{h,w} - b_{h,w}$$

This constraint keeps the total number of beds static but reassigns beds to other areas.

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