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ORIGINAL PAPER

A sustainable inventory system with price-sensitive demand and carbon emissions under partial trade credit and partial backordering

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Abstract
Companies and governments are actively looking for methods of curbing global warming. Management of inventory systems while considering environmental issues is an important problem. Therefore, this paper explores an Economic Order Quantity model by incorporating environmental issues under partial trade credit and partial backordering. The selling price and carbon emission-dependent demand function is adopted in this paper. The paper first formulates an inventory model with an exogenous price under cap-and-trade and carbon tax mechanisms. The study then extends the proposed problem when the selling price is an endogenous variable. These models are formulated as a nonlinear programming problem of profit maximization, and they are optimized applying global optimization of signomial geometric programming. Further, numerical examples and sensitivity analysis are presented to examine the effects of different shortage rates, credit periods, carbon tax, and price on the retailer's replenishment strategies.

Keywords Partial backordering · Inventory · Partial delay in payment · Carbon emissions-sensitive demand · Signomial geometric programming

Parameters

P Selling price unit (\$/unit)
C Purchasing cost unit (\$/unit)

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K Ordering cost (\$/order)
π Backordering cost (\$/unit/unit time).
g The cost of goodwill loss (\$/unit)
h The cost of holding an item in stock (\$/unit/unit time)
I_e Interest earned
I_p Interest charged, $I_p \geq I_e$
β The portion of shortages that will be backordered, $\beta \in [0, 1]$
α The portion of the purchasing cost that should be settled at the delivery time of items, $\alpha \in [0, 1]$
M Trade credit period
$\hat{C}$ The value of carbon emissions related per unit purchased
$\hat{K}$ The value of carbon emissions per order
$\hat{h}$ The value of carbon emissions per unit of held inventory per time
$\hat{\pi}$ The value of carbon emissions related per unit backordered per time
θ Price sensitivity
γ Maximum market demand
δ Carbon emission elasticity
t Carbon tax per unit
s Carbon price per unit

Decision variables

D Demand rate per year
B Maximum shortage level
F The portion of demand that will be satisfied from stock, $F \in [0, 1]$
T Inventory cycle time
Q Order quantity
CE The amount of carbon emissions

Other variables

SR Sales revenue per cycle
C_{P} Purchasing cost per cycle
C_{O} Ordering cost per cycle
C_{H} Holding cost per cycle
C_{B} Backordering cost per cycle
C_{G} Goodwill losses cost per cycle
CC Interest payable per cycle
IE Interest earned per cycle
ATP Average total profit per year

1 Introduction

According to the United Nations Intergovernmental Panel on Climate Change (IPCC), the recent global atmospheric concentration of greenhouse gases reached the highest point in the last 80 million years (Tsao et al. 2017). Companies are the main reason for carbon emissions as they contribute much to carbon emissions in the process of transportation, storage, production, and trading. In order to control carbon emissions, a wide range of mechanisms and environmental regulations have been enacted. Among many carbon emission policies, there are two most practiced carbon regulatory policies broadly applied for curbing carbon emissions in many countries and areas; carbon tax and cap-and-trade policies (Song et al. 2017).

Carbon tax policy is a cost-effective mechanism to limit carbon emissions in the world, which is extremely supported by international organizations and experts (Agency 1996; Oreskes 2011). Under carbon tax policy, companies have to pay a fixed fee for each unit of their emitted carbon. This policy has been imposed in many areas (Murray and Rivers 2015). For instance, Finland is the beginner country to implement carbon taxation in 1990 (Helynen 2004). British Colombia has adopted a carbon tax policy in 2008 and reduces emissions in the province by 9.9% (Murray and Rivers 2015). Australia has enacted a carbon tax policy in 2011 (Gale et al. 2013). The cap-and-trade system was suggested and became a general agreement by many carbon emissions trading markets such as Regional Greenhouse Gas Initiative (RGGI), Japan's Voluntary Emissions Trading Scheme (JVETS), and the European Union Emissions Trading Scheme (EU-ETS) (Song et al. 2017). Under this policy, each company receives the maximum value of emission credits, which is named the cap, from government agencies and the emission credits can be traded through the carbon trading market.

In addition to the environmental regulations imposed by government agencies, consumers' environmental awareness is another important way to stimulate companies to be environmental- friendly (Heydari et al. 2021). More customers are becoming interested in the environmental effects of goods (Heydari et al. 2021; Nouira et al. 2016; Yenipazarli 2019). They are intensely concerned about the carbon footprint and are inclined to give more money for eco-friendly products (Hammami et al. 2015). Relevant data indicate that 75% of Chinese and 83% of Europeans have used environmentally friendly products in the past years (Nouira et al. 2016; Zhou et al. 2018).

To answer the customer's concerns about environmental problems, companies are actively seeking means to control their carbon emissions level and exerting this criterion as a marketing tool to keep existing customers and attract new customers (Benjaafar et al. 2012). For example, some retailers have started to attach carbon emission labeling to their goods such as Tesco in the UK and Casino in France (Hovelaque and Bironneau 2015). The imposed environmental regulations and the increased pressure from environmental agencies and governments to reduce carbon emissions in conjunction with the increasing number of interested consumers in environmental effects indicate that demand rates for many products are becoming dependent on the level of their carbon emissions. Therefore, academia and industry cannot neglect the relation between demand rate and carbon emissions, especially in inventory replenishment decision-making problems (Hammami et al. 2018; Hovelaque and Bironneau 2015; Nouira et al. 2014, 2016).

The major goal of the sustainable supply chain is to present appropriate products and services with economized cost and a healthy environment at the same time to the consumers. To reach the environmental and economic facet of sustainability, the members of the supply chain should cooperate with each other for financial assistance and optimize their operations to control carbon emissions. Nowadays, trade credit policy is a corporate business strategy among members of the supply chain (Tiwari et al. 2018a). Financial Times reported that in 2007 near 90% of global tradeoff is performed through trade credit policy, which is around $25 trillion (Sarkar et al. 2018). Trade credit is one of the most effective sources of short-term financing and can decrease inventory holding costs, increase market share, and stimulate sales growth (Stokes 2005). Many countries and regions such as Taiwan, Hong Kong, Korea, UA, and Europe adopted a trade credit policy as a promotion of export (Cao and Yu 2018).

To reach economic profit also a healthy environment, two appropriate inventory models under a carbon tax and cap-and-trade mechanisms, which consider the combined impacts of partial trade credits and environmental issues on a single-item supply chain, are presented in this paper. In order to better show the real situation, a product's demand depends on the price and the number of carbon emissions. Reasonable pricing is a significant criterion in product purchase for consumers and improves the total profit of companies, especially for green-labeled products. A study of 2463 companies indicated that a 1% increase in price realization could lead to 11.1% contribution improvement (Maihami et al. 2017b). Therefore, the stochastic inventory policies for environmentally friendly products are important and interesting problems (Taleizadeh et al. 2008).

Accordingly, this current study considered two models. In the first model, we have formulated a sustainable inventory model with an exogenous price under partial trade credit and allowable shortage under cap-and-trade and carbon tax mechanisms. In the second model, we have extended the first model with an endogenous price. This model helps us find the optimal price of products and the effect of environmental regulations on the selling price, replenishment decisions, demand rate, and total profit (Taleizadeh et al. 2010, 2012, 2017). The proposed models present insights for companies and decision-makers. This paper also intends to solve the following managerial questions: what are the effects of the credit period and different shortage scenarios on the retailer's decisions? How do carbon price and carbon tax affect the optimal solutions in each model? Do these environmental regulations have the same impacts on the retailer, especially when the retailer makes the replenishment decisions and pricing simultaneously? What is the effect of customers' awareness on the optimal solutions for both models?

The remainder of this research is structured as follows. In the next section, we briefly discuss the relevant literature. A description of the problem with its notations is presented in Sect. 3. The model formulation is provided in Sect. 4. Then in Sect. 5, we solve the proposed models using a global optimization method. In Sect. 6, we provide a numerical example and sensitivity analysis for illustrating the applications of proposed models and the solution procedure. Finally, in Sect. 7, results and future works are described.

2 Literature review

This section provides a brief discussion of relevant literature under two main classifications. The first classification considers the joint trade credit and inventory models under different assumptions like pricing and shortages. The second classification reviews inventory management problems considering carbon emission issues.

2.1 Trade credit policy

Trade credit policies are famous subjects in the new competitive businesses, which have been well studied in the literature. As a pioneer researcher, Goyal (1985) developed the EOQ model incorporating trade credit policy. Teng (2002) revised the model of Goyal (1985) by setting the selling price instead of the unit cost, then discovered that it makes economic sense for buyers to order less amount for taking privilege of trade credit policy. Liao (2008) built an EPQ model for the exponential deteriorating items under two-level trade credit financing schemes. In response to the default (or bad debt) risks of providing trade credits, some researchers have used a partial trade credit policy. For instance, Huang (2007) formulated an inventory model in which a partial and conditional delay of payment is suggested to the buyer (retailer) if the amount of its orders is smaller than a specified amount. Wu et al. (2016) developed a single-supplier single retailer multiple-customers supply chain system in which the supplier offers upstream full trade credit to the retailer and customers get a downstream partial trade credit from the retailer. They assumed that the deterioration rate is non-decreasing and closer to its expiration date. They used discounted cash flow analysis to integrate the time value of money and inflation effects and calculate total costs. Lashgari et al. (2016) investigated the effects of partial delayed payments and partial advanced payments in inventory models under three shortage scenarios. In the first scenario, shortages are not allowed, but the other scenarios were modeled under full and partial shortage, respectively. Taleizadeh et al. (2008a) developed an EOQ model with incremental and total discounts where advanced payment is used. Also, Taleizadeh et al. (2009b) developed their previous research by considering multiple objective functions. Zhang et al. (2019) addressed an EOQ model for a retailer who obtained a partial trade credit from its supplier and gave a full or partial trade credit to its good or bad credit customers, respectively, based on the retailer's cost minimization.

Under a two-level trade credit policy, Maihami et al. (2017a) investigated a model for inventory control and pricing of non-instantaneous deteriorating items taking into account price-dependent stochastic demand and partial backlog. Tiwari et al. (2018a) considered a green production quantity model with random imperfect quantity items, service level constraints, and failure in reworking under trade credit policy and partial backordering. Cárdenas-Barrón et al. (2018) derived an EOQ model with nonlinear stock-dependent demand, nonlinear stock-dependent holding cost, and relaxed ending inventory level under conditions of trade credit policy and partial backordering. Under the condition of stochastic demand and lead-time-sensitive credit period, Mallick et al. (2018) established a non-instantaneous deteriorating inventory model adopting partially backlogged shortages. Otrodi et al. (2019) developed a possibilistic bi-objective model to integrate lot-sizing problems and pricing for perishable items with multiple demand classes under a two-level trade credit policy. In the interesting work of Banu and Mondal (2020), an uncertain inventory model was investigated for deteriorating items under two-level trade credit considering the impact of customers' credit period on the demand. For the first time, they developed a new type of fuzzy number known as a q-fuzzy number which considered both linear and non-linear membership functions together. Under credit-dependent demand, Mahato and Mahata (2021) developed a deteriorating inventory system with two-level trade credit when demand rate is dynamic and changes with product freshness condition and the length of credit period suggested customers.

2.2 Carbon emission control

The issues of carbon emission in inventory management have drawn more academic attention. For instance, Hua et al. (2011) studied how companies handle carbon footprint in inventory management under the cap-and-trade policy and analytically investigated the impacts of the carbon price, carbon cap, and carbon trade on ordering decisions. Bouchery et al. (2012) added various aspects of sustainability into the inventory model and addressed a multi-objective sustainable EOQ model. Zhang et al. (2014) studied the pricing strategies of a green supply chain in which the manufacturer produced both non-green and green products. Their results demonstrated that green products could create market demand.

Under carbon cap-and-trade and carbon offset mechanisms, Dye and Yang (2015) illustrated how environmental policies can be integrated into the EOQ model under trade credit and partial backordering. For the first time, Tsao et al. (2017) formulated newsvendor models considering product recycling, trade credit, and carbon emissions under carbon cap, carbon cap-and-trade, and carbon tax mechanisms. With numerical experiments, they demonstrated that a higher carbon tax leads to a reduction in order quantities, recycle price, and total profit, but an increase in the carbon cap causes an increase in order quantities, recycle price, and total profit. Tiwari et al. (2018b) presented a single-vendor–buyer integrated inventory model with deteriorating and imperfect quality items under a carbon tax policy. Integrating variable carbon emission cost and multi-delay-in-payments policies in a supplier-manufacturer-retailer supply chain was presented by Sarkar et al. (2018).

Additionally, Tang et al. (2018) investigated an inventory control (R, Q) model with three carbon reduction methods such as imposing carbon taxes (or cap-and-trade policy), setting a strict percentage reduction target, and purchasing carbon credits from carbon-offset projects. Under the cap-and-trade policy and governmental intervention, Mondal and Giri (2020) investigated the retailer's competition and coordination in a closed-loop green supply chain with two competing retailers and one manufacturer. They studied one centralized policy and three manufacturer-led decentralized policies: Cournot, Collusion, and Stackelberg depending on different competitive behaviors of the retailers. Their results showed that among the three decentralized cases, Cournot behavior is profitable to customers, manufacturers, and the whole supply chain, but Collusion behavior is profitable to the retailers only when the difference of their basic markets is small. Lee (2020) examined the joint investment in carbon emission reduction and inventory decisions in an EOQ model under a cap-and-price policy, in which the company is penalized (rewarded) if its carbon footprint is large (smaller) than a threshold. He studied the cap-and-offset, carbon tax, carbon cap, and cap-and-trade policies as special-cased of cap-and-price policy. In another research by Mondal and Giri (2021), they presented a two-echelon sustainable supply chain model where demand depends on the selling price and green activities of both members of the supply chain. They formulated four models under the cap-and-trade policy: centralized, decentralized, retailer-led revenue sharing, and bargaining revenue sharing. Sensitivity analysis indicated that a higher amount of carbon trading cost encouraged the manufacturer in improving the greening level and decreasing the carbon emissions.

Since the demand rate for many products is becoming sensitive to the carbon emissions amount, there are some helpful studies that consider the demand rate as an endogenous variable that depends on the carbon emissions. For example, under carbon cap-and-trade and carbon tax mechanism, Hovelaque and Bironneau (2015) presented a sustainable inventory model with price and environmental concerns dependent demand considering the scenarios with exogenous price and endogenous price. Their results show that an environmental policy is more important for cheaper and green-labeled products. They illustrated that increasing environmental sensitivity causes a reduction in carbon emissions and price under endogenous price, and carbon tax decreases the amount of carbon emissions but increases the marginal emissions in both scenarios. Zhang et al. (2015) studied the effect of consumer environmental awareness on the order quantities in a manufacturer-retailer supply chain, where two types of products are made: environmental products and traditional products. Products are different in their environmental quality and price. They assumed that the demand for a product would increase with environmental quantity and decrease with price linearly. Nouira et al. (2016) explored the effect of a carbon emission–dependent demand on decisions relative to the design of forwarding supply chains. Chen et al. (2017) developed a two-echelon sustainable supply chain model when the retailer's demand rate is price and emission-sensitive. Yet, when carbon emissions in inventory models are considered, a correlation between carbon emissions and demand is not clearly established. There is little literature on this subject, and more studies are needed to explore this issue in more detail.

For the convenience of the readers, a quick overview of the relevant previous research papers has been provided in the comparative Table 1. From this table, most mentioned articles on trade credit have failed to take carbon emission issues into account. Also, most articles on carbon emissions have failed to consider the issues of shortages, pricing, and trade credit policies. However, trade credit, carbon emissions, shortages, and price and carbon emission dependent demand exit simultaneously in real situations. Therefore, providing a model that will consider all of these realistic features is needed. To the best of our knowledge, our study is one of the first studies to consider the realistic features of partial trade credit, price and carbon emission dependent demand function, carbon emissions, shortages, and pricing at the same time.

Table 1 Summary of the literature review

Studies	Trade credit	Carbon emission issues	Carbon emission-sensitive demand	Shortages			Price	
				LS*	PB*	FB*	Exog*	End*
Hua et al. (2011)		*						
Zhang et al. (2014)		*	*					*
Hovelaque and Bironneau (2015)		*	*				*	*
Zhang et al. (2015)		*	*				*	
Chen et al. (2017)		*	*				*	
Wu et al. (2016)	*						*	
Lashgari et al. (2016)	*				*		*	
Maihami et al. (2017a)	*				*			*
Mallick et al. (2018)	*				*		*	
Otrodi et al. (2019)	*							*
Tsao et al. (2017)	*	*						
Dye and Yang (2015)	*	*			*		*	
Tiwari et al. (2018a)	*	*			*		*	
Sarkar et al. (2018)	*	*					*	
Mondal and Giri (2021)	*	*						*
This study	*	*	*	*	*	*	*	*

LS Lost sales, PB: partial backordering, *FB* Full backordering, *Exog* Exogenous, *End* Endogenous

3 Problem definition and notations

In this research, we regard sustainability issues in the context of integration, partial delayed payments, and inventory management models in a supplier-retailer supply chain system with price and carbon emission-dependent demand. The supplier produces a single product and delivers it to a particular retailer. The supplier can offer various trade credits to the retailer to delay the payments. One well-known mechanism widely applied in companies is partial delayed payment. Under this strategy, the retailer pays a determined portion, α percent, of the purchasing cost immediately and the remaining quantity, $(1-\alpha)$ percent, would be paid at time M, credit period, without additional charges (see Figs. 1 and 2).

In the proposed problem, we first formulate an inventory model with an exogenous price, and then, we consider an inventory model with an endogenous price for maximizing the retailer's total profit and minimizing carbon emissions under cap-and-trade and carbon tax mechanisms. In the first model, the retailer optimizes its profit only through replenishment decisions, while in the second model, the retailer optimizes its profit through price and replenishment decisions.

In order to show the obvious impact of partial delayed payments when incorporated with carbon emissions and shortages, the other assumptions in this study are regarded as follows:

Fig. 1 Behavior of EOQ system for Scenario (1): $M \leq FT$

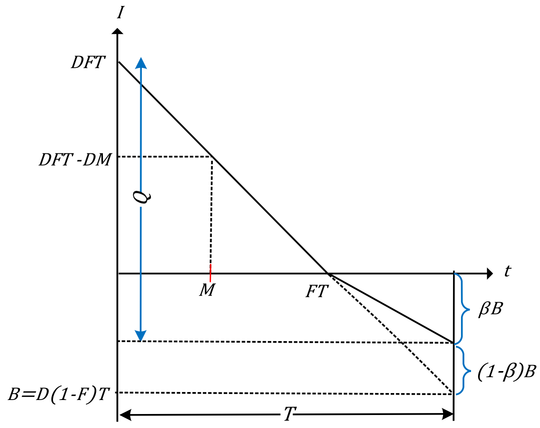
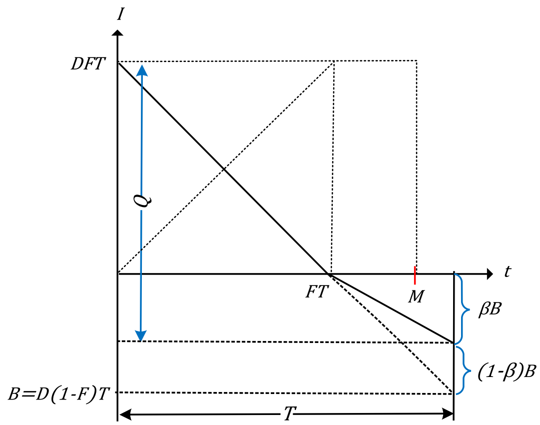


Fig. 2 Behavior of EOQ system for Scenario (2):$M \geq FT$



- Demand rate is a deterministic function and depends on carbon emission amounts and selling price.
- There is no lead time.
- Partial backlogged shortages are permitted.
- The time horizon is infinite.
- There is no deterioration.
- All parameters are precise and constant.
- The carbon emission function is computed with respect to the activities of the delivery, storage, purchase, and delivery of backordered items. To secure future business, backordered items are delivered with expedited delivery options that lead to extra carbon emissions.

The notations below are applied to develop the mathematical models:

4 The mathematical model

In this section, we have modeled two inventory models with price and carbon emission dependent demand, partial delayed payment, and partial backordering by considering the impacts of environmental issues based on an EOQ framework. In the first model, we first formulate an inventory model with an exogenous price under cap-and-trade and carbon tax mechanisms; in the second model, the proposed problem is formulated with an endogenous price.

The inventory system evolves as follows: Q units of items enter the system at the beginning of each cycle and are depleted to zero due to demand at the time FT. Next, shortages happen till the end of the order cycle. At the time of placing an order, the retailer obtains a partial trade credit from its supplier in which the retailer has to pay $\%\alpha$ of the total purchased quantity immediately and $\%(1-\alpha)$ of the total purchased amount to be paid at the time M. The whole process is repeated. The behavior of the considered inventory system is shown in Figs. 1 and 2.

As shown in Figs. 1 and 2, the order quantity per cycle is equal to the summation of the amount of backordered demands ($\beta D(1-F)T$) and the initial inventory on hand (DFT) which can be formulated as:

$$Q = \beta D(1-F)T + DFT = DT(F + \beta(1-F)) \tag{1}$$

The total profit without considering carbon emissions per cycle consists of the following components:

1. Sales revenue (SR)

$$SR = P \times Q = PD(F + \beta(1-F))\mathrm{T} \tag{2}$$

2. Holding cost $\left(C_H\right)$

$$C_H = h \times \left(\frac{DFT \times FT}{2}\right) = \frac{hDF^2T^2}{2} \tag{3}$$

3. Purchasing cost $\left(C_P\right)$

$$C_P = C \times Q = CD(F + \beta(1-F))T \tag{4}$$

4. Ordering cost $\left(C_O\right)$

$$C_O = K \tag{5}$$

5. Total shortages cost

As shown in Figs. 1 and 2, during the time interval $[FT,\ T]$ the inventory system has shortages and faces a mixture of lost sales and backorders. Since the maximum shortage amount is $B = D(1-F)T$ and β is the fraction of demand backordered during $[FT,\ T]$, goodwill cost for lost sales $\left(C_G\right)$ and the shortage cost for backorders $\left(C_B\right)$ are as follows, respectively:

$$C_G = G \times ((1-\beta)D(1-F)T) = G(1-\beta)D(1-F)T \tag{6}$$

And

$$C_B = \pi \times \left(\frac{\beta D(1-F)T \times (1-F)T}{2} \right) = \frac{\beta \pi D(1-F)^2 T^2}{2} \tag{7}$$

6. The interest earned and interest payable is calculated based on the relationship between the delay period offered to the retailer (M) and the length of time in which no inventory shortage happens (FT); thus, we consider the following two scenarios.

4.1 Scenario (1):$M \leq FT$

Under the partial delayed payment strategy, the retailer pays the α percentage of the entire purchasing cost immediately, instant payments, and the rest of purchasing cost, credit payments, is paid at the time M. In this scenario, the interest payable ($CC(1)$) is calculated as follows (see Fig. 1):

$$\begin{aligned} CC(1) &= \left[I_c \times \alpha C \times \frac{DFT \times FT}{2} \right] + \left[I_c \times (1-\alpha) C \times \frac{D(FT-M) \times (FT-M)}{2} \right] \\ &= \frac{\alpha C I_c D F^2 T^2}{2} + \frac{(1-\alpha) C I_c D (FT-M)^2}{2} \end{aligned} \tag{8}$$

At the beginning of the cycle, the amount $\beta DT(1-F)$ is consumed for satisfying the backordered demand. As mentioned earlier, the retailer should pay only the α percent of the purchasing cost of these items immediately, so the interest earned per cycle on the rest purchasing cost is $\left(\beta DT(1-F)(1-\alpha)C \times M \times I_e\right)$. On the other hand, the amount of new demands at a time M is DM (see Fig. 1), so the interest earned per cycle from new demands is $\frac{DM \times M}{2}(1-\alpha)CI_e$. Therefore, the total interest earned ($IE(1)$) for Scenario (1) is obtained as follows:

$$\begin{aligned} IE(1) &= \left[\beta DT(1-F)(1-\alpha)C \times M \times I_e \right] + \left[\frac{DM \times M}{2}(1-\alpha)CI_e \right] \\ &= \beta(1-\alpha)CI_e D(1-F)TM + \frac{(1-\alpha)CI_e DM^2}{2} \end{aligned} \tag{9}$$

4.2 Scenario (2):$M \geq FT$

For this Scenario, the total interest payable per cycle ($CC(2)$) is calculated by the following equation (see Fig. 2):

$$CC(2) = I_c \times \alpha\, C \times \frac{DFT \times FT}{2} = \frac{\alpha\, CI_c DF^2 T^2}{2} \tag{10}$$

The total interest earned per cycle ($IE(2)$) is obtained by the following equation:

$$
\begin{aligned}
IE(2) = & \underbrace{\left[\beta DT(1-F)(1-\alpha)C\times M\times I_e\right]}_{\textit{interest earned on backordered demands}} \\
& + \underbrace{\left[(1-\alpha)CI_e\times\frac{DFT\times FT}{2}\right]+\left[(1-\alpha)CI_e\times(DFT\times(M-FT))\right]}_{\textit{Interest earned on new demand}}\Bigg) \\
= & \left[I_e(1-\alpha)C\beta D(1-F)TM\right]+\left[I_e(1-\alpha)C\frac{DF^2T^2}{2}\right] \\
& + \left[I_e(1-\alpha)CDFT(M-FT)\right]
\end{aligned}
\tag{11}
$$

4.3 Carbon emissions and demand functions

Referring to Dye and Yang (2015) and Tang et al. (2015), the carbon emission amount (CE) can be determined by the following equation:

$$
\begin{aligned}
CE = & \underbrace{\frac{\hat{C}Q}{T}}_{CEP}+\underbrace{\frac{\hat{K}}{T}}_{CEO}+\underbrace{\frac{\hat{h}DF^2T^2}{2T}}_{CES}+\underbrace{\frac{\beta\hat{\pi}D(1-F)^2T^2}{2T}}_{CEB} \\
= & \hat{C}D(F+\beta(1-F))+\frac{\hat{K}}{T}+\frac{\hat{h}DF^2T}{2}+\frac{\beta D\hat{\pi}(1-F)^2T}{2}
\end{aligned}
\tag{12}
$$

In Eq. 12, the terms 1 to 4 represent the amount of carbon emissions from purchases (CEP), replenishment orders (CEO), storage (CES), and backorders (CEB) per year, respectively. To secure future business, backordered items are delivered with expedited delivery options that lead to extra carbon emissions.

As discussed earlier, this research is developed with a linear demand function as follows:

$$D=\gamma-\theta P-\delta\, CE \tag{13}$$

where γ and θ are the maximum market demand and price sensitivity, respectively; CE is the carbon emission amount with elasticity δ, which is calculated by Eq. (12). We can rewrite the demand function by using Eqs. (12) and (13) as follows:

$$D=\gamma-\theta P-\delta\left(\hat{C}D(F+\beta(1-F))+\frac{\hat{K}}{T}+\frac{\hat{h}DF^2T}{2}+\frac{\beta\hat{\pi}D(1-F)^2T}{2}\right) \tag{14}$$

Expanding the above equation:

$$\left(1+\hat{C}\delta(F+\beta(1-F))+0.5\delta\hat{h}F^2T+0.5\beta\hat{\pi}\left(F^2-2F+1\right)T\right)D=\gamma-\theta P-\frac{\delta\hat{K}}{T} \tag{15}$$

Finally, we have

$$D = \frac{(\gamma - \theta P)T - \delta \hat{K}}{\left(1 + \delta\beta\hat{C}\right)T + \delta\hat{C}(1-\beta)FT + 0.5\delta\left(\hat{h} + \beta\hat{\pi}\right)F^2T^2 - \delta\beta\hat{\pi}FT^2 + 0.5\beta\delta\hat{\pi}T^2} \tag{16}$$

As mentioned previously, the first model is formulated with an exogenous price while the second model is presented with an endogenous price. In this subsection, we rewrite the demand function for each model by defining additional variables as follows:

- Demand function with an exogenous price (Model 1):

$$\begin{aligned} D &= \frac{(\gamma - \theta P)T - \delta \hat{K}}{\left(1 + \delta\beta\hat{C}\right)T + \delta\hat{C}(1-\beta)FT + 0.5\delta\left(\hat{h} + \beta\hat{\pi}\right)F^2T^2 - \delta\beta\hat{\pi}FT^2 + 0.5\beta\delta\hat{\pi}T^2} \\ &= \frac{(\gamma - \theta P)T - \delta \hat{K}}{\omega_1 T + \omega_2 F T + \omega_3 F^2 T^2 - \omega_4 F T^2 + \omega_5 T^2} = \frac{aT - b}{\varpi} \end{aligned} \tag{17}$$

where:

$$a = (\gamma - \theta P) > 0 \tag{18}$$

$$b = \delta \hat{K} > 0 \tag{19}$$

$$\varpi = \omega_1 T + \omega_2 F T + \omega_3 F^2 T^2 - \omega_4 F T^2 + \omega_5 T^2 \tag{20}$$

$$\omega_1 = 1 + \delta\beta\hat{C} > 0 \tag{21}$$

$$\omega_2 = \delta\hat{C}(1-\beta) > 0 \tag{22}$$

$$\omega_3 = 0.5\delta(\hat{h} + \beta\hat{\pi}) > 0 \tag{23}$$

$$\omega_4 = \delta\beta\hat{\pi} > 0 \tag{24}$$

$$\omega_5 = 0.5\delta\beta\hat{\pi} > 0 \tag{25}$$

- Demand function with an endogenous price (Model 2):

$$\begin{aligned} D &= \frac{(\gamma - \theta P)T - \delta \hat{K}}{\left(1 + \delta\beta\hat{C}\right)T + \delta\hat{C}(1-\beta)FT + 0.5\delta\left(\hat{h} + \beta\hat{\pi}\right)F^2T^2 - \delta\beta\hat{\pi}FT^2 + 0.5\beta\delta\hat{\pi}T^2} \\ &= \frac{(\gamma - \theta P)T - \delta \hat{K}}{\omega_1 T + \omega_2 FT + \omega_3 F^2T^2 - \omega_4 FT^2 + \omega_5 T^2} = \frac{\gamma T - \theta PT - b}{\varpi} \end{aligned} \tag{26}$$

where b and ϖ are the same as Eqs. 19 and 20 respectively, and $\omega_i (i = 1, 2, ..., 5)$ are the same as Eqs. 21–25.

4.4 The inventory system under cap–and–trade and carbon tax mechanisms

The main objectives of our models are maximizing the retailer's profit and minimizing carbon emissions under a carbon tax and cap-and-trade mechanisms. Under the cap-and-trade mechanism, retailers have a limit on emissions which is modeled by $CE + X = Y$, where X is the amount of carbon emissions that can be transferred (positive, negative, or zero) and Y is the total carbon emissions permits. When $X > 0$, the retailer sells X unit of permits with price s per unit of emission; when $X < 0$, the retailer buys $|X|$ units of permits at s; when $X = 0$, the retailer neither sells nor buys any permits. (Hua et al. 2011). Carbon tax regulation is one of the main mechanisms in the world to reduce carbon emissions and promote energy-saving (Xu et al. 2016). Under this regulation, companies are charged for each unit of their carbon emissions with a constant tax rate. This regulation has been imposed in many regions, such as Australia in 2011 and British Columbia in 2008, which led to a 9.9% decrease in carbon emissions in that province. Therefore, the total profit with considering the amount of carbon emissions under the two mentioned policies can be calculated as:

$$\text{Total profit} = \left\{ \begin{array}{l} \text{sales revenue} - \text{holding cost} - \text{purchasing cost} - \text{ordering cost} \\ -\text{back ordering cost} - \text{good will losses cost} - \text{capital cost} + \text{interest earned} \\ +(\text{carbon price} \times \text{the transfer amount of carbon emissions}) \\ -(\text{carbon tax} \times \text{carbon emissions}) \end{array} \right\} \tag{27}$$

In the following, we first formulate the objective functions for each model by using Eq. 27, then transform them into a constrained signomial geometric programming problem.

4.4.1 Model 1: EOQ model with an exogenous price

For this model, the average annual total profit is calculated as follows:

$$ATP_1 = \begin{cases} ATP_1(1) \ if \ M \le FT \\ ATP_1(2) \ if \ M \ge FT \end{cases} \tag{28}$$

where

$$\begin{aligned} ATP_1(i) = &\frac{1}{T}\left(SR - C_p - C_o - C_h - C_b - C_G - CC(i) + IE(i)\right) \\ &- t \times CE + s \times (Y - CE) i = 1, 2 \end{aligned} \tag{29}$$

The following results are obtained after simplifying (See Appendix A):

$$\begin{aligned} ATP_1(1) &= \left(\mu_0 - \mu_2 TF^2 + \mu_3 FT + \mu_4 F - \mu_5 T - \mu_6 - \mu_7 T^{-1}\right)(aT - b)\varpi^{-1} - \mu_1 T^{-1} + sY \\ &= \left(\mu_0 a + \mu_5 b\right) T\varpi^{-1} - \left(\mu_0 b + \mu_7 a\right)\varpi^{-1} - \mu_1 T^{-1} - \mu_2 a \ F^2 T^2 \varpi^{-1} \\ &\quad + \mu_2 bF^2 T\varpi^{-1} + \mu_3 a \ F \ T^2 \varpi^{-1} - \mu_3 b \ F \ T\varpi^{-1} + \mu_4 a \ F \ T\varpi^{-1} \\ &\quad - \mu_4 bF\varpi^{-1} - \mu_5 aT^2 \varpi^{-1} - \mu_6 aT\varpi^{-1} + \mu_6 b\varpi^{-1} + \mu_7 bT^{-1}\varpi^{-1} + sY \end{aligned} \tag{30}$$

where:

$$\mu_0 = \left(P - C' + \beta(1-\alpha)CI_eM\right) \geq 0 \tag{31}$$

$$\mu_1 = K' \geq 0 \tag{32}$$

$$\mu_2 = 0.5\left(h' + \beta\pi'' + \alpha CI_c + (1-\alpha)CI_c\right) \geq 0 \tag{33}$$

$$\mu_3 = \beta\pi'' \geq 0 \tag{34}$$

$$\mu_4 = \left(\left(P - C' + G\right)(1-\beta) + \left(I_c - \beta I_e\right)(1-\alpha)CM\right) \geq 0 \tag{35}$$

$$\mu_5 = 0.5\beta\pi'' \geq 0 \tag{36}$$

$$\mu_6 = \left(P - C' + G\right)(1-\beta) \geq 0 \tag{37}$$

$$\mu_7 = 0.5\left(I_c - I_e\right)(1-\alpha)CM^2 \geq 0 \tag{38}$$

And

$$\begin{aligned} ATP_1(2) &= \left(\tau_0 - \tau_2TF^2 + \tau_3FT + \tau_4F - \tau_5T - \tau_6\right)(aT-b)\varpi^{-1} - \tau_1T^{-1} + sY \\ &= \left(\tau_0a + \tau_5b\right)T\varpi^{-1} - \tau_0b\varpi^{-1} - \tau_1T^{-1} - \tau_2aF^2T^2\varpi^{-1} + \tau_2bF^2T\varpi^{-1} + \tau_3aFT^2\varpi^{-1} \\ &\quad - \tau_3bFT\varpi^{-1} + \tau_4aFT\varpi^{-1} - \tau_4bF\varpi^{-1} - \tau_5aT^2\varpi^{-1} - \tau_6aT\varpi^{-1} + \tau_6b\varpi^{-1} + sY \end{aligned} \tag{39}$$

where

$$\tau_0 = \left(P - C' + \beta(1-\alpha)CI_eM\right) \geq 0 \tag{40}$$

$$\tau_1 = K' \geq 0 \tag{41}$$

$$\tau_2 = 0.5\left(h' + \beta\pi'' + \alpha CI_c + (1-\alpha)CI_e\right) \geq 0 \tag{42}$$

$$\tau_3 = \beta\pi'' \geq 0 \tag{43}$$

$$\tau_4 = \left(\left(P - C' + G\right) + \left((1-\alpha)CI_eM\right)\right)(1-\beta) \geq 0 \tag{44}$$

$$\tau_5 = 0.5\beta\pi'' \geq 0 \tag{45}$$

$$\tau_6 = \left(P - C' + G\right)(1-\beta) \geq 0 \tag{46}$$

The main objective of Model (1) is to obtain the optimal replenishment strategy ($F$and$T$) to maximize the total profit $\left(ATP_1\right)$. For determining the optimal solutions, the two following optimization problems first must be solved, then their objective functions are compared to each other in order to determine which is higher, e.g.max $\left(ATP_1(1), ATP_1(2)\right)$; those amounts related to the higher profit should be selected as the optimal solutions.

- The optimization problem of Model 1 and Scenario (1):

$$Max\ ATP_1(1) \tag{47}$$

$$\text{s.t.}\quad M \le FT \tag{48}$$

$$\varpi = \omega_1 T + \omega_2 F T + \omega_3 F^2 T^2 - \omega_4 F T^2 + \omega_5 T^2 \tag{49}$$

$$0 < F \le 1 \tag{50}$$

$$T, \varpi > 0 \tag{51}$$

where $ATP_1(1)$ is given by the Eq. 30 and $\omega_i (i = 1, 2, ..., 5)$ are shown by the Eqs. 21–25.
- The optimization problem of Model 1 and Scenario (2):

$$Max\ ATP_1(2) \tag{52}$$

$$s.t.\quad M \ge FT \tag{53}$$

and constraints 49–51.where $ATP_1(2)$ is given by the Eq. 39.

4.4.2 Model 2: EOQ model with an endogenous price

For this model, the average annual total profit is calculated as follows:

$$ATP_2 = \begin{cases} ATP_2(1) & if\ \ M \le FT \\ ATP_2(2) & if\ \ M \ge FT \end{cases} \tag{54}$$

where

$$\begin{aligned} ATP_2(i) =& \frac{1}{T}\left(SR - C_p - C_o - C_h - C_b - C_G - CC(i) + IE(i)\right) \\ &- t \times CE + s \times (Y - CE) \quad i = 1, 2 \end{aligned} \tag{55}$$

The following results are obtained after simplifying (See Appendix A):

$$\begin{aligned}ATP_2(1) = {} & \kappa_0\gamma PF\varpi^{-1} - \kappa_0\theta P^2F\varpi^{-1} - \kappa_0 bPFT^{-1}\varpi^{-1} + \kappa_1\gamma P\varpi^{-1} - \kappa_1\theta P^2\varpi^{-1} - \kappa_1 bPT^{-1}\varpi^{-1} \\ & - \kappa_2\gamma\varpi^{-1} + \kappa_2\theta P\varpi^{-1} + \kappa_2 bT^{-1}\varpi^{-1} - \kappa_3\gamma F\varpi^{-1} + \kappa_3\theta FP\varpi^{-1} + \kappa_3 bFT^{-1}\varpi^{-1} \\ & - \kappa_4\gamma F^2T\varpi^{-1} + \kappa_4\theta F^2TP\varpi^{-1} + \kappa_4 bF^2\varpi^{-1} + \kappa_5\gamma FT\varpi^{-1} - \kappa_5\theta FTP\varpi^{-1} \\ & - \kappa_5 bF\varpi^{-1} - \kappa_6\gamma T\varpi^{-1} + \kappa_6\theta TP\varpi^{-1} + \kappa_6 b\varpi^{-1} - \kappa_7\gamma T^{-1}\varpi^{-1} + \kappa_7\theta PT^{-1}\varpi^{-1} \\ & + \kappa_7 bT^{-2}\varpi^{-1} + \kappa_8\gamma\varpi^{-1} - \kappa_8\theta P\varpi^{-1} - \kappa_8 bT^{-1}\varpi^{-1} + \kappa_9\gamma F\varpi^{-1} - \kappa_9\theta FP\varpi^{-1} \\ & - \kappa_9 bFT^{-1}\varpi^{-1} - \kappa_{10}T^{-1} + sY\end{aligned} \tag{56}$$

where:

$$\kappa_0 = (1-\beta) \geq 0 \tag{57}$$

$$\kappa_1 = \beta \geq 0 \tag{58}$$

$$\kappa_2 = \left(\beta C' + G(1-\beta)\right) \geq 0 \tag{59}$$

$$\kappa_3 = \left(C' - G\right)(1-\beta) \geq 0 \tag{60}$$

$$\kappa_4 = 0.5\left(h' + \beta\pi'' + CI_c\right) \geq 0 \tag{61}$$

$$\kappa_5 = \beta\pi'' \geq 0 \tag{62}$$

$$\kappa_6 = 0.5\beta\pi'' \geq 0 \tag{63}$$

$$\kappa_7 = 0.5(1-\alpha)C\left(I_c - I_e\right)M^2 \geq 0 \tag{64}$$

$$\kappa_8 = \beta(1-\alpha)CI_eM \geq 0 \tag{65}$$

$$\kappa_9 = \left(I_c - \beta I_e\right)(1-\alpha)CM \geq 0 \tag{66}$$

$$\kappa_{10} = K' \geq 0 \tag{67}$$

And

$$\begin{aligned}ATP_2(2) = {} & \eta_0\gamma PF\varpi^{-1} - \eta_0\theta P^2F\varpi^{-1} - \eta_0 bPFT^{-1}\varpi^{-1} + \eta_1\gamma P\varpi^{-1} - \eta_1\theta P^2\varpi^{-1} - \eta_1 bPT^{-1}\varpi^{-1} \\ & - \eta_2\gamma\varpi^{-1} + \eta_2\theta P\varpi^{-1} + \eta_2 bT^{-1}\varpi^{-1} - \eta_3\gamma F\varpi^{-1} + \eta_3\theta FP\varpi^{-1} + \eta_3 bFT^{-1}\varpi^{-1} \\ & - \eta_4\gamma F^2T\varpi^{-1} + \eta_4\theta F^2TP\varpi^{-1} + \eta_4 bF^2\varpi^{-1} + \eta_5\gamma FT\varpi^{-1} - \eta_5\theta FTP\varpi^{-1} \\ & - \eta_5 bF\varpi^{-1} - \eta_6\gamma T\varpi^{-1} + \eta_6\theta TP\varpi^{-1} + \eta_6 b\varpi^{-1} + \eta_7\gamma\varpi^{-1} - \eta_7\theta P\varpi^{-1} \\ & - \eta_7 bT^{-1}\varpi^{-1} + \eta_8\gamma F\varpi^{-1} - \eta_8\theta FP\varpi^{-1} - \eta_8 bFT^{-1}\varpi^{-1} - \eta_9 T^{-1} + sY\end{aligned} \tag{68}$$

where

$$\eta_0 = (1-\beta) \geq 0 \tag{69}$$

$$\eta_1 = \beta \geq 0 \tag{70}$$

$$\eta_2 = \left(\beta C' + G(1-\beta)\right) \geq 0 \tag{71}$$

$$\eta_3 = \left(C' - G\right)(1-\beta) \geq 0 \tag{72}$$

$$\eta_4 = 0.5\left(h' + \beta\pi'' + \alpha CI_c + (1-\alpha)CI_e\right) \geq 0 \tag{73}$$

$$\eta_5 = \beta\pi'' \geq 0 \tag{74}$$

$$\eta_6 = 0.5\beta\pi'' \geq 0 \tag{75}$$

$$\eta_7 = \beta(1-\alpha)CI_eM \geq 0 \tag{76}$$

$$\eta_8 = (1-\beta)(1-\alpha)CI_eM \geq 0 \tag{77}$$

$$\eta_9 = K' \geq 0 \tag{78}$$

In this model, the retailer's profit is maximized through F,T and selling price (P). As mentioned earlier, to determine the optimal solutions, first we have to solve the two following optimization problems, then their objective functions are compared to each other and those amounts related to the higher profit are selected as the optimal solutions.

- The optimization problem of Model 2 and Scenario (1):

$$Max\ ATP_2(1) \tag{79}$$

$$s.t.\ P > 0 \tag{80}$$

and constraints 49–51.where $ATP_2(1)$ is given by the Eq. 56.

- The optimization problem of Model 2 and Scenario (2):

$$Max\ ATP_2(2) \tag{81}$$

s.t. constraints (49–51), (53), (80)

where $ATP_2(2)$ is given by the Eq. 68.

5 Solution methodology

An examination of assumptions related to trade credit, price and carbon emissions dependent demand, and shortages in EOQ models transformed the models into non-linear programming (NLP) problems. A useful method for solving these types of NLP problems is the Geometric Programming (GP) approach (Liu 2007; Mandal et al. 2005; Samadi et al. 2013). This technique has very theoretical and effective computational features for solving complex optimization problems in various scopes, such as engineering, management, science, etc. (Jung and Klein 2005; Shen et al. 2008). Signomial Geometric Programming (SGP) problems are the initial developments of GP problems (Passy and Wilde 1967). These kinds of problems are categorized in the class of non-convex optimization problems that are inherently intractable NP-hard problems (Xu 2014). SGP problems are well applied to solve inventory models in the literature (Bayati et al. 2013; Jung and Klein 2005; Karimian et al. 2020; Kim and Lee 1998; Mandal et al. 2006; Moradi et al. 2021; Rabbani and Aliabadi 2019; Samadi et al. 2013). An important factor of SGP is the used methods. In the last decades, several approaches have been provided to solve SGP problems. For instance, branch-and-bound (Floudas 2000), heuristic strategy (Chiang et al. 2005), and quasi geometric programming problems (Toscano and Amouri 2012) are common approaches.

This section checks the efficiency of the suggested models using a global optimization method proposed by Xu (2014). In this approach, first, some algebra and conversion tactics are applied to transform the initial SGP problems into a sequence of standard GP problems that are nonlinear convex problems and can be efficiently solved, then the proposed approach is presented as an iterative algorithm to obtain the global solution. Xu (2014) illustrated the effectiveness and tractability of the suggested method by seven examples. The obtained outcomes of examples indicated that the suggested method needs much lower CP time to get global optimal solutions of SGP problems with fewer errors in objective functions and constraints than the recent methods do.

Therefore, we first rewrite the optimization problems calculated in the previous section in the form of an SGP problem, we then discuss how to use the proposed approach of Xu (2014) for finding the optimal solutions. Since the processes of solving for four optimization problems proposed in the previous section are the same, in this section only the steps of solving the optimization problem of Model 1 and Scenario (1), Eqs. 47–51, are given. Other optimization problems are solved similar to Model 1 and Scenario (1).

An SGP program is equivalent to the following optimization problem:

$$\min \ \varphi_0(z) = \sum_{k=1}^{k_0} a_{0k} b_{0k} \prod_{i=1}^{I} z_i^{\alpha_{oik}}$$

$$\text{s.t.} \ \ \varphi_j(z) = \sum_{k=1}^{k_j} a_{jk} b_{jk} \prod_{i=1}^{I} z_i^{\alpha_{jik}} \leq 1 \quad \forall \, j = 1, 2, \ldots, a$$

$$\varphi_j(z) = \sum_{k=1}^{k_j} a_{jk} b_{jk} \prod_{i=1}^{I} z_i^{\alpha_{jik}} = 1 \quad \forall\ j = a+1,\ a+2, ..., b$$

$$z_i > 0\ \forall\ i = 1, 2, ..., I$$

where $b_{jk} > 0$, $a_{jk} = \pm 1$, $\alpha_{jik} \in R, k_j(j = 0, 1, 2, 3, ..., b)$ indicates the number of terms of objective function and constraints, $\prod_{i=1}^{I} z_i^{\alpha_{jik}}$ and $\varphi_j(j = 0, 1, 2, 3, ..., b)$, respectively, are monomial and signomial functions. In the SGP problem given in above, the objective function is to minimize the sum (both positive and negative) of non-linear functions, also each constraint is the sum (both positive and negative) of non-linear functions and is equal or less than one. Therefore, to transform the optimization problem (47–51) into the presented SGP problem, the objective function (47) needs to be rewritten in the minimization form and constraints (48) and (49) become equal or less than one. These conversions are presented in the following form:

$$\xrightarrow{\textit{constraint}\,(49)} \quad \varpi = \omega_1\, T + \omega_2\, F\, T + \omega_3\, F^2\, T^2 - \omega_4\, F\, T^2 + \omega_5\, T^2$$

$$\xrightarrow{\text{two sides of equation} \times \varpi^{-1}} \quad \underbrace{\varpi^{-1} \times \varpi}_{1} = \varpi^{-1}$$
$$\times \left(\omega_1\, T + \omega_2\, FT + \omega_3\, F^2T^2 - \omega_4\, FT^2 + \omega_5\, T\right)$$

$$\Rightarrow \quad \omega_1\, T\varpi^{-1} + \omega_2\, FT\varpi^{-1} + \omega_3\, F^2T^2\varpi^{-1} - \omega_4\, FT^2\varpi^{-1} + \omega_5\, T\varpi^{-1} = 1$$

In a similar way, Eq. 48 becomes as $MF^{-1}\, T^{-1} \leq 1$. Also, it is known that $Max\ ATP_1(1) = -Min\ \left(-ATP_1(1)\right)$. After these transformations, the optimization problem (47–51) can be rewritten as the following equivalent optimization problem, which is a constrained SGP problem (neglecting the constant terms):

$$\begin{aligned} Min\ \left(-ATP_1(1)\right) = &- \left(\mu_0 a + \mu_5 b\right)T\varpi^{-1} + \left(\mu_0 b + \mu_7 a\right)\varpi^{-1} + \mu_1 T^{-1} + \mu_2 aF^2T^2\varpi^{-1} \\ &- \mu_2 bF^2T\varpi^{-1} - \mu_3 aFT^2\varpi^{-1} + \mu_3 bFT\varpi^{-1} - \mu_4 aFT\varpi^{-1} + \mu_4 bF\varpi^{-1} \\ &+ \mu_5 aT^2\varpi^{-1} + \mu_6 aT\varpi^{-1} - \mu_6 b\varpi^{-1} - \mu_7 bT^{-1}\varpi^{-1} \end{aligned} \tag{82}$$

$$s.t.\, MF^{-1}\, T^{-1} \leq 1 \tag{83}$$

$$\omega_1 T\varpi^{-1} + \omega_2\, FT\varpi^{-1} + \omega_3\, F^2T^2\varpi^{-1} - \omega_4\, FT^2\varpi^{-1} + \omega_5\, T\varpi^{-1} = 1 \tag{84}$$

$$0 < F \leq 1 \tag{85}$$

$$T, \varpi > 0 \tag{86}$$

Now Eqs. 82–86 can be solved using the global optimization approach given in the next subsection.

5.1 Global optimization approach

In this work, the global optimization approach of Xu (2014) is applied to solve our models. The suggested approach depends on posing the non-convex optimization problem (82–86), a constrained SGP problem, as a serious GP problem that can be transformed into a nonlinear convex problem, and thus it can be optimized effectively through some easy conversion and convexification techniques.

Referring to Xu (2014), we rewrite the problem (82–86) as:

$$\min\ Z_0 = Z_0^+ - Z_0^- \tag{87}$$

$$s.t.\ Z_1^+ - Z_1^- = 1 \tag{88}$$

and constraints (83), (85), and (86) where Z_0^+ and Z_1^+ are positive terms of the objective function (82) and constraint (84), respectively, Z_0^- and Z_1^- are negative terms of the objective function (82) and constraint (84), respectively, formulated as:

$$\begin{aligned} Z_0 = &-(\mu_0 a + \mu_5 b)T\varpi^{-1} + (\mu_0 b + \mu_7 a)\varpi^{-1} + \mu_1 T^{-1} \\ &+\mu_2 aF^2T^2\varpi^{-1} + \mu_3 bFT\varpi^{-1} - \mu_4 aFT\varpi^{-1} \\ &+a\mu_4 bF\varpi^{-1} + \mu_5 aT^2\varpi^{-1} + \mu_6 aT\varpi^{-1} - \mu_6 b\varpi^{-1} - \mu_7 bT^{-1}\varpi^{-1} \end{aligned} \tag{89}$$

$$\begin{aligned} Z_0^+ = &\left(\mu_0 b + \mu_7 a\right)\varpi^{-1} + \mu_1 T^{-1} + \mu_2 aF^2T^2\varpi^{-1} \\ &+ \mu_3 bFT\varpi^{-1} + \mu_4 bF\varpi^{-1} + \mu_5 aT^2\varpi^{-1} + \mu_6 aT\varpi^{-1} \end{aligned} \tag{90}$$

$$\begin{aligned} Z_0^- = &\left(\mu_0 a + \mu_5 b\right)T\varpi^{-1} + \mu_2 bTF^2\varpi^{-1} + \mu_3 aT^2F\varpi^{-1} \\ &+ \mu_4 aFT\varpi^{-1} + \mu_6 b\varpi^{-1} + \mu_7 bT^{-1}\varpi^{-1} \end{aligned} \tag{91}$$

$$Z_1^+ = \omega_1 T\varpi^{-1} + \omega_2 FT\varpi^{-1} + \omega_3 F^2T^2\varpi^{-1} + \omega_5 T\varpi^{-1} \tag{92}$$

$$Z_1^- = \omega_4 FT^2\varpi^{-1} \tag{93}$$

Then an adequately great number $GN > 0$ is defined so that $Z_0^+ - Z_0^- + GN > 0$ and the problem (87–88) can be rewritten as follows:

$$\min\ Z_0^+ - Z_0^- + GN \tag{94}$$

s.t. and constraints (83), (85), (86) and (88)

In order to represent constraint (88) and objective function (94) as fractional and linear forms, respectively, we apply an extra variable R and rewrite the above problem in the following form:

$$\min \quad R \tag{95}$$

$$s.t.\ \frac{Z_0^+ + GN}{Z_0^- + R} \le 1 \tag{96}$$

$$\frac{Z_1^+}{Z_1^- + 1} = 1 \tag{97}$$

constraints (83), (85) and (86)

Equations 83, 85, 86, and 95–97 are equivalent to a complementary geometric programming problem (Chiang et al. 2007) that belong to a class of NP-hard non-convex problems. According to Xu (2014), we introduce the extra variable S and reformulate the optimization problem (83), (85), (86) and (95–97) as:

$$\min\ R + wS \tag{98}$$

$$s.t.\ \frac{Z_1^+}{Z_1^- + 1} \le 1 \tag{99}$$

$$\frac{Z_1^+}{Z_1^- + 1} \ge 1 - S \tag{100}$$

$$0 \le S \le 1 \tag{101}$$

and constraints (83), (85), (86), and (96)

Note that w is the weighting factor with an adequately large amount. The variable S in the above problem generates negative optimization variables. Thus, an extra variable E is defined for transforming the variable S into a positive variable:

$$E = \frac{1}{1 - S} \ge 1 \tag{102}$$

By this transformation strategy, we have:

$$\min\ R + wE \tag{103}$$

$$s.t.\ \frac{(E)^{-1}\left(Z_1^- + 1\right)}{Z_1^+} \le 1 \tag{104}$$

$$E \ge 1 \tag{105}$$

and constraints (83), (85), (86), (96), and (99)

In the above-formulated problem, Eq. 103 is a posynomial function, and constraints (83), (85), (86), and (105) are monomial inequalities. They are all permissible equations needed in standard GP problems, while Eqs. 96, 99, and 104 are not permissible in standard GP problems. To cope with this issue, Xu (2014) applied arithmetic–geometric mean approximation for estimating each denominator of Eqs. (96, 99, and 104) by monomial functions. Assume $f(m)$ is a posynomial function as $f(m) = \sum_u v_u(m)$ which $v_u(m)$ are monomial terms. So, the following equation is obtained with the arithmetic–geometric mean approximation:

$$f(m) \geq \hat{f}(m) = \prod_u \left(\frac{v_u(m)}{\alpha_u(n)} \right)^{\alpha_u(n)} \tag{106}$$

where n is a fixed point with $n > 0$ and the parameters $\alpha_u(n)$ are calculated by the following equation:

$$\alpha_u(n) = \frac{v_u(n)}{f(n)} \quad \forall u \tag{107}$$

Boyd et al. (2007) showed that $\hat{f}(n)$ is the best local monomial approximation of $f(m)$ near n. Therefore, an inequality restriction on a proportion of two posynomials as $\frac{g(m)}{f(m)} \leq 1$ can be approximated by $\frac{g(m)}{\hat{f}(m)} \leq 1$ while $\frac{g(m)}{f(m)} \leq \frac{g(m)}{\hat{f}(m)} \leq 1$ holds (Xu 2014). Using the suggested monomial estimation technique to every denominator of Eqs. 96, 99, and 104, the following optimization problem is obtained at the r^{th} iteration:

$$\min \ R + wE \tag{108}$$

$$s.t. \ \frac{Z_0^+ + GN}{\hat{Z}_0^-} \leq 1 \tag{109}$$

$$\frac{Z_1^+}{\hat{Z}_1^-} \leq 1 \tag{110}$$

$$\frac{(E)^{-1}\left(Z_1^- + 1\right)}{\hat{Z}_1^+} \leq 1 \tag{111}$$

and constraints (83), (85), (86), and (105).

where $\hat{Z}_0^-$,$\hat{Z}_1^-$ and $\hat{Z}_1^+$ are the corresponding monomial functions approximated by Eq. 106. So, optimization problem (83), (85), (86), (105), (108–111) is a standard GP problem that can be optimized effectively (Boyd et al. 2007).

Briefing the above results, an iterative algorithm can be established for obtaining the optimal solutions of two models. The basic steps of the used algorithm are depicted as a flowchart in Fig. 3.

Referring to Xu (2014), the main features of the proposed algorithm can be described as below:

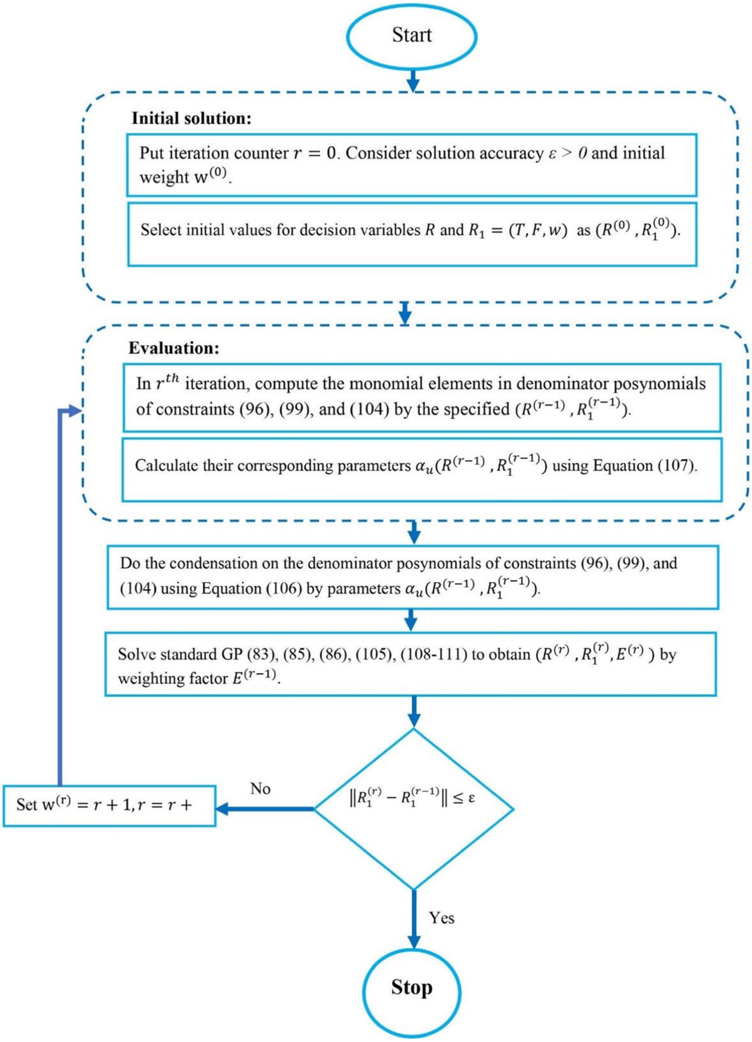


Fig. 3 Flowchart of the solution procedure

- The number of iterations and CPU time to reach optimal solutions as well as the amount of errors in objective and constraints functions are less than the other solving techniques of SGP problems.
- In the execution of the algorithm, one can select any point (infeasible or feasible point) as an initial solution. This is because the presented algorithm can find a feasible point rapidly.
- The proposed algorithm provides a rapid convergence behavior. For problem (83), (85), (86), (96), (99), (103–105), and also problem (83), (85), (86), (105), (108–111), the following conditions are kept using Eq. 106:

(a) $\frac{Z_0^+ + GN}{Z_0^- + R} \leq \frac{Z_0^+ + GN}{\hat{Z}_0^-}$

$$\frac{Z_1^+}{Z_1^- + 1} \leq \frac{Z_1^+}{\hat{Z}_1^-} \Rightarrow \frac{(E)^{-1}\left(Z_1^- + 1\right)}{Z_1^+} \leq \frac{(E)^{-1}\left(Z_1^- + 1\right)}{\hat{Z}_1^+}$$

(b) In rth iteration:

$$\nabla\left(\frac{Z_0^{+(r)} + GN}{Z_0^{-(r)} + R}\right) = \nabla\left(\frac{Z_0^{+(r)} + GN}{\hat{Z}_0^{-(r)}}\right) \Rightarrow \nabla\left(\frac{Z_1^{+(r)}}{Z_1^{-(r)} + 1}\right) = \nabla\left(\frac{Z_1^{+(r)}}{\hat{Z}_1^{-(r)}}\right)$$

$$\nabla\left(\frac{\left(E^{(r)}\right)^{-1}\left(Z_1^{-(r)} + 1\right)}{Z_1^{+(r)}}\right) = \nabla\left(\frac{\left(E^{(r)}\right)^{-1}\left(Z_1^{-(r)} + 1\right)}{\hat{Z}_1^{+(r)}}\right)$$

(c) In rth iteration:

$$\frac{Z_0^{+(r)} + GN}{Z_0^{-(r)} + R} = \frac{Z_0^{+(r)} + GN}{\hat{Z}_0^{-(r)}}$$

$$\frac{Z_1^{+(r)}}{Z_1^{-(r)} + 1} = \frac{Z_1^{+(r)}}{\hat{Z}_1^{-(r)}} \quad \Rightarrow \frac{\left(E^{(r)}\right)^{-1}\left(Z_1^{-(r)} + 1\right)}{Z_1^{+(r)}} = \frac{\left(E^{(r)}\right)^{-1}\left(Z_1^{-(r)} + 1\right)}{\hat{Z}_1^{+(r)}}$$

where ∇ shows the gradient of a function. $Z_i^{+(r)}, Z_i^{-(r)}, Z_i^{-(r)}, Z_i^{+(r)} (i = 1, 2)$ show the calculated amount of $Z_i^+, Z_i^-, Z_i^-, Z_i^+ (i = 1, 2)$ in rth iteration. Also, the increasing compensation $w^{(r)}$ will force the extra variable E to reach one as the suggested algorithm goes toward the ultimate point. Therefore, it can be concluded that the sequential solutions of the optimization problem (83), (85), (86), (105), (108–111) converge to a point by fulfilling the KKT conditions of the original problem (Marks and Wright 1978).

6 Numerical example

To demonstrate the implementation of proposed models and the solution procedure, we design the following numerical example to obtain the optimal solutions. These models are optimized by MATLAB software and executed on an Intel Core i5 PC with a 1.4 GHz CPU and 4.00 GB RAM and GGPLAB solver (Mutapcic et al. 2006). The function and parameters used in both model and algorithm parameters are given in Tables 2 and 3 respectively.

The numerical example is solved for various amounts of β and M as: $\beta = 0$, lost sale case,$\beta = 0.8$, partial backordering case, $\beta = 1$, and full backordering case, $M = 0$, without considering delayed payments, $M = 0.5$, with considering

Table 2 Used functions and parameters in example 1 for Models 1 and 2

Function / Parameter	Value	Function / Parameter	Value
D	$1000 - 0.4P - 0.1\,CE$	π	2
h	0.9	G	2.5
C	3	I_p	0.1
P	10	I_e	0.05
t	0.1	$\hat{h}$	1
s	0.1	$\hat{\pi}$	1
α	0.1	$\hat{K}$	40
K	31	$\hat{C}$	1
Y	1000		

Table 3 Algorithm parameters of the numerical example

Algorithm parameters	Value	Algorithm parameters	Value
$F_{11}^{(0)}$ and $F_{12}^{(0)}$	0.1	GN_{12} and GN_{22}	950
$F_{21}^{(0)}$ and $F_{22}^{(0)}$	1	$R^{(0)}$	1
$T_{11}^{(0)}$ and $T_{12}^{(0)}$	0.12	$\varpi^{(0)}$	1
$T_{21}^{(0)}$ and $T_{22}^{(0)}$	0.13	ε	10^{-4}
$P_{21}^{(0)}$ and $P_{22}^{(0)}$	3	GN_{21}	700
GN_{11}	400		

Table 4 The obtained results for each Scenario in Model 1

Scenario i			T_{i1}^*	F_{i1}^*	$ATP_1^*(i)$	Iteration
1	$\beta = 0$	$M = 0$	0.9284	0.9999	541.8218	12
		$M = 0.5$	0.9532	1	552.0290	10
	$\beta = 0.8$	$M = 0$	0.8591	0.8139	543.7493	9
		$M = 0.5$	0.8779	0.8189	559.7172	9
	$\beta = 1$	$M = 0$	0.7343	0.0638	545.8987	3
		$M = 0.5$	1.0038	0.5053	587.4436	4
2	$\beta = 0$	$M = 0$	0.9284	0.9999	541.8218	12
		$M = 0.5$	0.5	1	509.7156	15
	$\beta = 0.8$	$M = 0$	0.8591	0.8139	543.7493	9
		$M = 0.5$	0.5972	0.8373	485.9279	18
	$\beta = 1$	$M = 0$	0.7343	0.0638	545.8987	3
		$M = 0.5$	0.9809	0.5098	551.1357	11

partial delayed payments. The optimal solutions for both models are reported in Tables 4 and 5. In this section, we denote the optimal solutions of Scenarios (1) and (2) in Model 1 as $\left(F_{11}^*, T_{11}^*, ATP_1^*(1)\right)$ and $\left(F_{21}^*, T_{21}^*, ATP_1^*(2)\right)$ respectively, and

Table 5 The obtained results for each Scenario in Model 2

Scenario i			T^*_{i2}	F^*_{i2}	P^*_{i2}	$ATP^*_2(i)$	Iteration
1	$\beta = 0$	$M = 0$	0.1789	0.9492	62.1720	2582.1437	8
		$M = 0.5$	1.4748	0.9972	63.8760	3503.5846	398
	$\beta = 0.8$	$M = 0$	1.6935	0.5011	65.9746	3481.4004	13
		$M = 0.5$	1.7065	0.5722	66.1542	3496.5763	17
	$\beta = 1$	$M = 0$	1.6894	0.4027	63.4904	3742.6270	14
		$M = 0.5$	1.7935	0.4520	63.8406	3737.9396	17
2	$\beta = 0$	$M = 0$	0.1789	0.9492	62.1720	2582.1437	8
		$M = 0.5$	0.4185	0.9976	61.5972	3353.8453	287
	$\beta = 0.8$	$M = 0$	1.6935	0.5011	65.9746	3481.4004	13
		$M = 0.5$	1.2502	0.3999	65.4486	3432.5912	13
	$\beta = 1$	$M = 0$	1.6894	0.4027	63.4904	3742.6270	14
		$M = 0.5$	1.4819	0.3374	62.7919	3741.7944	14

optimal solutions of Scenarios (1) and (2) in Model 2 as $\left(F^*_{12}, T^*_{12}, P^*_{12}, ATP^*_2(1)\right)$ and $\left(F^*_{22}, T^*_{22}, P^*_{22}, ATP^*_2(2)\right)$ respectively. Also, GN_{ij} are sufficiently large constants that are considered in the mentioned algorithm for Scenario $i (i = 1, 2)$ and Model $j (j = 1, 2)$.

According to Taleizadeh et al. (2013), for determining the optimal solution in each model, we must compare $ATP_1(1)$ and $ATP_1(2)$ for Model 1 and also $ATP_2(1)$ and $ATP_2(2)$ for Model 2 in order to determine which is higher; those amounts related with the higher profit should be selected as the optimal solutions. For example, we determine the optimal solution for Model 1 when $\beta = 0$ and $M = 0.5$ as $ATP_1 = \max_{i=1,2}\left\{ATP^*_1(i)\right\} = \max\{552.0290,\ 509.7156\} = 552.0290 = ATP^*_1(1)$, so $(T^*,\ F^*) = \left(T^*_{11}, F^*_{11}\right) = (0.9532,\ 1)$ and

$$\begin{aligned} D^* &= \frac{(\gamma - \theta P)T^* - \delta\hat{K}}{\left(1 + \delta\beta\hat{C}\right)T^* + \delta\hat{C}(1-\beta)F^*T^* + 0.5\delta\left(\hat{h} + \beta\hat{\pi}\right)(F^*)^2(T^*)^2 - \delta\beta\hat{\pi}F^*(T^*)^2 + 0.5\beta\delta\hat{\pi}(T^*)^2} \\ &= 79.9923 \end{aligned}$$

$$Q^* = D^*(F^*T^* + \beta(1 - F^*)T^*) = 76.2509$$

$$CE^* = \hat{C}D^*(F^* + \beta(1 - F^*)) + \frac{\hat{K}}{T^*} + \frac{\hat{h}D^*(F^*)^2T^*}{2} + \frac{\beta\hat{\pi}D^*(1 - F^*)^2T^*}{2} = 160.0775$$

In the same way, the optimal solutions of Models 1 and 2 can be obtained. These results are given in Tables 6 and 7.

Figures 4, 5, and 6 briefly show the effects of β and M on the optimal total profit, the optimal carbon emissions, and the optimal selling price for Models 1 and 2. In Model 1, we can observe from Fig. 4, for fixed M, when β increases, the optimal total profit increases and the optimal carbon emissions decrease, this means that when shortages happen in full backordering form, the retailer's total profit is at maximum level and carbon emissions are at a minimum level. Moreover, for fixed β and $0 \leq \beta < 1$ the optimal carbon emissions decrease as M increases, and when $\beta = 1$,

Table 6 Optimal solutions of Model 1

		T^*	F^*	D^*	Q^*	ATP_1^*	CE^*
$\beta = 0$	$M = 0$	0.9284	0.9999	79.9822	74.2468	541.8218	160.1778
	$M = 0.5$	0.9532	1	79.9923	76.2509	552.0290	160.0775
$\beta = 0.8$	$M = 0$	0.8591	0.8139	81.1281	67.1026	543.7493	148.7193
	$M = 0.5$	0.8779	0.8189	81.1412	68.6514	552.7172	148.5883
$\beta = 1$	$M = 0$	0.7343	0.0638	81.8952	95.6632	548.2710	141.0477
	$M = 0.5$	1.0038	0.5053	81.7842	82.0989	587.4436	142.1579

Table 7 Optimal solutions of Model 2

		T^*	F^*	P^*	D^*	Q^*	ATP_2^*	CE^*
$\beta = 0$	$M = 0$	0.1789	0.9492	62.1720	52.7564	13.3501	3042.6906	217.0857
	$M = 0.5$	1.4748	0.9972	63.8760	57.5570	84.6455	3503.5846	126.7187
$\beta = 0.8$	$M = 0$	1.6935	0.5011	65.9746	60.8271	89.6752	3487.6242	102.4512
	$M = 0.5$	1.7065	0.5722	66.1542	60.2236	93.9794	3496.5763	102.8596
$\beta = 1$	$M = 0$	1.6894	0.4027	63.4904	61.6753	104.1916	3742.6270	112.3868
	$M = 0.5$	1.4819	0.3374	62.7919	62.3441	92.3888	3741.7944	114.8767

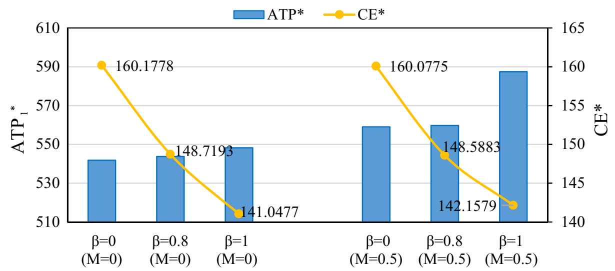


Fig. 4 The impacts of β and M on the ATP_1^* and CE^* in Model 1

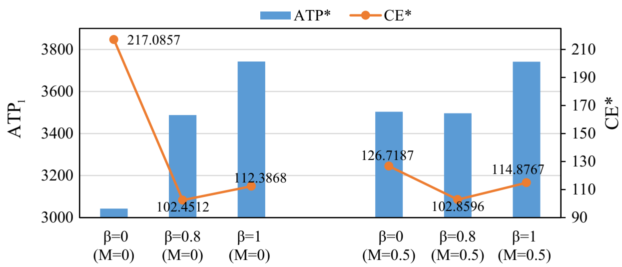


Fig. 5 The impacts of β and M on the ATP_2^* and CE^* in Model 2

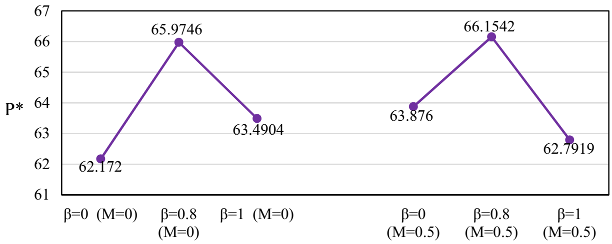


Fig. 6 The impacts of β and M on the optimal selling price in Model 2

the optimal carbon emissions increase as M increases. As we can see from Fig. 4, for fixed β, when M increases the optimal total profit increases. These results can assist the retailer to evaluate the behavior of the system in the different states of encountering shortages and credit periods offered to him in order to achieve both economic and environmental aspects of sustainability.

In Model 2, we can observe from Fig. 5, for fixed M, when β increases, the optimal total profit increases, meanwhile the optimal carbon emissions first decrease, and then increase as β increases. This means that when shortages happen in the partial form, the amount of carbon emissions is lower than when shortages are fully backordered ($\beta = 1$) and lost sales ($\beta = 0$). Moreover, from Fig. 5, it is observed that for fixed β and $0 \leq \beta < 1$, when M increases, the optimal total profit increases, and when $\beta = 1$, the optimal total profit decreases as M increases. Moreover, for fixed β and $0 < \beta \leq 1$, when M increases, the optimal carbon emissions increase, and when $\beta = 0$, the optimal carbon emissions decrease as M increases. As mentioned earlier, the retailer optimizes its profit through the replenishment decisions and selling price in Model 2. In this model, the retailer can find the optimal selling price under environmental regulations, different values of credit period, and different situations of shortages. For fixed M, we can find from Fig. 6 that the optimal selling price first increases, and then decreases as β increases. This identifies that when shortages happen in partial form, the optimal selling price has a higher value. On the other hand, for fixed β and $0 \leq \beta < 1$, when M increases, the optimal selling price increases, and when $\beta = 1$, the optimal selling price decreases as M increases.

6.1 Sensitivity analysis

In order to investigate the effect of changes in the main parameters on the optimal solutions obtained by the global optimization approach, we use a sensitivity analysis for both models using similar data in a numerical example when $M = 0.5$ and $\beta = 0.8$. This sensitivity analysis indicates some managerial insights.

We first investigate the effect of the carbon tax and carbon price on the optimal solutions of both models; the computed results are reported in Tables 8 and 9, respectively. In Model 1, as given in Table 8, when carbon price and carbon tax increase, the values of T^*,Q^*, and D^* increase and the values of F^* and CE^* decrease. On the other hand, the value of ATP^* increases as carbon price increases, while the value of ATP^* decreases as carbon tax increases. This finding

Table 8 The impacts of the carbon tax and carbon price on the optimal solutions of Model 1

		T^*	F^*	D^*	Q^*	ATP^*	CE^*
$s = 0.1$	$t = 0$	0.8526	0.8454	80.9336	66.8679	568.7342	150.6639
	$t = 0.5$	1.0151	0.8838	80.9550	80.2188	496.8051	150.5501
	$t = 1$	1.1022	0.7760	81.6936	88.3584	423.5268	143.0643
	$t = 1.5$	1.2052	0.7091	82.1229	93.2173	353.1618	138.7708
$t = 0.1$	$s = 0$	0.8526	0.8454	80.9336	66.8679	468.7342	150.6639
	$s = 0.5$	1.0151	0.8838	80.9550	80.2188	896.8051	150.5501
	$s = 1$	1.1323	0.7760	81.6936	88.3584	1323.5268	143.0643
	$s = 1.5$	1.2052	0.7091	82.1229	93.2173	1753.1618	138.7708

Table 9 The impacts of the carbon tax and carbon price on the optimal solutions of Model 2

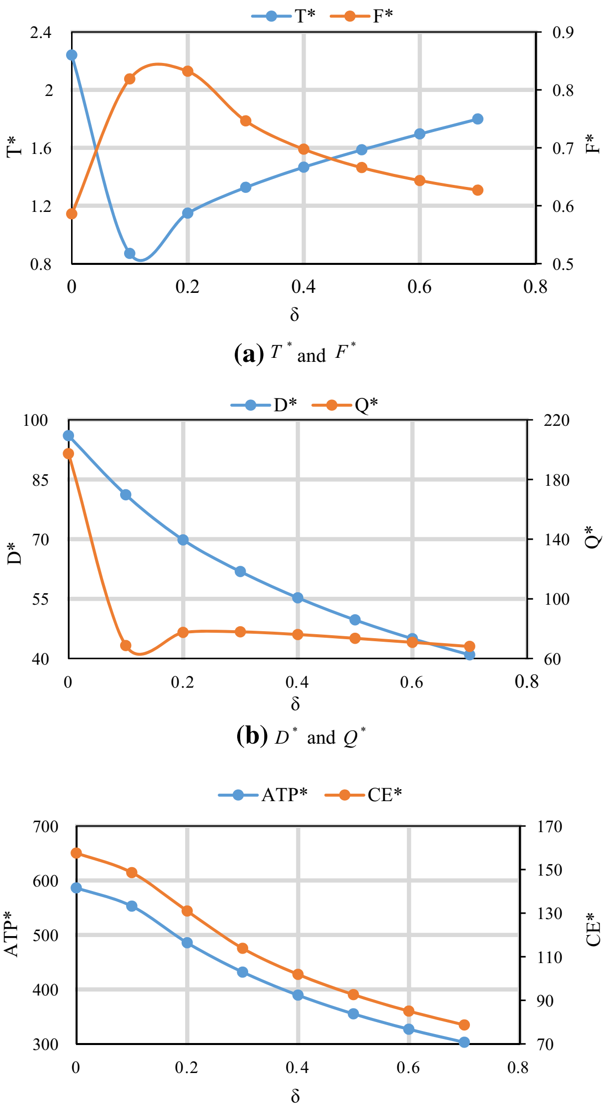
		T^*	F^*	P^*	D^*	Q^*	ATP^*	CE^*
$s = 0.1$	$t = 0$	1.7415	0.5772	65.8527	60.2097	95.9871	3493.0376	103.0517
	$t = 0.5$	1.5500	0.5665	67.2729	60.1306	85.1211	3508.4285	102.6845
	$t = 1$	1.5434	0.5353	69.0550	59.8530	83.7944	3513.9516	101.4225
	$t = 1.5$	1.5387	0.5140	70.7737	59.4730	82.6126	3516.8403	100.4214
$t = 0.1$	$s = 0$	1.7415	0.5772	65.8527	60.2097	95.9871	3393.0376	103.0517
	$s = 0.5$	1.5500	0.5665	67.2729	60.1306	85.1211	3908.4285	102.6845
	$s = 1$	1.5434	0.5353	69.0550	59.8530	83.7944	4413.9516	101.4225
	$s = 1.5$	1.5387	0.5240	70.7737	59.4730	82.6126	4918.8403	100.4214

indicates that two mechanisms (carbon tax and carbon price) do not have the same effect on the retailer's profit as well as a carbon tax and cap-and-trade policies have a dual benefit: to improve the sustainability of inventory management and also to increase consumer demand towards more sustainable items.

In Model 2, as reported in Table 9, when carbon price and carbon tax increase, the values of P^* and ATP^* increase and the values of T^*,Q^*,D^*,F^*, and CE^* decrease. This identifies that two mechanisms (carbon tax and carbon price) have the same effect on the retailer in Model 2. Similar to the results obtained from the study of Hovelaque and Bironneau (2015), we can observe that for both models carbon tax is always beneficial to the environment but unfavorable to the retailer due to lower carbon emissions and lower profit. Moreover, by increasing the carbon tax, the optimal selling price increases and so this led to a decrease in attractiveness for all consumers.

Next, we investigate the effect of carbon emissions elasticity δ on the optimal solutions of Models 1 and 2; the computed results are depicted in Figs. 7 and 8. According to Fig. 7, the following results can be extracted for Model 1:

- From Fig. 7a, when δ increases, the value of F^* first increases and then decreases. Moreover, the value of T^* first decreases and then increases as δ increases. This shows that for an initial low elasticity δ, the retailer can increase the number of orders.



(a) T^* and F^*

(b) D^* and Q^*

(c) ATP^* and CE^*

Fig. 7 Effect of carbon emissions elasticity on the optimal solutions of Model 1

- From Fig. 7b, the optimal value of the demand rate D^* decreases by increasing δ. On the other hand, the optimal order quantity first decreases, then increases, and next remains constant as δ increases. When δ is high, the retailer should decrease the order quantity.
- Fig. 7c, shows optimal total profit and optimal carbon emissions decrease as δ increase. So, growing carbon emissions elasticity δ has a positive effect on the environment, but a negative effect on the retailer's profit.

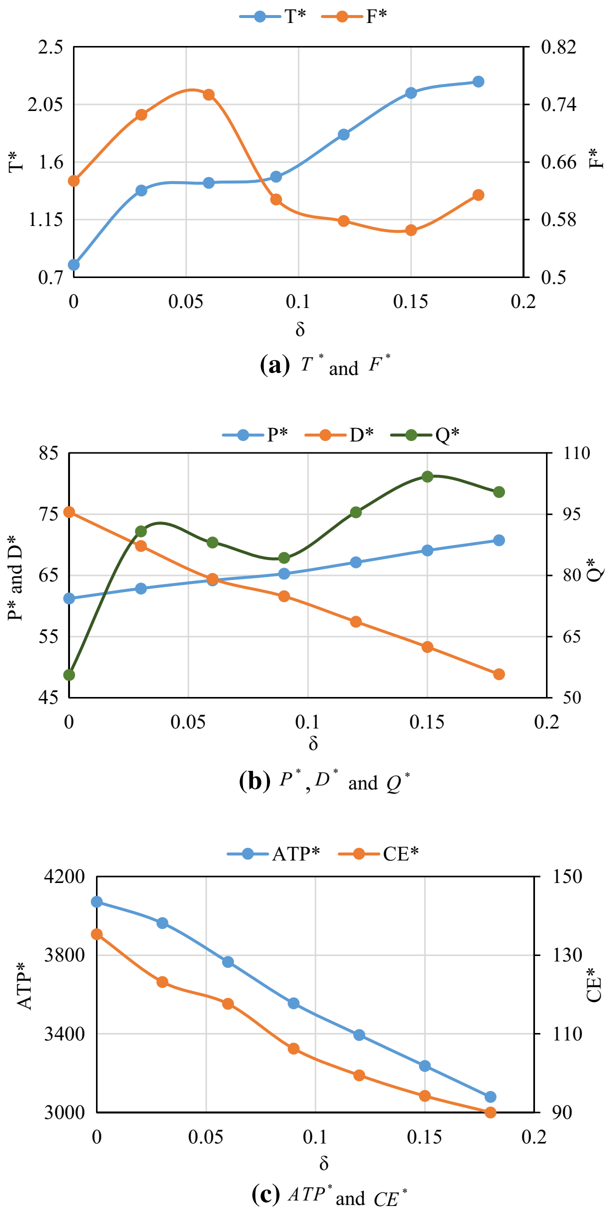


Fig. 8 Effect of carbon emissions elasticity on the optimal solutions of Model 2

We have the following results for Model 2 according to Fig. 8:

- From Fig. 8a, when δ increases, the value of F^* first increases and then decreases. The value T^* increases as δ increases.

- From Fig. 8b, an increase δ leads to an increase in selling price and order quantity as well as a decrease in the demand rate. Therefore, when the selling price is an endogenous variable, the retailer should increase order quantity and decrease the number of orders when δ is high. Our results are contrary to the findings of Hovelaque and Bironneau (2015) about the effects of carbon emissions elasticity δ on the optimal selling price and order quantity, which find that an increase in carbon emissions elasticity δ generates a decrease in selling price and quantity. Since trade credit is allowed in our proposed models, the retailer uses this opportunity and can order more items as well as increase the selling price to make more profit.
- From Fig. 8c, the optimal total profit and the optimal carbon emissions decrease as δ increase. Therefore, carbon emissions elasticity δ has the same effects on the total profit and carbon emissions in both models.

Finally, we do sensitivity analysis for parameters h , g , π , K , I_e , I_c, and α, changing each parameter in the appropriate unit for both models. The results are reported in Tables 10 and 11. According to Table 10, the following managerial insights can be obtained for Mode1 1:

- An increase in holding cost h leads to a decrease in T^*, F^*, Q^*, ATP^*, and CE^* as well as an increase in demand rate D^*. This shows that an increase in unit holding cost has a helpful impact on the environment. In addition, when the holding cost is high, the retailer should reduce the order quantity and replenishment cycle time in order to avoid higher holding costs.
- When the amount of lost sales cost g and backorder costs π increase, the values of F^* and CE^* increase whiles the values of T^*, Q^*, D^*, and ATP^* decrease. It means that the retailer should try to diminish shortages when the lost sales cost and backorder costs are high.
- The values of T^*, D^*, and Q^* increase as the ordering cost K increases; moreover, the values of F^*, ATP^*, and CE^* decrease as the ordering cost K increases. This indicates that an increase in unit ordering costs has a helpful impact on the environment. From a managerial interpretation, it is concluded that the retailer should increase the order quantity when the ordering cost is high.
- A higher value of I_c causes lower values of F^*, D^*, Q^*ATP^*, and CE^* as well as a higher value of T^*. A higher value of I_e causes higher values of D^* and ATP^*, but lower values of T^*, F^*, Q^*, and CE^*. These results illustrate that the retailer should decrease the order quantity when the interest payable is high. Moreover, when the interest earned is high, the total profit is high.
- When the number of initial payment α increases, the values of T^*, F^*, Q^*, ATP^*, and CE^* decrease and the value of D^* increases. It means that an increase in customer's initial payment can have a helpful effect on the environment although the retailer's profit decreases.

For Model 2, the effects of these changes are reported in Table 11 and the following managerial insights can be extracted from them:

Table 10 Computed results of sensitivity analysis for Model 1

	T^*	F^*	D^*	Q^*	ATP^*	CE^*
$h = 0.72$	0.8822	0.8493	80.9716	69.28	559.2381	150.2836
$h = 0.9$	0.8779	0.8189	81.1412	68.6514	552.7172	148.5883
$h = 1.08$	0.875	0.7943	81.2723	68.1895	546.9408	147.2774
$g = 2$	0.8929	0.7958	81.3023	69.6327	553.2077	146.9769
$g = 2.5$	0.8779	0.8189	81.1412	68.6514	552.7172	148.5883
$g = 3$	0.8626	0.8425	80.9724	67.6507	552.4927	150.2759
$\pi = 1.6$	0.9199	0.8036	81.3099	71.8567	553.1068	146.9013
$\pi = 2$	0.8779	0.8189	81.1412	68.6514	552.7172	148.5883
$\pi = 2.4$	0.8449	0.8344	80.9794	66.1553	552.4876	150.2055
$K = 24.8$	0.8217	0.8465	80.8523	64.397	560.3395	151.4767
$K = 31$	0.8779	0.8189	81.1412	68.6514	552.7172	148.5883
$K = 37.2$	0.9284	0.8001	81.3445	72.5001	545.8976	146.5553
$I_p = 0.08$	0.878	0.8246	81.109	68.7141	553.3453	148.9099
$I_p = 0.1$	0.8779	0.8189	81.1412	68.6514	552.7172	148.5883
$I_p = 0.12$	0.8779	0.8137	81.1711	68.6074	552.1249	148.2892
$I_e = 0.04$	0.8784	0.8207	81.1321	68.7114	552.3747	148.6789
$I_e = 0.05$	0.8779	0.8189	81.1412	68.6514	552.7172	148.5883
$I_e = 0.06$	0.8773	0.8172	81.1502	68.5908	553.0619	148.4982
$\alpha = 0.08$	0.8779	0.8191	81.1403	68.6563	552.9116	148.5973
$\alpha = 0.1$	0.8779	0.8189	81.1412	68.6514	552.7172	148.5883
$\alpha = 0.12$	0.8778	0.8188	81.1421	68.6464	552.5229	148.5793

- When the number of lost sales cost g, the values of F^*, T^*, Q^*, P^*, ATP^*, and CE^* increase, while the value of D^* decreases. This shows that the retailer should try to diminish lost sales to reduce the amount of carbon emissions.
- Values of F^*, ATP^*, and CE^* increase as the ordering cost K increases; moreover, the values D^* decrease as the ordering cost K increases. This indicates that an increase in unit ordering cost has a harmful impact on the environment when the retailer optimizes its profit with an endogenous price.
- The values of T^*, F^*, Q^*, ATP^*, and CE^* decrease as holding cost h increases, while the values of P^* and D^* increase as holding cost h increases. These results signify that an increase in unit holding cost can have a helpful effect on the environment although the retailer's total profit decreases. Moreover, when the holding cost is high, the retailer should reduce the order quantity and replenishment cycle time to avoid higher holding costs.
- A higher value of I_c causes lower values of F^*, Q^*, ATP^*, and CE^* as well as higher values of D^* and P^*. Moreover, a higher value of I_e causes higher values of T^*, D^*, and Q^*, but lower values of F^*,P^*, ATP^*, and CE^*. These results illustrate that the retailer should decrease the order quantity when the interest payable is high.
- When the number of initial payments α increases, the values of T^*, F^*, Q^*, ATP^*, and CE^* decrease and the values of D^* and P^* increase. It means that an increase in the customer's initial payment can have a helpful effect on the environment although the

Table 11 Computed results of sensitivity analysis for Model 2

	T^*	F^*	P^*	D^*	Q^*	ATP^*	CE^*
$h = 0.72$	1.7084	0.6131	66.0918	59.7297	94.1473	3500.232	103.8111
$h = 0.9$	1.7065	0.5722	66.1542	60.2236	93.9794	3496.5763	102.8596
$h = 1.08$	1.6949	0.5375	66.2049	60.622	93.2449	3491.642	102.2474
$g = 2$	1.6809	0.5529	65.9937	60.5413	92.6642	3493.325	102.6155
$g = 2.5$	1.7065	0.5722	66.1542	60.2236	93.9794	3496.5763	102.8596
$g = 3$	1.7308	0.5912	66.3142	59.8973	95.1964	3498.97	103.1583
$\pi = 1.6$	1.6494	0.5679	65.6313	60.5376	91.2234	3485.565	103.1159
$\pi = 2$	1.7065	0.5722	66.1542	60.2236	93.9794	3496.5763	102.8596
$\pi = 2.4$	1.6001	0.6018	66.2687	59.9948	88.3512	3511.5	103.6874
$K = 24.8$	1.7028	0.556	66.0508	60.4545	93.8037	3494.805	102.6069
$K = 31$	1.7065	0.5722	66.1542	60.2236	93.9794	3496.5763	102.8596
$K = 37.2$	1.6282	0.5987	66.0675	60.0561	89.9351	3499.867	103.6259
$I_p = 0.08$	1.7064	0.5795	66.1341	60.1435	93.9949	3496.926	103.016
$I_p = 0.1$	1.7065	0.5722	66.1542	60.2236	93.9794	3496.5763	102.8596
$I_p = 0.12$	1.7062	0.5655	66.1736	60.2979	93.9406	3496.208	102.7198
$I_e = 0.04$	1.7058	0.5753	66.1634	60.1852	93.9418	3496.901	102.9176
$I_e = 0.05$	1.7065	0.5722	66.1542	60.2236	93.9794	3496.5763	102.8596
$I_e = 0.06$	1.7071	0.5692	66.1449	60.2622	94.0105	3496.242	102.8029
$\alpha = 0.08$	1.7068	0.5725	66.1533	60.2201	93.9985	3496.662	102.865
$\alpha = 0.1$	1.7065	0.5722	66.1542	60.2236	93.9794	3496.5763	102.8596
$\alpha = 0.12$	1.7062	0.572	66.1551	60.2272	93.9603	3496.491	102.8542

retailer's profit decreases. Therefore, to reduce the selling price and to generate profit, the retailer should try to obtain a greater discount from his/her supplier.

7 Conclusion and future research

This paper developed a sustainable EOQ model under partial trade credit and shortages. This study incorporated environmental issues and shortage issues into a joint partial delay in payments and inventory management models. We considered a correlation between the demand rate and carbon emissions so that the demand rate is sensitive to the selling price and to carbon emissions. We first modeled the proposed problem with an exogenous price, then extended the inventory model when the selling price is an endogenous variable. These models have been formulated as a nonlinear programming problem of profit maximization for the retailer. To find optimal solutions, we first transformed these models to a signomial geometric programming problem, then we applied an optimization approach presented by Xu (2014) to solve our models. Numerical illustrations are presented to show the impacts of different types of shortages in both inventory systems (inventory system with fixed selling price (Model 1) and inventory system with pricing (Model 2)) as well as different credit periods under a carbon tax and cap-and-trade regulations. Several managerial insights have been extracted from a numerical example with

$M = 0.5$ and $\beta = 0.8$. For example, in Model 2, when shortages happen in partial form, the amount of carbon emissions is lower and the optimal value of selling price is higher than when shortages happen in backordered and lost sales forms. Meanwhile, in Model 1, we observed that the minimum carbon emissions happen in the backordered type. In addition, maximum profit for the retailer in both models happens when shortages are in backordered forms and the retailer is offered a partial delay payment. We can also observe that for both models carbon tax is always beneficial to the environment but unfavorable to the retailer due to lower carbon emissions and lower profit. Therefore, both models help the retailers how to react in different situations according to shortages, pricing, trade credit strategies, and carbon regulations to reach economic profits as well as healthy environments. Our study can be extended to multiple products, deteriorating items, and fuzzy environments, and it can consider other relationships between carbon emissions, demand rate, and realistic restrictions such as space, budget, etc.

Appendix A. Developing objective functions for Models 1 and 2

A.1. According to Eqs. (2–9) and (12), the average annual total profit in Model 1 and Scenario (1), when $M \leq FT$, is calculated as follows:

$$\begin{aligned}
ATP_1(1) &= \frac{1}{T}\left(SR - C_p - C_o - C_h - C_b - C_G - CC(1) + IE(1)\right) - tCE + s(Y - CE) \\
&= \frac{1}{T}\left\{ \underbrace{PD(F + \beta(1-F))T}_{SR} - \underbrace{CD(F + \beta(1-F))T}_{C_P} - \underbrace{K}_{C_O} - \underbrace{0.5hDF^2T^2}_{C_H} - \underbrace{0.5\beta\pi D(1-F)^2T^2}_{C_B} - \underbrace{G(1-\beta)D(1-F)T}_{C_G} \right. \\
&\quad \left. - \underbrace{\left(0.5\alpha CI_cDF^2T^2 + 0.5(1-\alpha)CI_cD(FT-M)^2\right)}_{CC(1)} + \underbrace{\left(\beta(1-\alpha)CI_eD(1-F)TM + 0.5(1-\alpha)CI_eDM^2\right)}_{IE(1)} \right\} \\
&\quad \underbrace{- (t+s)\frac{\left(\hat{C}D(F + \beta(1-F))T + \hat{K} + 0.5\hat{h}DF^2T^2 + 0.5\beta\hat{\pi}D(1-F)^2T^2\right)}{T} + sY}_{Emission\ \ \cos t}
\end{aligned} \tag{112}$$

Expanding terms:

$$\begin{aligned}
ATP_1(1) &= \frac{1}{T}\left\{(PF + \beta P - \beta PF)DT - (CF + \beta C - \beta CF)DT - K - 0.5hDF^2T^2\right. \\
&\quad - \left(0.5\beta\pi F^2 - \beta\pi F + 0.5\beta\pi\right)DT^2 - (G(1-\beta) - G(1-\beta)F)DT \\
&\quad - \left(0.5\alpha CI_cF^2T^2 + 0.5(1-\alpha)CI_c\left(F^2T^2 - 2FTM + M^2\right)\right)D \\
&\quad + \left(\beta(1-\alpha)CI_eTM - \beta(1-\alpha)CI_eTFM + 0.5(1-\alpha)CI_eM^2\right)D \\
&\quad - \frac{(t+s)}{T}\left(\left(\hat{C}F + \beta\hat{C} - \beta\hat{C}F\right)DT + \hat{K} + 0.5\hat{h}DF^2T^2 + \left(0.5\beta\hat{\pi}F^2 - \beta\hat{\pi}F + 0.5\beta\hat{\pi}\right)DT^2\right) \\
&\quad + sY = \underbrace{\left(P - C' + \beta(1-\alpha)CI_eM\right)}_{\mu_0}D - \underbrace{K'}_{\mu_1}T^{-1} - \underbrace{\left(0.5\left(h' + \beta\pi'' + \alpha CI_c + (1-\alpha)CI_c\right)\right)}_{\mu_2}DF^2T \\
&\quad + \underbrace{\left(\beta\pi''\right)}_{\mu_3}DFT + \underbrace{\left(\left(P - C' + G\right)(1-\beta) + \left(I_c - \beta I_e\right)(1-\alpha)CM\right)}_{\mu_4}DF - \underbrace{\left(0.5\beta\pi''\right)}_{\mu_5}DT \\
&\quad - \underbrace{\left(\left(P - C' + G\right)(1-\beta)\right)}_{\mu_6}D - \underbrace{\left(0.5\left(I_c - I_e\right)(1-\alpha)CM^2\right)}_{\mu_7}DT^{-1} + sY
\end{aligned} \tag{113}$$

In above equation, we assume that $(t+s)\hat{C}+C=C'$, $(t+s)\hat{h}+h=h'$, $(t+s)\hat{\pi}+\pi=\pi''$ and $(t+s)\hat{K}+K=K'$, so the average total profit per year in Model 1 and Scenario (1) is:

$$ATP_1(1)=\left(\mu_0-\mu_2TF^2+\mu_3FT+\mu_4F-\mu_5T-\mu_6-\mu_7T^{-1}\right)D-\mu_1T^{-1}+sY \tag{114}$$

where

$$\mu_0=\left(P-C'+\beta(1-\alpha)CI_eM\right)\geq 0 \tag{115}$$

$$\mu_1=K'\geq 0 \tag{116}$$

$$\mu_2=0.5\left(h'+\beta\pi''+\alpha CI_c+(1-\alpha)CI_c\right)\geq 0 \tag{117}$$

$$\mu_3=\beta\pi''\geq 0 \tag{118}$$

$$\mu_4=\left(\left(P-C'+G\right)(1-\beta)+\left(I_c-\beta I_e\right)(1-\alpha)CM\right)\geq 0 \tag{119}$$

$$\mu_5=0.5\beta\pi''\geq 0 \tag{120}$$

$$\mu_6=\left(P-C'+G\right)(1-\beta)\geq 0 \tag{121}$$

$$\mu_7=0.5\left(I_c-I_e\right)(1-\alpha)CM^2\geq 0 \tag{122}$$

After replacing demand function by using Eqs. 17–25 in Eq. 114, the average total profit per year in Model 1 and Scenario (1) is:

$$\begin{aligned}ATP_1(1)&=\left(\mu_0-\mu_2TF^2+\mu_3FT+\mu_4F-\mu_5T-\mu_6-\mu_7T^{-1}\right)(aT-b)\varpi^{-1}-\mu_1T^{-1}+sY\\&=\left(\mu_0a+\mu_5b\right)T\varpi^{-1}-\left(\mu_0b+\mu_7a\right)\varpi^{-1}-\mu_1T^{-1}-\mu_2aF^2T^2\varpi^{-1}+\mu_2bF^2T\varpi^{-1}\\&\quad+\mu_3aFT^2\varpi^{-1}-\mu_3bFT\varpi^{-1}+\mu_4aFT\varpi^{-1}-\mu_4bF\varpi^{-1}-\mu_5aT^2\varpi^{-1}-\mu_6aT\varpi^{-1}\\&\quad+\mu_6b\varpi^{-1}+\mu_7bT^{-1}\varpi^{-1}+sY\end{aligned} \tag{123}$$

In next subsections, another objective functions will be calculated by performing the similar steps applied in this subsection.

A.2. According to Eqs. (2–7), (10, 11), and (12), the average annual total profit in Model 1 and Scenario (2), when $M\geq FT$, is calculated as follows:

$$ATP_1(2) = \frac{1}{T}\left(SR - C_p - C_o - C_h - C_b - C_G - CC(2) + IE(2)\right) - tCE + s(Y - CE)$$
$$= \frac{1}{T}\left\{\underbrace{PD(F + \beta(1-F))\mathrm{T}}_{SR} - \underbrace{CD(F + \beta(1-F))T}_{C_P} - \underbrace{K}_{C_O} - \underbrace{0.5hDF^2T^2}_{C_H}\right.$$
$$- \underbrace{0.5\beta\pi D(1-F)^2T^2}_{C_B} - \underbrace{G(1-\beta)D(1-F)T}_{C_G} - \underbrace{\left(0.5\alpha CI_cDF^2T^2\right)}_{CC(2)}$$
$$\left. + \underbrace{\left(I_e(1-\alpha)C\beta D(1-F)TM + 0.5(1-\alpha)CI_eDF^2T^2 + I_e(1-\alpha)CDFT(M-FT)\right)}_{IE(2)}\right\}$$
$$\underbrace{- (t+s)\frac{\left(\hat{C}D(F + \beta(1-F))T + \hat{K} + 0.5\hat{h}DF^2T^2 + 0.5\beta\hat{\pi}D(1-F)^2T^2\right)}{T} + sY}_{Emission\ \cos t} \tag{124}$$

Expanding terms:

$$ATP_1(2) = \frac{1}{T}\left\{(PF + \beta P - \beta PF)DT - (CF + \beta C - \beta CF)DT - K - 0.5hDF^2T^2\right.$$
$$- \left(0.5\beta\pi F^2 - \beta\pi F + 0.5\beta\pi\right)DT^2 - (G(1-\beta) - G(1-\beta)F)DT$$
$$- \left(0.5\alpha CI_cF^2\right)DT^2 + \left(\beta(1-\alpha)CI_eM - \beta(1-\alpha)CI_eFM\right)DT$$
$$\left. + \left(0.5(1-\alpha)CI_eF^2T + I_e(1-\alpha)CFM - I_e(1-\alpha)CF^2T\right)DT\right\}$$
$$- \frac{(t+s)}{T}\left(\left(\hat{C}F + \beta\hat{C} - \beta\hat{C}F\right)DT + \hat{K} + 0.5\hat{h}DF^2T^2\right)$$
$$- \frac{(t+s)}{T}\left(0.5\beta\hat{\pi}F^2 - \beta\hat{\pi}F + 0.5\beta\hat{\pi}\right)DT^2 + sY$$
$$= \underbrace{\left(P - C' + \beta(1-\alpha)CI_eM\right)}_{\tau_0} D - \underbrace{K'}_{\tau_1} T^{-1}$$
$$- \underbrace{\left(0.5\left(h' + \beta\pi'' + \alpha CI_c + (1-\alpha)CI_e\right)\right)}_{\tau_2} DF^2T + \underbrace{\left(\beta\pi''\right)}_{\tau_3} DFT$$
$$+ \underbrace{\left(\left(P - C' + G\right)(1-\beta) + (1-\beta)(1-\alpha)CI_eM\right)}_{\tau_4} DF - \underbrace{\left(0.5\beta\pi''\right)}_{\tau_5} DT$$
$$- \underbrace{\left(\left(P - C' + G\right)(1-\beta)\right)}_{\tau_6} D + sY \tag{125}$$

In above equation, we assume that $(t+s)\hat{C} + C = C'$, $(t+s)\hat{h} + h = h'$, $(t+s)\hat{\pi} + \pi = \pi''$ and $(t+s)\hat{K} + K = K'$, so the average total profit per year in Model 1 and Scenario (2) is:

$$ATP_1(2) = \left(\tau_0 - \tau_2TF^2 + \tau_3FT + \tau_4F - \tau_5T - \tau_6\right)D - \tau_1T^{-1} + sY \tag{126}$$

where

$$\tau_0 = \left(P - C' + \beta(1-\alpha)CI_eM\right) \geq 0 \tag{127}$$

$$\tau_1 = K' \geq 0 \tag{128}$$

$$\tau_2 = 0.5\left(h' + \beta\pi'' + \alpha CI_c + (1-\alpha)CI_e\right) \geq 0 \tag{129}$$

$$\tau_3 = \beta\pi'' \geq 0 \tag{130}$$

$$\tau_4 = \left(\left(P - C' + G\right) + \left((1-\alpha)CI_eM\right)\right)(1-\beta) \geq 0 \tag{131}$$

$$\tau_5 = 0.5\beta\pi'' \geq 0 \tag{132}$$

$$\tau_6 = \left(P - C' + G\right)(1-\beta) \geq 0 \tag{133}$$

After replacing demand function by using Eqs. (17–25) in Eq. 126, the average total profit per year in Model 1 and Scenario (2) is:

$$\begin{aligned} ATP_1(2) &= \left(\tau_0 - \tau_2TF^2 + \tau_3FT + \tau_4F - \tau_5T - \tau_6\right)(aT-b)\varpi^{-1} - \tau_1T^{-1} + sY \\ &= \left(\tau_0a + \tau_5b\right)T\varpi^{-1} - \tau_0b\varpi^{-1} - \tau_1T^{-1} - \tau_2aF^2T^2\varpi^{-1} + \tau_2bF^2T\varpi^{-1} + \tau_3aFT^2\varpi^{-1} \\ &\quad - \tau_3bFT\varpi^{-1} + \tau_4aFT\varpi^{-1} - \tau_4bF\varpi^{-1} - \tau_5aT^2\varpi^{-1} - \tau_6aT\varpi^{-1} + \tau_6b\varpi^{-1} + sY \end{aligned} \tag{134}$$

A.3. According to Eqs. (2–9) and (12) the average annual total profit in Model 2 and Scenario (1), when $M \leq FT$, is calculated as follows:

$$\begin{aligned} ATP_2(1) &= \frac{1}{T}\left(SR - C_p - C_o - C_h - C_b - C_G - CC(1) + IE(1)\right) \\ &\quad - tCE'(T,F) + s\left(Y - CE'(T,F)\right) \\ &= \frac{1}{T}\left\{\underbrace{PD(F+\beta(1-F))\mathrm{T}}_{SR} - \underbrace{CD(F+\beta(1-F))T}_{C_P} - \underbrace{K}_{C_O} - \underbrace{0.5hDF^2T^2}_{C_H}\right. \\ &\quad - \underbrace{0.5\beta\pi D(1-F)^2T^2}_{C_B} - \underbrace{G(1-\beta)D(1-F)T}_{C_G} \\ &\quad - \underbrace{\left(0.5\alpha CI_cDF^2T^2 + 0.5(1-\alpha)CI_cD(FT-M)^2\right)}_{CC(1)} \\ &\quad + \underbrace{\left(\beta(1-\alpha)CI_eD(1-F)TM + 0.5(1-\alpha)CI_eDM^2\right)}_{IE(1)} \\ &\quad \underbrace{-(t+s)\frac{\left(\hat{C}D(F+\beta(1-F))T + \hat{K} + 0.5\hat{h}DF^2T^2 + 0.5\beta\hat{\pi}D(1-F)^2T^2\right)}{T} +}_{Emission\ \cos t} \end{aligned} \tag{135}$$

Expanding terms:

$$\begin{aligned}ATP_2(1) = &\frac{1}{T}\big\{(PF + \beta P - \beta PF)DT - (CF + \beta C - \beta CF)DT - K - 0.5hDF^2T^2\\&- \left(0.5\beta\pi F^2 - \beta\pi F + 0.5\beta\pi\right)DT^2 - (G(1-\beta) - G(1-\beta)F)DT\\&- \left(0.5\alpha CI_cF^2T^2 + 0.5(1-\alpha)CI_c\left(F^2T^2 - 2FTM + M^2\right)\right)D\\&+\left(\beta(1-\alpha)CI_eTM - \beta(1-\alpha)CI_eTFM + 0.5(1-\alpha)CI_eM^2\right)D\big\}\\&- \frac{(t+s)}{T}\left(\left(\hat{C}F + \beta\hat{C} - \beta\hat{C}F\right)DT + \hat{K} + 0.5\hat{h}DF^2T^2 + \left(0.5\beta\hat{\pi}F^2 - \beta\hat{\pi}F + 0.5\beta\hat{\pi}\right)DT^2\right)\\&+ sY = \underbrace{(1-\beta)}_{\kappa_0}PFD + \underbrace{\beta}_{\kappa_1}PD - \underbrace{\left(\beta C' + G(1-\beta)\right)}_{\kappa_2}D - \underbrace{\left(C' - G\right)(1-\beta)}_{\kappa_3}FD\\&- \underbrace{0.5\left(h' + \beta\pi'' + CI_c\right)}_{\kappa_4}DF^2T + \underbrace{\beta\pi''}_{\kappa_5}DFT - \underbrace{0.5\beta\pi''}_{\kappa_6}DT\\&- \underbrace{0.5(1-\alpha)C\left(I_c - I_e\right)M^2}_{\kappa_7}DT^{-1} + \underbrace{\beta(1-\alpha)CI_eM}_{\kappa_8}D + \underbrace{\left(I_c - \beta I_e\right)(1-\alpha)CM}_{\kappa_9}DF - \underbrace{K'}_{\kappa_{10}}T^{-1} + sY\end{aligned} \tag{136}$$

In above equation, we assume that $(t+s)\hat{C} + C = C'$, $(t+s)\hat{h} + h = h'$, $(t+s)\hat{\pi} + \pi = \pi''$ and $(t+s)\hat{K} + K = K'$, so the average total profit per year in Model 2 and Scenario (1) is:

$$\begin{aligned}ATP_2(1) =& \big(\kappa_0PF + \kappa_1P - \kappa_2 - \kappa_3F - \kappa_4F^2T + \kappa_5FT\\&-\kappa_6T - \kappa_7T^{-1} + \kappa_8 + \kappa_9F\big)D - \kappa_{10}T^{-1} + sY\end{aligned} \tag{137}$$

where

$$\kappa_0 = (1-\beta) \geq 0 \tag{138}$$

$$\kappa_1 = \beta \geq 0 \tag{139}$$

$$\kappa_2 = \left(\beta C' + G(1-\beta)\right) \geq 0 \tag{140}$$

$$\kappa_3 = \left(C' - G\right)(1-\beta) \geq 0 \tag{141}$$

$$\kappa_4 = 0.5\left(h' + \beta\pi'' + CI_c\right) \geq 0 \tag{142}$$

$$\kappa_5 = \beta\pi'' \geq 0 \tag{143}$$

$$\kappa_6 = 0.5\beta\pi'' \geq 0 \tag{144}$$

$$\kappa_7 = 0.5(1-\alpha)C\left(I_c - I_e\right)M^2 \geq 0 \tag{145}$$

$$\kappa_8 = \beta(1-\alpha)CI_eM \geq 0 \tag{146}$$

$$\kappa_9 = \left(I_c - \beta I_e\right)(1-\alpha)CM \geq 0 \tag{147}$$

$$\kappa_{10} = K' \geq 0 \tag{148}$$

After replacing demand function by using Eqs. (20–26) in Eq. 137, the average total profit per year in Model 2 and Scenario (1) is:

$$\begin{aligned} ATP_2(1) &= \left(\kappa_0 PF + \kappa_1 P - \kappa_2 - \kappa_3 F - \kappa_4 F^2 T + \kappa_5 FT - \kappa_6 T - \kappa_7 T^{-1} + \kappa_8 + \kappa_9 F\left(\gamma - \theta P - bT^{-1}\right)\varpi^{-1}\right. \\ &\quad - \kappa_{10} T^{-1} + sY \;= \kappa_0 \gamma PF\varpi^{-1} - \kappa_0 \theta P^2 F\varpi^{-1} - \kappa_0 bPFT^{-1}\varpi^{-1} \\ &\quad + \kappa_1 \gamma P\varpi^{-1} - \kappa_1 \theta P^2 \varpi^{-1} - \kappa_1 bPT^{-1}\varpi^{-1} - \kappa_2 \gamma \varpi^{-1} + \kappa_2 \theta P\varpi^{-1} \\ &\quad + \kappa_2 bT^{-1}\varpi^{-1} - \kappa_3 \gamma F\varpi^{-1} + \kappa_3 \theta FP\varpi^{-1} \;\; + \kappa_3 bFT^{-1}\varpi^{-1} - \kappa_4 \gamma F^2 T\varpi^{-1} \\ &\quad + \kappa_4 \theta F^2 TP\varpi^{-1} + \kappa_4 bF^2 \varpi^{-1} + \kappa_5 \gamma FT\varpi^{-1} - \kappa_5 \theta FTP\varpi^{-1} \\ &\quad - \kappa_5 bF\varpi^{-1} \;\; - \kappa_6 \gamma T\varpi^{-1} + \kappa_6 \theta TP\varpi^{-1} + \kappa_6 b\varpi^{-1} - \kappa_7 \gamma T^{-1}\varpi^{-1} \\ &\quad + \kappa_7 \theta PT^{-1}\varpi^{-1} + \kappa_7 bT^{-2}\kappa_8 \gamma \varpi^{-1} - \kappa_8 \theta P\varpi^{-1} - \kappa_8 bT^{-1}\varpi^{-1} \\ &\quad + \kappa_9 \gamma F\varpi^{-1} - \kappa_9 \theta FP\varpi^{-1} - \kappa_9 bFT^{-1}\varpi^{-1} - \kappa_{10} T^{-1} + sY \end{aligned} \tag{149}$$

A.4. According to Eqs. (2–7), (10–11), and (12) the average annual total profit in Model 2 and Scenario (2), when $M \geq FT$, is calculated as follows:

$$\begin{aligned} ATP_2(2) &= \frac{1}{T}\left(SR - C_p - C_o - C_h - C_b - C_G - CC(2) + IE(2)\right) - tCE + s(Y - CE) \\ &= \frac{1}{T}\left\{ \underbrace{PD(F+\beta(1-F))\mathrm{T}}_{SR} - \underbrace{CD(F+\beta(1-F))T}_{C_P} - \underbrace{K}_{C_O} - \underbrace{0.5hDF^2T^2}_{C_H} \right. \\ &\quad - \underbrace{0.5\beta\pi D(1-F)^2T^2}_{C_B} - \underbrace{G(1-\beta)D(1-F)T}_{C_G} - \underbrace{\left(0.5\alpha CI_cDF^2T^2\right)}_{CC(2)} \\ &\quad \left. + \underbrace{\left(I_e(1-\alpha)C\beta D(1-F)TM + 0.5(1-\alpha)CI_eDF^2T^2 + I_e(1-\alpha)CDFT(M-FT)\right)}_{IE(2)} \right\} \\ &\quad \underbrace{- (t+s)\frac{\left(\hat{C}D(F+\beta(1-F))T + \hat{K} + 0.5\hat{h}DF^2T^2 + 0.5\beta\hat{\pi}D(1-F)^2T^2\right)}{T} + sY}_{Emission\ \cos t} \end{aligned} \tag{150}$$

Expanding terms:

$$\begin{aligned}
ATP_2(2) = &\frac{1}{T}\big\{(PF + \beta P - \beta PF)DT - (CF + \beta C - \beta CF)DT - K - 0.5hDF^2T^2 \\
&- \big(0.5\beta\pi F^2 - \beta\pi F + 0.5\beta\pi\big)DT^2 - (G(1-\beta) - G(1-\beta)F)DT \\
&- \big(0.5\alpha CI_cF^2\big)DT^2 + \big(\beta(1-\alpha)CI_eM - \beta(1-\alpha)CI_eFM\big)DT \\
&+ \big(0.5(1-\alpha)CI_eF^2T + I_e(1-\alpha)CFM - I_e(1-\alpha)CF^2T\big)DT \\
&- \frac{(t+s)}{T}\big(\big(\hat{C}F + \beta\hat{C} - \beta\hat{C}F\big)DT + \hat{K} + 0.5\hat{h}DF^2T^2\big) \\
&- \frac{(t+s)}{T}\ \big(0.5\beta\hat{\pi}F^2 - \beta\hat{\pi}F + 0.5\beta\hat{\pi}\big)DT^2 + sY \\
= &\underbrace{(1-\beta)}_{\eta_0}PFD + \underbrace{\beta}_{\eta_1}PD - \underbrace{\big(\beta C' + G(1-\beta)\big)}_{\eta_2}D - \underbrace{\big(C' - G\big)(1-\beta)}_{\eta_3}FD \\
&- \underbrace{0.5\big(h' + \beta\pi'' + CI_c\big)}_{\eta_4}DF^2T + \underbrace{\beta\pi''}_{\eta_5}DFT - \underbrace{0.5\beta\pi''}_{\eta_6}DT \\
&+ \underbrace{\beta(1-\alpha)CI_eM}_{\eta\ 7}D + \underbrace{\big(I_c - \beta I_e\big)(1-\alpha)CM}_{\eta_8}DF - \underbrace{K'}_{\eta_9}T^{-1} + sY
\end{aligned} \tag{151}$$

In above equation, we assume that $(t+s)\hat{C} + C = C'$, $(t+s)\hat{h} + h = h'$, $(t+s)\hat{\pi} + \pi = \pi''$ and $(t+s)\hat{K} + K = K'$, so the average total profit per year in Model 2 and Scenario (2) is:

$$ATP_2(2) = \big(\eta_0PF + \eta_1P - \eta_2 - \eta_3F - \eta_4F^2T + \eta_5FT - \eta_6T + \eta_7 + \eta_8F\big)D \quad - \kappa_{10}T^{-1} + sY \tag{152}$$

where

$$\eta_0 = (1-\beta) \geq 0 \tag{153}$$

$$\eta_1 = \beta \geq 0 \tag{154}$$

$$\eta_2 = \big(\beta C' + G(1-\beta)\big) \geq 0 \tag{155}$$

$$\eta_3 = \big(C' - G\big)(1-\beta) \geq 0 \tag{156}$$

$$\eta_4 = 0.5\big(h' + \beta\pi'' + \alpha CI_c + (1-\alpha)CI_e\big) \geq 0 \tag{157}$$

$$\eta_5 = \beta\pi'' \geq 0 \tag{158}$$

$$\eta_6 = 0.5\beta\pi'' \geq 0 \tag{159}$$

$$\eta_7 = \beta(1-\alpha)CI_eM \geq 0 \tag{160}$$

$$\eta_8 = (1-\beta)(1-\alpha)CI_eM \geq 0 \tag{161}$$

$$\eta_9 = K' \geq 0 \tag{162}$$

After replacing demand function by using Eqs. 20–26 in Eq. 149, the average total profit per year in Model 2 and Scenario (2) is:

$$\begin{aligned} ATP_2(2) = {} & \left(\eta_0 PF + \eta_1 P - \eta_2 - \eta_3 F - \eta_4 F^2 T + \eta_5 FT - \eta_6 T + \eta_7 + \eta_8 F\right)\left(\gamma - \theta P - bT^{-1}\right)\varpi^{-1} \\ & - \eta_9 T^{-1} + sY = \eta_0 \gamma PF\varpi^{-1} - \eta_0 \theta P^2 F\varpi^{-1} - \eta_0 bPFT^{-1}\varpi^{-1} + \eta_1 \gamma P\varpi^{-1} - \eta_1 \theta P^2 \varpi^{-1} \\ & - \eta_1 bPT^{-1}\varpi^{-1} - \eta_2 \gamma \varpi^{-1} + \eta_2 \theta P\varpi^{-1} + \eta_2 bT^{-1}\varpi^{-1} - \eta_3 \gamma F\varpi^{-1} + \eta_3 \theta FP\varpi^{-1} \\ & + \eta_3 bFT^{-1}\varpi^{-1} - \eta_4 \gamma F^2 T\varpi^{-1} + \eta_4 \theta F^2 TP\varpi^{-1} + \eta_4 bF^2 \varpi^{-1} + \eta_5 \gamma FT\varpi^{-1} \\ & - \eta_5 \theta FTP\varpi^{-1} - \eta_5 bF\varpi^{-1} - \eta_6 \gamma T\varpi^{-1} + \eta_6 \theta TP\varpi^{-1} + \eta_6 b\varpi^{-1} + \eta_7 \gamma \varpi^{-1} - \eta_7 \theta P\varpi^{-1} \\ & - \eta_7 bT^{-1}\varpi^{-1} + \eta_8 \gamma F\varpi^{-1} - \eta_8 \theta FP\varpi^{-1} - \eta_8 bFT^{-1}\varpi^{-1} - \eta_9 T^{-1} + sY \end{aligned} \tag{163}$$

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