

University of Southern Queensland



**Test of Hypotheses for
Linear Regression Models with
Non-Sample Prior Information**

A Dissertation Submitted by

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Abstract

Classical inferences about population parameters are usually drawn from the sample data alone. This applies to methods used in parameter estimation and hypothesis testing. Inferences about population parameters could be improved using non-sample prior information (NSPI) on the value of another related parameter. However, any NSPI on the value of any parameter is likely to be uncertain (or unsure). The NSPI can be classified as (i) unknown (unspecified), (ii) known (specified), and (iii) uncertain if the suspected value is unsure. For the three different scenarios, three different statistical tests: (i) unrestricted test (UT), (ii) restricted test (RT) and (iii) preliminary test test (PTT) are defined. The current research is to test the intercept parameter(s) when NSPI is available on the slope parameter(s). The test statistics, their sampling distributions, and power functions of the tests are derived. Comparison of power functions of the tests are used to recommend a best test. In this thesis, we test (1) the intercept of the simple regression model (SRM) when there is NSPI on the slope, (2) the intercept vector of the multivariate simple regression model (MSRM) when there is NSPI on the slope vector, (3) a subset of regression parameters of the multiple regression model (MRM) when NSPI is available on another subset of the regression parameters, and (4) the equality of the intercepts for $p (\geq 2)$ lines of the parallel regression model (PRM) when there is NSPI on the slopes.

For each of the above four regression models, the following steps are carried out: (1) derived the test statistics of the UT, RT and PTT for both known and unknown variance, (2) derived the sampling distributions of the test statistics of the UT, RT and PTT, (3) derived and compared the power function and the size of the UT, RT and PTT. For known variance, under a sequence of an alternative hypothesis, the sampling distributions of the UT and RT of the simple regression model follows a normal distribution. However, the PTT follows a bivariate normal distribution. For unknown variance, the sampling distribution of the UT and RT of the simple regression model

follows a Student's t distribution but the PTT follows a correlated bivariate Student's t distribution. For the multivariate simple regression, multiple regression and parallel regression models, the sampling distribution of the UT and RT follows a univariate noncentral F distribution under the alternative hypothesis. However, the PTT follows a correlated bivariate noncentral F distribution. For the four regression models above, there is a correlation between the UT and PT but there is no such correlation between the RT and PT. To evaluate the power function of the PTT the probability integral of the bivariate normal, bivariate Student's t and bivariate noncentral F distributions are used. For the computations of the power function of the PTT of the MSRM, MRM and PRM require the cumulative distribution function (cdf) of a correlated bivariate noncentral F (BNCF) distribution. But the correlated BNCF distribution is not available in the literature, and hence we derive the probability density function (pdf) and cdf of the BNCF distribution. The R package is used for all computations and graphical analyses.

The statistical criteria that are used to compare the performance of the UT, RT and PTT are the size and power of the tests. A test that minimizes the size and maximizes the power is preferred over any other tests. In reality, the size of a test is fixed, and then the choice of the best test is based on its maximum power.

The study shows that the power of the RT is always higher than that of the UT and PTT, and the power of the PTT lies between the power of the RT and UT. The size of the UT is smaller than that of the RT and PTT. Among the three tests, the UT has the lowest power and lowest size. In terms of power it is the worst and in terms of size it is the best. The RT has maximum power and size. The PTT has smaller size than the RT and the RT has larger power than the PTT. The PTT protects against maximum size of the RT and minimum power of the UT. Thus the the PTT attains a reasonable dominance over the UT and RT for all regression models when the suspected value of the slope parameter(s) suggested by the NSPI is not too far away from that under the null hypothesis.

Certification of Dissertation

I certify that the ideas, mathematical derivation of the formulas, findings, analyses and conclusions reported in this dissertation are the result of my own work, except where otherwise acknowledged. I also certify that the thesis is original and has not been previously submitted for any other award to any other university, except where otherwise acknowledged.

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List of Abbreviations

BCF	bivariate central F
BNCC	bivariate noncentral chi-square
BNCF	bivariate noncentral F
cdf	cumulative distribution function
d.f.	degrees of freedom
LR	likelihood ratio
LRT	likelihood ratio test
LSE	least-square estimator
MLE	maximum likelihood estimator
MLUE	maximum likelihood unrestricted estimator
MRM	multiple regression model
MSRM	multivariate simple regression model
NSPI	non-sample prior information
pdf	probability density function
PRM	parallel regression model
PT	preliminary test
PTE	preliminary test estimator
PTT	preliminary test test
RE	restricted estimator
RT	restricted test
SRM	simple regression model
UE	unrestricted estimator
UT	unrestricted test

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Chapter 1

Overview

1.1 Introduction

As a common practice, classical inferences about population parameters are always drawn from the sample data alone. This applies to methods used in parameter estimation and hypothesis testing. Inferences about population parameters could be improved using non-sample prior information (NSPI) from trusted sources (cf Bancroft, 1944). Such information, which is usually available from previous studies or expert knowledge or experience of the researchers, is un-related to the sample data. It is expected that the inclusion of NSPI in addition to the sample data improves the quality of the estimator and the performance of the test. However, any NSPI on the value of any parameter is likely to be uncertain (or unsure). In this case, the information can be articulated in the form of a null hypothesis. An appropriate statistical test on this null hypothesis will be useful to eliminate the uncertainty on the suspected information. Then the outcome of the preliminary testing (pre-testing) on the uncertain NSPI is used in the hypothesis testing or estimation. This approach is likely to improve the quality of the estimator and the performance of the statistical test (see Khan and Saleh, 2001; Saleh, 2006, p. 1; Yunus, 2010; Yunus and Khan, 2011a).

The NSPI can be classified as (i) unknown (unspecified) if NSPI on the value of

the parameter(s) is unavailable, (ii) known (certain or specified) if the exact value of the parameter(s) is available, and (iii) uncertain if the suspected value is unsure (that is, suspected to be a fixed quantity). For the three different scenarios, three different estimators, namely the (i) unrestricted estimator (UE), (ii) restricted estimator (RE) and (iii) preliminary test estimator (PTE) are defined in the literature (see, e.g., Judge and Bock, 1978; Saleh, 2006, p. 58). Khan (2003), and Khan and Hoque (2003) provide the UE, RE, and PTE for different linear models. For the testing purpose, three different statistical tests, namely the (i) unrestricted test (UT), (ii) restricted test (RT) and (iii) pre-test test (PTT) are defined along the same line as the three different estimators. The UE and UT use the sample data alone but the RE and RT do not use the sample data alone. The PTE and PTT use both the NSPI and the sample data. The PTE is a choice between the UE and RE, whereas the PTT is a choice between the UT and RT. The choice depends on the outcome of the pre-testing on the uncertain NSPI value. Note that by definition the test statistics of the PT and UT are correlated but that of the PT and RT are uncorrelated, indeed independent.

Many authors have contributed to this area to the estimation of parameter(s) in the presence of uncertain NSPI. Bancroft (1944, 1964, 1965) and Han and Bancroft (1968) introduced a preliminary test estimation of parameters to estimate the parameters of a model with uncertain prior information. Later, Sclove et al. (1972), Stein (1981), Maatta and Casella (1990), Bhoj and Ahsanullah (1994), Khan (2003, 2005, 2006a, 2006b, 2008), Khan and Saleh (1995, 1997, 2001, 2005, 2008), Khan et al. (2002a, 2002b, 2005), Khan and Hoque (2003), and Saleh (2006, p. 55) covered various work in the area of improved estimation using NSPI. But there is a very limited number of studies on the testing of parameters in the presence of uncertain NSPI. Although Tamura (1965), Saleh and Sen (1978, 1982), Yunus and Khan (2008, 2011a, 2011b), and Yunus (2010) used the NSPI for testing hypothesis using nonparametric

methods, the problem has not been addressed in the parametric context. Some authors have studied the UE, RE and PTE for parametric cases (for instance Bechhofer (1951), Bozivich et al. (1956), Bancroft (1964) and Saleh (2006)), but not the tests.

The current research is to test the intercept parameter(s) when NSPI is available on the slope parameter(s) of various linear models. The test statistics, their sampling distributions, and power function of the tests are derived. Comparison of power function of the tests are used to recommend a best test. In this thesis, we test

1. the intercept of the *simple regression model* (SRM) when there is NSPI on the slope,
2. the intercept vector of the *multivariate simple regression model* (MSRM) when there is NSPI on the slope vector,
3. a subset of regression parameters of the *multiple regression model* (MRM) when NSPI is available on another subset of the regression parameters, and
4. the equality of the intercepts for $p (\geq 2)$ lines of the *parallel regression model* (PRM) when there is NSPI on the slopes.

For each of the above four regression models, the following tasks are carried out:

1. derive the test statistics of the UT, RT and PTT for both known and unknown variance,
2. derive the sampling distribution of the test statistics of the UT, RT and PTT, and
3. derive and compare the power function and the size of the UT, RT and PTT.

For known variance, under a sequence of the alternative hypotheses, the sampling distribution of the test statistics of the UT and RT of the simple regression model

follow normal distributions. However, the test statistic of the PTT follows a bivariate normal distribution. For unknown variance, the sampling distribution of the test statistics of the UT and RT of the simple regression model follow Student's t distribution but that of the PTT follows a correlated bivariate Student's t distribution. For the MSRM, MRM and PRM, the sampling distribution of the test statistics of the UT and RT follow a univariate noncentral F distribution under alternative hypothesis. However, the test statistic of the PTT follows a correlated bivariate noncentral F (BNCF) distribution. For the above four regression models, there is a correlation between the UT and PT but there is no correlation between the RT and PT. To evaluate the power function of the PTT the probability integral of the bivariate normal, bivariate Student's t and BNCF distributions are used. However, the bivariate probability integrals of the above three distributions are very complicated. Computational formulas to evaluate the probability density function (pdf) and cumulative distribution function (cdf) of the distributions are provided. The R package is used for all computations and graphical analyses.

The computations of the power function of the PTT of the regression models (MSRM, MRM and PRM) require the cdf of a correlated BNCF distribution. But the correlated BNCF distribution is not available in the literature, and hence we derive the pdf and cdf of the BNCF distribution. The pdf and cdf of the *doubly* BNCF distribution are derived by mixing correlated bivariate noncentral chi-square (BNCC) and central chi-square distributions. Whereas, the pdf and cdf of the *singly* BNCF distribution are obtained by compounding the Poisson distribution with the bivariate central F (BCF) distribution. For the computation the probability integral of the bivariate Student's t distribution, we refer to the pdf of the multivariate t distribution given by Kotz and Nadarajah (2004, p. 1).

The statistical criteria that are used to compare the performance of the UT, RT

and PTT are the size and power of the tests. A statistical test that has a minimum size is preferable because it ensures a smaller probability of a type I error. Also, if more than one statistical tests have the same power, the test that has a minimum size is preferable. Furthermore, a test (or among the tests of the same size) that has maximum power is preferred over any other tests because it guarantees the highest probability of rejecting any false null hypothesis. A test that minimizes the size and maximizes the power is preferred over any other tests. In reality, the size of a test is fixed, and then the choice of the best test is based on the criterion of maximum power.

The study shows that the power of the RT is always higher than that of the UT and PTT, and the power of the PTT lies between the power of the RT and UT. The size of the UT is smaller than that of the RT and PTT. Among the three tests, the UT has the lowest power and lowest size. In terms of power it is the worst and in terms of size it is the best. The RT has maximum power and size. The PTT has smaller size than the RT and the RT has larger power than the PTT. The PTT protects against maximum size of the RT and minimum power of the UT. Thus, PTT attains a reasonable dominance over the other two tests for all four regression models when the suspected value of the parameter(s) suggested by the NSPI is not too far away from its true value.

1.2 Main Contributions of the Thesis

The main contributions of the dissertation are as follow.

1. To use the uncertain NSPI in addition to the sample data to improve the performance of the test.
2. To propose the UT, RT and PTT for the following tests.

- To test the intercept when NSPI is available on the value of the slope for the simple regression model.
 - To test the intercept when NSPI is available on the value of the slope for the multivariate simple regression model.
 - To test a subset of regression parameters when NSPI on another subset of regression parameters is available for the multiple regression model.
 - To test the equality of the two intercepts when NSPI on the equality of the two slopes is available for the parallel regression model.
3. To determine the sampling distribution of the test statistics of the UT, RT and PTT for the four different regression models.
 4. To derive the power functions of the UT, RT and PTT for the four different regression models.
 5. To compare the power functions and the sizes of the UT, RT and PTT for the four different regression models.
 6. To search for an optimum test that minimizes the size and maximizes the power and recommend the best performing test with maximum power (for fixed size).

For the SRM, we consider two cases for known and unknown variance, but for the other models only the case of unknown variance/covariance is considered.

1.3 Thesis Outlines

The thesis consists of seven chapters, an overview is presented in Chapter 1. Chapter 2 describes a literature review of previous works, some related distributions, the bivariate noncentral F distribution and methodology of analysis. Testing of the intercept for both cases of known and unknown variance by the UT, RT and PTT in the presence of uncertain NSPI on the slope of the simple regression model is introduced

in Chapter 3. Chapter 4 is devoted to testing the intercept by the UT, RT and PTT for the multivariate simple regression model. Testing of a subset of regression parameters by the UT, RT and PTT for the multiple regression model is provided in Chapter 5. Chapter 6 describes testing of the equality of the two intercepts by the UT, RT and PTT for the parallel regression model. In each chapter, the study discusses the test statistics, their distributions, power function of the tests and comparison of power of the UT, RT and PTT. An illustrative example is given using simulated data. The graphical representation of the power and size of the tests are also provided. Finally the power function and size of the UT, RT and PTT are compared. The conclusion and discussion are provided in Chapter 7.

Chapter 2

Literature Review, the Bivariate Noncentral F Distribution and Methodology of Analysis

2.1 Literature Review

2.1.1 The Preliminary Test Estimation

From Section 1.1, we see the use of prior information in the parameter estimation or hypothesis test may improve the quality of the estimator or the performance of the test. However, such a prior information is usually uncertain. This has warranted the preliminary testing on the suspected value of the parameter(s) to remove the uncertainty. The outcome of the pre-test is then incorporated into the procedure of estimation or test on another parameter (cf Yunus, 2010).

Judge and Bock (1978), Khan and Saleh (2001), and Saleh (2006) have discussed detail about the theory of pre-test in the area of parameter estimation including UE, RE and PTE. There are a large number of published articles on the PTE given by Saleh (2006); Khan (1998); Khan and Saleh (2001); Kabir and Khan (2009), among others, whereas the idea of the PTE is introduced by Bancroft (1944, 1964). Later, Bancroft (1965) implemented the idea of PTE in the ANOVA to study the effect of pre-testing on the estimation of variance.

To understand the concept of the PTE, we can refer to the problem of estimating the population mean (μ) of any population based on a set of sample data when it is *a priori* suspected that $\mu = \mu_0$. Let X_1, \dots, X_n be a random sample from $N(\mu, \sigma^2)$. Then the sample mean $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ is an unrestricted estimator (UE) of μ . Similarly the sample variance $S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$ is an unbiased estimator of σ^2 . Based on the exclusive sample data, $\mu^{UE} = \bar{X}$. If the uncertain NSPI on μ is given by μ_0 , then the restricted estimator (RE) of the mean is defined as $\mu^{RE} = \mu_0$. Note μ_0 does not depend on the sample data. To remove the uncertainty in the suspected values of μ a statistical test is performed. In this case we test $H_0 : \mu = \mu_0$ versus $H_a : \mu \neq \mu_0$ using the test statistic

$$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$$

which follows a Student's t distribution with $(n - 1)$ degrees of freedom. Then the pre-test estimator (PTE) of μ is defined as

$$\hat{\mu}^{PTE} = \mu^{UE} I(t_0 \geq t_{\frac{\alpha}{2}, n-1}) + \mu^{RE} I(t_0 < t_{\frac{\alpha}{2}, n-1}),$$

where α is the level of significance and t_0 is the observed value of the t statistic and $I(B)$ is the indicator function of the set B . Clearly PTE depends on the value of α and it is either the UE or RE depending on the outcome of the pre-test on the NSPI. The PTE can also be explained for the SRM,

$$Y_i = \beta_0 + \beta_1 X_i + e_i,$$

where the error variable e_i for $i = 1, 2, \dots, n$ are i.i.d $N(0, \sigma^2)$ (Wackerly et al., 2008, p.581). If β_0 and β_1 be the unknown intercept and slope parameters, respectively, with $\beta_1 = \beta_{10}$ (suspected), then the UE, RE and PTE are given as:

- (i) the UE of β_0 is the least square estimator (LSE) or maximum likelihood estimator (MLE), $\tilde{\beta}_0^{UE} = \bar{Y} - \tilde{\beta}_1 \bar{X}$, where $\tilde{\beta}_1$ is the maximum likelihood estimator or, equivalently, least square estimator of β_1 , $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ and $\bar{Y} = \frac{1}{n} \sum_{i=1}^n Y_i$,

(ii) the RE of β_0 is $\hat{\beta}_0^{RE} = \bar{Y} - \beta_{10}\bar{X}$, and

(iii) if the NSPI on β_1 is uncertain, the uncertainty is removed by testing $H_0^* : \beta_1 = \beta_{10}$ with the test statistic,

$$T^2 = \frac{(\tilde{\beta}_1 - \beta_{10})^2 \sum_{i=1}^n (X_i - \bar{X})^2}{SE(\tilde{\beta}_1)},$$

where $T^2 \sim F_{1,n-2}$ under H_0^* . Based on the rejection or acceptance of H_0^* , the PTE of the intercept β_0 is a choice between the UE and RE. The PTE of β_0 is

$$\tilde{\beta}_0^{PTE} = \hat{\beta}_0 I(T^2 < F_{\alpha,1,n-2}) + \tilde{\beta}_0 I(T^2 \geq F_{\alpha,1,n-2}),$$

where $F_{\alpha,1,n-2}$ is the α -level upper critical value of a central F distribution with $(1, n-2)$ degrees of freedom. Hoque et al. (2009) explained that the PTE outperforms the UE if the uncertain NSPI about the value of the slope is not too far from its true value under the linex loss function. The PTE was also studied for other regression models such as the MSRM (Sen and Saleh, 1979; Ahmed, 1992; Khan, 2005, 2006a) and PRM (Akritas et al., 1984; Lambert et al., 1985a; Khan, 2003). So far, a lot of authors have covered various works in the area of improved estimation using NSPI, but only a limited number of authors have studied the UT, RT and PTT for parametric setup. However, studies of the PTE found that there is no uniform domination among the UE, RE and PTE, and the PTE is an extreme choice between the UE and RE. The question remains whether the UT, RT and PTT for parametric regression models possesses the same kind of properties.

In the analysis of regression model, estimation of parameters and hypothesis testing of parameters are two important aspects. Researchers are interested in estimating both the slope and the intercept parameters. However the estimation of the intercept parameter is more complicated than estimating the slope, as the former depends on the estimation of the slope parameter. In this study the errors of the regression mod-

els are assumed to be normal. For normal models, the maximum likelihood estimator is identical to the least square estimator.

2.1.2 The Pre-test Test

There is a limited number of studies on the testing of parameters in the presence of uncertain NSPI for the regression models. There are few articles found in the literature that use pre-test in the analysis of variance (Bechhofer, 1951; Bozivich et al., 1956; Paull, 1950 among others). Ohtani and Toyoda (1985, 1986) considered the problem of testing the linear hypothesis of regression coefficients after pre-testing the disturbance variance on the parametric linear regression model. Ohtani (1998), and Ohtani and Giles (1993) extended the ideas related to this parametric problems. In the same spirit, Lambert et al. (1985b) studied the performance of the UT, RT and PTT on the parallelism model. Then, Yunus (2010), and Yunus and Khan (2011a, 2011b) studied the theory of pre-test in the area of hypothesis testing including the UT, RT and PTT in the nonparametric context. So far, there is no study on the performance of the power function of the PTT using the BNCF distribution for multivariate parametric regression models, namely the MSRM, MRM and PRM.

2.2 The Bivariate Central F Distribution

The BCF distribution is discussed in Krishnaiah (1964), Amos and Bulgren (1972), Schuurmann et al. (1975), and El-Bassiouny and Jones (2009). Following Krishnaiah (1964) for $X_i = F_i \sim F_{\nu_1, \nu_2}$ with $i = 1, 2$ and correlation coefficient ρ , the pdf and cdf of the BCF distribution are defined, respectively, as

$$f(x_1, x_2) = \left(\frac{\nu_2^{\nu_2/2} (1 - \rho^2)^{(\nu_1 + \nu_2)/2}}{\Gamma(\nu_1/2) \Gamma(\nu_2/2)} \right) \times \sum_{j=0}^{\infty} \left(\frac{\rho^{2j} \Gamma(\nu_1 + (\nu_2/2) + 2j)}{j! \Gamma((\nu_1/2) + j)} \right) \nu_1^{\nu_1 + 2j} \\ \times \left(\frac{(x_1 x_2)^{(\nu_1/2) + j - 1}}{[\nu_2(1 - \rho^2) + \nu_1(x_1 + x_2)]^{\nu_1 + (\nu_2/2) + 2j}} \right), \text{ and} \quad (2.2.1)$$

$$P(X_1 < d, X_2 < d) = \left(\frac{(1 - \rho^2)^{\nu_1/2}}{\Gamma(\nu_1/2)\Gamma(\nu_2/2)} \right) \sum_{j=0}^{\infty} \left(\frac{\rho^{2j}\Gamma(\nu_1 + (\nu_2/2) + 2j)}{j!\Gamma((\nu_1/2) + j)} \right) L_j, \quad (2.2.2)$$

where

$$L_j = \int_0^h \int_0^h \frac{(x_1 x_2)^{(\nu_1/2)+j-1} dx_1 dx_2}{(1 + x_1 + x_2)^{\nu_1 + (\nu_2/2) + 2j}},$$

with $h = \frac{d\nu_1}{\nu_2(1-\rho^2)}$. An approximation to the value of the cdf of the BCF distribution is also found in Amos and Bulgren (1972).

Krishnaiah (1964, 1965), and Krishnaiah and Armitage (1965) studied the multivariate central F distribution. Hewett and Bulgren (1971) studied about prediction interval for failure times in certain life testing experiments using the multivariate central F distribution. Later, Schuurmann et al. (1975) presented a statistical table for the critical values of some selected multivariate central F distributions.

2.3 The Bivariate Noncentral F Distribution

Following Johnson et al. (1995, p. 480), we note that the *doubly* noncentral F variable with (ν_1, ν_2) degrees of freedom and noncentrality parameters λ_1 and λ_2 is defined as

$$F''_{\nu_1, \nu_2}(\lambda_1, \lambda_2) = \frac{\chi_{\nu_1}^2(\lambda_1)/\nu_1}{\chi_{\nu_2}^2(\lambda_2)/\nu_2}, \quad (2.3.1)$$

where the two noncentral chi-square distributions are independent. In many applications $\lambda_2 = 0$, which arises when there is a central χ^2 variable in the denominator of F''_{ν_1, ν_2} . This is called a *singly* noncentral (or simply noncentral) F variable with (ν_1, ν_2) degrees of freedom and noncentrality parameter λ_1 . The case for $\lambda_1 = 0, \lambda_2 \neq 0$, is not considered here, but note that

$$F''_{\nu_1, \nu_2}(0, \lambda_2) = \frac{1}{F'_{\nu_1, \nu_2}(\lambda_2)}, \quad E[F'_{\nu_1, \nu_2}(\lambda_1)] = \frac{\nu_2(\nu_1 + \lambda_1)}{\nu_1(\nu_2 - 2)}, \quad \nu_2 > 2, \text{ and}$$

$$Var[F'_{\nu_1, \nu_2}(\lambda_1)] = 2 \left(\frac{\nu_2}{\nu_1} \right)^2 \left(\frac{(\nu_1 + \lambda_1)^2 + (\nu_1 + 2\lambda_1)(\nu_2 - 2)}{(\nu_2 - 4)(\nu_2 - 2)^2} \right), \quad \nu_2 > 4.$$

Moreover, Johnson et al. (1995, p. 499) described

$$G''_{\nu_1, \nu_2}(\lambda_1, \lambda_2) = \frac{\chi'_{\nu_1}(\lambda_1)}{\chi'_{\nu_2}(\lambda_2)} \quad (2.3.2)$$

as a mixture of G_{ν_1+2j, ν_2+2k} distribution in proportion of

$$\left(\frac{e^{-\lambda_1/2} \left(\frac{\lambda_1}{2}\right)^j}{j!} \right) \times \left(\frac{e^{-\lambda_2/2} \left(\frac{\lambda_2}{2}\right)^k}{k!} \right),$$

representing product of two independent Poisson distributions. The details of the probability density function and cumulative density function of G'' are also found in Johnson et al. (1995, p. 500). Some discussions on the approximation of the noncentral F distribution are found in Mudholkar et al. (1976). An approximation to the multivariate noncentral F distribution is found in Tiku (1966).

2.3.1 The Singly Bivariate Noncentral F Distribution

Following Krishnaiah (1964) and Johnson et al. (1995, p. 499), for $X_i = F_i \sim F_{\nu_1, \nu_2}$ with $i = 1, 2$, and $R \sim \text{Pois}(\lambda)$ the *singly* BNCF variable is given as

$$\sum_{r=0}^{\infty} \left(\frac{e^{-\lambda/2} \left(\frac{\lambda}{2}\right)^r}{r!} \right) \times F_{\nu_r, \nu_2}, \quad (2.3.3)$$

where $\nu_r = \nu_1 + 2r$. Furthermore, the pdf of the *singly* BNCF distribution with noncentrality parameter λ is given by the pdf

$$f(x_1, x_2, \nu_r, \nu_2, \lambda) = \sum_{r=0}^{\infty} \left(\frac{e^{-\lambda/2} \left(\frac{\lambda}{2}\right)^r}{r!} \right) f_1(x_1, x_2, \nu_r, \nu_2), \quad (2.3.4)$$

where $f_1(x_1, x_2, \nu_r, \nu_2)$ is the pdf of a BCF distribution with ν_r and ν_2 degrees of freedom, that is,

$$\begin{aligned} f_1(x_1, x_2, \nu_r, \nu_2) &= \left(\frac{\nu_2^{\nu_2/2} (1 - \rho^2)^{(\nu_r + \nu_2)/2}}{\Gamma(\nu_r/2) \Gamma(\nu_2/2)} \right) \sum_{j=0}^{\infty} \left(\frac{\rho^{2j} \Gamma(\nu_r + (\nu_2/2) + 2j)}{j! \Gamma((\nu_r/2) + j)} \right) \\ &\times (\nu_r^{\nu_r + 2j}) \left(\frac{(x_1 x_2)^{(\nu_r/2) + j - 1}}{[\nu_2(1 - \rho^2) + \nu_r(x_1 + x_2)]^{\nu_r + (\nu_2/2) + 2j}} \right). \end{aligned} \quad (2.3.5)$$

Then the cdf of the *singly* BNCF distribution is defined as

$$P(\cdot) = P(X_1 < d, X_2 < d, \nu_r, \nu_2, \lambda) = \sum_{r=0}^{\infty} \left(\frac{e^{-\lambda/2} \left(\frac{\lambda}{2}\right)^r}{r!} \right) \times P_2(X_1 < d, X_2 < d, \nu_r, \nu_2), \quad (2.3.6)$$

where

$$P_2(X_1 < d, X_2 < d, \nu_r, \nu_2) = \left(\frac{(1 - \rho^2)^{\nu_r/2}}{\Gamma(\nu_r/2)\Gamma(\nu_2/2)} \right) \times \sum_{j=0}^{\infty} \left(\frac{\rho^{2j} \Gamma(\nu_r + (\nu_2/2) + 2j)}{j! \Gamma((\nu_r/2) + j)} \right) L_{jr} \quad (2.3.7)$$

in which L_{jr} is defined as

$$L_{jr} = \int_0^{h_r} \int_0^{h_r} \frac{(x_1 x_2)^{(\nu_r/2)+j-1} dx_1 dx_2}{(1 + x_1 + x_2)^{\nu_r + (\nu_2/2) + 2j}}$$

with $h_r = \frac{d\nu_r}{\nu_2(1-\rho^2)}$.

For the computation of the value of the cdf of the *singly* BNCF distribution, R codes are used. To make the computation easy, we rewrite the formula of the cdf of the *singly* BNCF distribution as follows:

$$\begin{aligned}
P(\cdot) &= \sum_{r=0}^{\infty} \left(\frac{e^{-\lambda/2} \left(\frac{\lambda}{2}\right)^r}{r!} \right) \left(\frac{(1-\rho^2)^{\nu_r/2}}{\Gamma(\nu_r/2)\Gamma(\nu_2/2)} \right) \\
&\quad \times \sum_{j=0}^{\infty} \left(\frac{\rho^{2j}\Gamma(\nu_r + (\nu_2/2) + 2j)}{j!\Gamma((\nu_r/2) + j)} \right) L_{jr} \\
&= \sum_{r=0}^{\infty} T_r \left[\left(\frac{\Gamma(\nu_r + \nu_2/2)}{0!\Gamma(\nu_r/2)} \right) L_{0r} + \left(\frac{\rho^2\Gamma(\nu_r + \nu_2/2 + 2)}{1!\Gamma(\nu_r/2 + 1)} \right) L_{1r} + \dots \right] \\
&= \sum_{r=0}^{\infty} T_r [H_{0r}L_{0r} + H_{1r}L_{1r} + H_{2r}L_{2r} + \dots] \\
&= \sum_{r=0}^{\infty} T_r H_{0r}L_{0r} + T_r H_{1r}L_{1r} + T_r H_{2r}L_{2r} + \dots \\
&= [T_0 H_{00}L_{00} + T_0 H_{10}L_{10} + T_0 H_{20}L_{20} + \dots] + \\
&\quad [T_1 H_{01}L_{01} + T_1 H_{11}L_{11} + T_1 H_{21}L_{21} + \dots] + \\
&\quad [T_2 H_{02}L_{02} + T_2 H_{12}L_{12} + T_2 H_{22}L_{22} + \dots] + \dots, \tag{2.3.8}
\end{aligned}$$

where

$$\begin{aligned}
T_r &= \left(\frac{e^{-\lambda/2} \left(\frac{\lambda}{2}\right)^r}{r!} \right) \left(\frac{(1-\rho^2)^{\nu_r/2}}{\Gamma(\nu_r/2)\Gamma(\nu_2/2)} \right), \\
H_{0r} &= \frac{1\Gamma(\nu_r + (\nu_2/2))}{0!\Gamma((\nu_r/2))}, \quad H_{1r} = \frac{\rho^2\Gamma(\nu_r + (\nu_2/2) + 2)}{1!\Gamma((\nu_r/2) + 1)}, \\
H_{2r} &= \frac{\rho^4\Gamma(\nu_r + (\nu_2/2) + 4)}{2!\Gamma((\nu_r/2) + 2)}, \dots,
\end{aligned}$$

and P_0 is defined as

$$\begin{aligned}
P_0 &= \sum_{r=0}^{\infty} T_r H_{0r}L_{0r} = T_0 H_{00}L_{00} + T_1 H_{01}L_{01} + T_2 H_{02}L_{02} + \dots, \text{ for } j = 0, \\
&= \left(\frac{e^{-\lambda/2}}{0!} \times \frac{(1-\rho^2)^{\nu_1/2}}{\Gamma(\nu_1/2)\Gamma(\nu_2/2)} \right) \left(\frac{1\Gamma(\nu_1 + \nu_2/2)}{0!\Gamma(\nu_1/2)} \right) L_{00} + \\
&\quad \left(\frac{e^{-\lambda/2} \left(\frac{\lambda}{2}\right)}{1} \times \frac{(1-\rho^2)^{\nu_1+2/2}}{\Gamma((\nu_1+2)/2)\Gamma(\nu_2/2)} \right) \left(\frac{1\Gamma(\nu_1 + 2 + (\nu_2/2))}{0!\Gamma((\nu_1+2)/2)} \right) L_{01} \\
&\quad + \dots. \tag{2.3.9}
\end{aligned}$$

Similarly, we obtain the expressions for

$$P_1 = \sum_{r=0}^{\infty} T_r H_{1r}L_{1r}, \quad P_2 = \sum_{r=0}^{\infty} T_r H_{2r}L_{2r} \text{ and so on.}$$

Finally we obtain

$$P(.) = P_0 + P_1 + P_2 + P_3 + \dots = \sum_{j=0}^{\infty} P_j.$$

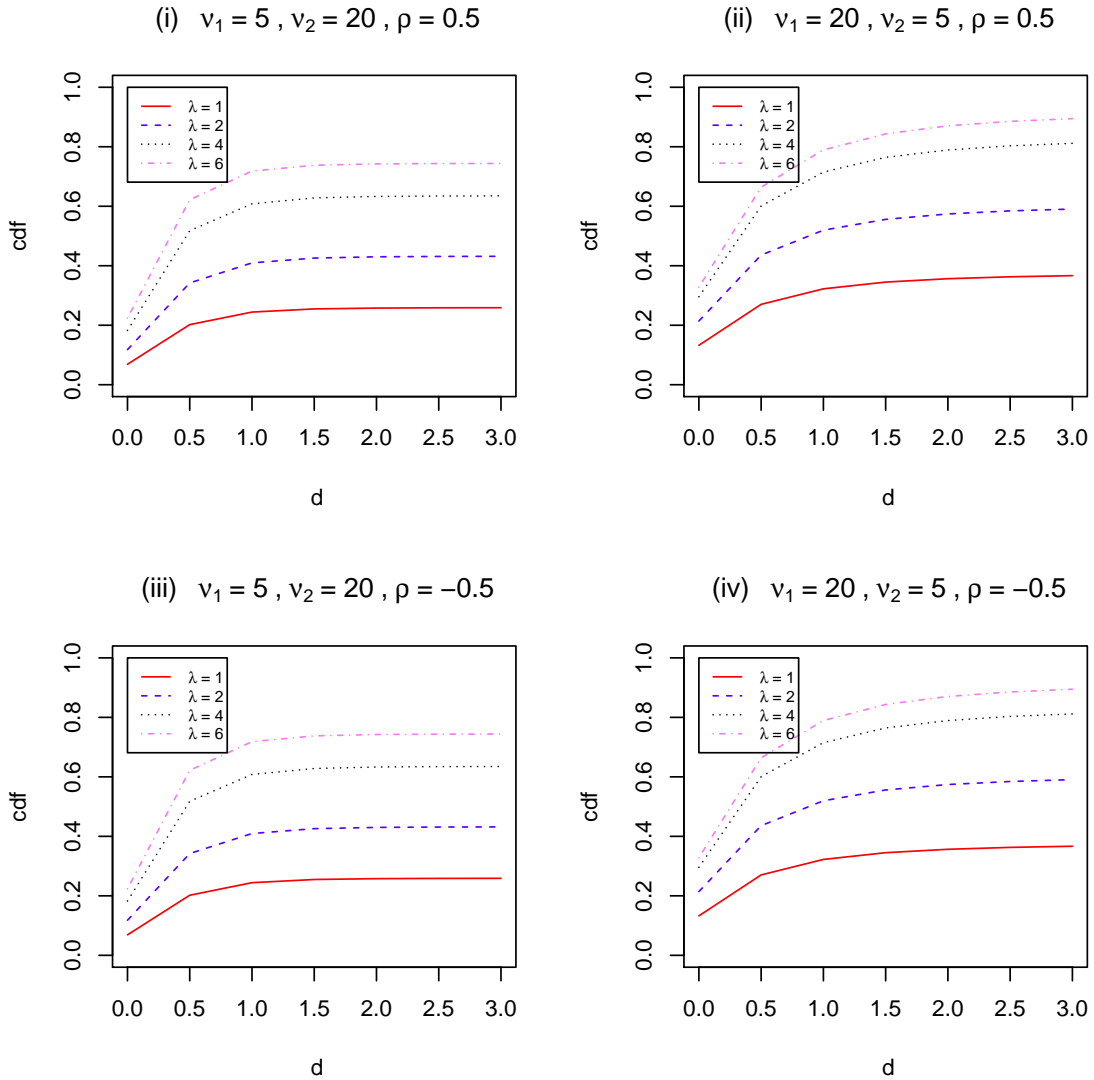


Figure 2.1: The cdf of the *singly* bivariate noncentral F distribution.

The graph of the cdf of the *singly* BNCF distributions is presented in Figure 2.1. This figure shows that the value of the cdf of the *singly* BNCF distribution increases as the value of any of the parameters, degrees of freedom ν_1 (for fixed ν_2), λ , and d , increases. From Equation (2.3.6) we see that the cdf of the *singly* BNCF distribution

also depends on ρ . For both $\rho < 0.5$ and $\rho > 0.5$, the curves of the cdf of the *singly* BNCF distribution are lower than that for $\rho = 0.5$. For $\rho = -0.5$ and $\rho = 0.5$, the graph of the cdf of the *singly* BNCF distribution is identical. The R codes that produce Figures 2.1 are provided in Appendix A.1.

2.3.2 The Doubly Bivariate Noncentral F Distribution

The *doubly* BNCF distribution is defined by compounding the pdf of the BNCC distribution of X_1 and X_2 with m degrees of freedom and noncentrality parameters θ_1 and θ_2 , and central chi-square distribution of Z with n degrees of freedom (see Yunus and Khan, 2011c, and Amos and Bulgren, 1972). The pdf of the correlated BNCC variables X_1 and X_2 is given by

$$\begin{aligned}
 g(x_1, x_2) &= \sum_{j=0}^{\infty} \sum_{r_1=0}^{\infty} \sum_{r_2=0}^{\infty} [\rho^{2j} (1 - \rho^2)^{m/2} \Gamma(m/2 + j)] \\
 &\times \left[\frac{(x_1)^{m/2+j+r_1-1} e^{-\frac{(x_1)}{2(1-\rho^2)}}}{[2(1-\rho^2)]^{m/2+j+r_1} \Gamma(m/2 + j + r_1)} \times \frac{e^{-\theta_1/2} (\theta_1/2)^{r_1}}{r_1!} \right] \\
 &\times \left[\frac{(x_2)^{m/2+j+r_2-1} e^{-\frac{(x_2)}{2(1-\rho^2)}}}{[2(1-\rho^2)]^{m/2+j+r_2} \Gamma(m/2 + j + r_2)} \times \frac{e^{-\theta_2/2} (\theta_2/2)^{r_2}}{r_2!} \right],
 \end{aligned} \tag{2.3.10}$$

and the pdf of a central chi-square variable Z with n degrees of freedom is

$$f(z) = \frac{z^{(n/2)-1} e^{-z/2}}{2^{n/2} \Gamma(n/2)}, z > 0, \tag{2.3.11}$$

where Z is independent of X_1 and X_2 . Then using transformation of variable method for multivariable case (see for instance, Wackerly et al. 2008, p. 325), we obtain the joint pdf of $\mathbf{y} = [y_1, y_2]'$ and z variables as

$$f(\mathbf{y}, z) = f(\mathbf{x})f(z) | J((\mathbf{x}, z) \rightarrow (\mathbf{y}, z)) |, \tag{2.3.12}$$

where $y_1 = \frac{nx_1}{mz}$, $y_2 = \frac{nx_2}{mz}$ and the Jacobian of the transformation $(x_1, x_2, z) \rightarrow (y_1, y_2, z)$, is given by

$$\det. \begin{pmatrix} \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2} & \frac{\partial x_1}{\partial z} \\ \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial y_2} & \frac{\partial x_2}{\partial z} \\ \frac{\partial z}{\partial y_1} & \frac{\partial z}{\partial y_2} & \frac{\partial z}{\partial z} \end{pmatrix} = \det. \begin{pmatrix} \frac{m}{n}z & 0 & \frac{m}{n}y_1 \\ 0 & \frac{m}{n}z & \frac{m}{n}y_2 \\ 0 & 0 & 1 \end{pmatrix} = \left(\frac{m}{n}z\right)^2. \quad (2.3.13)$$

Therefore, the joint pdf of \mathbf{y} and z is given by

$$\begin{aligned} f(\mathbf{y}, z) &= \sum_{j=0}^{\infty} \sum_{r_1=0}^{\infty} \sum_{r_2=0}^{\infty} [\rho^{2j}(1-\rho^2)^{m/2}\Gamma(m/2+j)] \\ &\times \left[\frac{\left(\frac{m}{n}y_1z\right)^{m/2+j+r_1-1} e^{-\frac{(\frac{m}{n}y_1z)}{2(1-\rho^2)}}}{[2(1-\rho^2)]^{m/2+j+r_1}\Gamma(m/2+j+r_1)} \times \frac{e^{-\theta_1/2}(\theta_1/2)^{r_1}}{r_1!} \right] \\ &\times \left[\frac{\left(\frac{m}{n}y_2z\right)^{m/2+j+r_2-1} e^{-\frac{(\frac{m}{n}y_2z)}{2(1-\rho^2)}}}{[2(1-\rho^2)]^{m/2+j+r_2}\Gamma(m/2+j+r_2)} \times \frac{e^{-\theta_2/2}(\theta_2/2)^{r_2}}{r_2!} \right] \\ &\times \frac{z^{(n/2)-1}e^{-z/2}}{2^{n/2}\Gamma(n/2)} \times \left(\frac{m}{n}z\right)^2. \end{aligned} \quad (2.3.14)$$

Then the joint pdf of (Y_1, Y_2) , where

$$Y_i = \frac{X_i/m}{Z/n}, \text{ for } i = 1, 2, \quad (2.3.15)$$

is obtained as

$$f(\mathbf{y}) = f(y_1, y_2) = \int_z f(\mathbf{y}, z) dz, \quad (2.3.16)$$

that is,

$$\begin{aligned}
f(y_1, y_2) &= \left(\frac{m}{n}\right)^2 \left[\frac{(1-\rho^2)^{m/2}}{2^{n/2}\Gamma(n/2)} \right] \\
&\times \sum_{j=0}^{\infty} \sum_{r_1=0}^{\infty} \sum_{r_2=0}^{\infty} [\rho^{2j}\Gamma(m/2+j)] \left(\frac{e^{-\theta_1/2}(\theta_1/2)^{r_1}}{r_1!} \right) \left(\frac{e^{-\theta_2/2}(\theta_2/2)^{r_2}}{r_2!} \right) \\
&\times \left[\frac{\left(\frac{m}{n}y_1\right)^{m/2+j+r_1-1}}{[2(1-\rho^2)]^{m/2+j+r_1}\Gamma(m/2+j+r_1)} \right] \\
&\times \left[\frac{\left(\frac{m}{n}y_2\right)^{m/2+j+r_2-1}}{[2(1-\rho^2)]^{m/2+j+r_2}\Gamma(m/2+j+r_2)} \right] \\
&\times \int_0^{\infty} z^{m+n/2+2j+r_1+r_2-1} e^{-z/2\left(\frac{(m/n)y_1}{1-\rho^2} + \frac{(m/n)y_2}{1-\rho^2} + 1\right)} dz \\
&= \left(\frac{m}{n}\right)^2 \left[\frac{(1-\rho^2)^{m/2}}{2^{n/2}\Gamma(n/2)} \right] \\
&\times \sum_{j=0}^{\infty} \sum_{r_1=0}^{\infty} \sum_{r_2=0}^{\infty} [\rho^{2j}\Gamma(m/2+j)] \left(\frac{e^{-\theta_1/2}(\theta_1/2)^{r_1}}{r_1!} \right) \left(\frac{e^{-\theta_2/2}(\theta_2/2)^{r_2}}{r_2!} \right) \\
&\times \frac{\left(\frac{m}{n}\right)^{m+2j+r_1+r_2-2} \left[y_1^{m/2+j+r_1-1} y_2^{m/2+j+r_2-1} \right]}{[2(1-\rho^2)]^{m+2j+r_1+r_2}\Gamma(m/2+j+r_1)\Gamma(m/2+j+r_2)} \\
&\times \left(\frac{2}{w_y}\right)^{q_{rj}} \Gamma(q_{rj}) \\
&= \left(\frac{m}{n}\right)^m \left[\frac{(1-\rho^2)^{\frac{m+n}{2}}}{\Gamma(n/2)} \right] \sum_{j=0}^{\infty} \sum_{r_1=0}^{\infty} \sum_{r_2=0}^{\infty} \left[\rho^{2j} \left(\frac{m}{n}\right)^{2j} \Gamma(m/2+j) \right] \\
&\times \left[\left(\frac{e^{-\theta_1/2}(\theta_1/2)^{r_1}}{r_1!} \right) \left(\frac{\left(\frac{m}{n}\right)^{r_1}}{\Gamma(m/2+j+r_1)} \right) \left(y_1^{m/2+j+r_1-1} \right) \right] \\
&\times \left[\left(\frac{e^{-\theta_2/2}(\theta_2/2)^{r_2}}{r_2!} \right) \left(\frac{\left(\frac{m}{n}\right)^{r_2}}{\Gamma(m/2+j+r_2)} \right) \left(y_2^{m/2+j+r_2-1} \right) \right] \\
&\times \Gamma(q_{rj}) \left[(1-\rho^2) + \frac{m}{n}y_1 + \frac{m}{n}y_2 \right]^{-q_{rj}}, \tag{2.3.17}
\end{aligned}$$

where $w_y = \frac{(m/n)y_1}{1-\rho^2} + \frac{(m/n)y_2}{1-\rho^2} + 1$ and $q_{rj} = m + n/2 + 2j + r_1 + r_2$. The cdf of the doubly BNCF distribution is then defined as

$$P(Y_1 < a, Y_2 < b) = \int_0^a \int_0^b f(y_1, y_2) dy_1 dy_2, \tag{2.3.18}$$

where a and b are positive real numbers.

The above cdf can be expressed as

$$P(Y_1 \leq a, Y_2 \leq b) = \int_0^\infty f(z) \int_0^{\frac{bmy}{n}} \int_0^{\frac{amy}{n}} g(x_1, x_2) dx_1 dx_2 dz, \quad (2.3.19)$$

where $g(x_1, x_2)$ is the pdf of a BNCC distribution which is given in Equation (2.3.10), $f(z)$ is the pdf of the central chi-square variable which is given in Equation (2.3.11), and Y_i for $i = 1, 2$, are given in Equation (2.3.15).

Equation (2.3.19) can be written as

$$\begin{aligned} P(Y_1 \leq a, Y_2 \leq b) &= (1 - \rho^2)^{\frac{m}{2}} \sum_{r_1=0}^{\infty} \sum_{r_2=0}^{\infty} \sum_{j=0}^{\infty} \frac{\left(\frac{m}{2}\right)_j}{j!} \rho^{2j} I_2(\tilde{\alpha}_j, \tilde{c}, \beta) \\ &\quad \times \frac{e^{-\theta_1/2} (\theta_1/2)^{r_1}}{r_1!} \frac{e^{-\theta_2/2} (\theta_2/2)^{r_2}}{r_2!}, \end{aligned} \quad (2.3.20)$$

where

$$I_2(\tilde{\alpha}_j, \tilde{c}, \beta) = \int_0^\infty \frac{e^{-z} z^{\beta-1}}{\Gamma(\beta)} \frac{\gamma(\alpha_1, c_1 z)}{\Gamma(\alpha_1)} \frac{\gamma(\alpha_2, c_2 z)}{\Gamma(\alpha_2)} dz \quad (2.3.21)$$

and

$$\beta = \frac{n}{2}, \quad \tilde{c} = \left(\frac{am}{n(1-\rho^2)}, \frac{bm}{n(1-\rho^2)} \right), \quad \tilde{\alpha}_j = \left(\frac{m}{2} + j + r_1, \frac{m}{2} + j + r_2 \right). \quad (2.3.22)$$

Here, $\gamma(\alpha, x) = \int_0^x e^{-t} t^{\alpha-1} dt$, and $\Gamma(\alpha) = \int_0^\infty e^{-t} t^{\alpha-1} dt$.

Amos and Bulgren (1972) used various representations of $\gamma(\alpha_1, c_1 z)$ and $\gamma(\alpha_2, c_2 z)$ and derived the series form of $I_2(\tilde{\alpha}_j, \tilde{c}, \beta)$ in terms of regularized beta functions. In this paper we use the I_2 as given by Amos and Bulgren (1972), that is,

$$\begin{aligned} I_2(\tilde{\alpha}_j, \tilde{c}, \beta) &= I_u(\alpha_1, \beta) - \frac{(1-u)^\beta \Gamma(\beta + \alpha_1)}{\alpha_1 \Gamma(\beta) \Gamma(\alpha_1)} \\ &\quad \times \sum_{r=0}^{\infty} \frac{(\beta + \alpha_1)_r}{(1 + \alpha_1)_r} u^{r+\alpha_1} I_{1-y}(r + \beta + \alpha_1, \alpha_2), \end{aligned} \quad (2.3.23)$$

with

$$u = c_1/(1 + c_1), \quad 1 - y = (1 + c_1)/(1 + c_1 + c_2), \quad (2.3.24)$$

and

$$\int_0^\infty \frac{e^{-z} z^{\beta-1}}{\Gamma(\beta)} \frac{\gamma(\alpha, cz)}{\Gamma(\alpha)} dz = I_z(\alpha, \beta) \text{ and}$$

$$\int_0^\infty \frac{e^{-z} z^{\beta-1}}{\Gamma(\beta)} \frac{\Gamma(\alpha, cz)}{\Gamma(\alpha)} dz = I_{1-z}(\beta, \alpha)$$

are the regularized beta functions, with $\alpha > 0, \beta > 0, x = c/(1+c)$, and $1-x = 1/(1+c)$.

The infinite series is truncated after $t_1 + 1, t_2 + 1$ and $t_3 + 1$ terms and thus giving a bound on the truncation error, say r_t . For any finite a and b ,

$$k_j = (1 - \rho^2)^{\frac{m}{2}} \sum_{j=0}^{\infty} \frac{\binom{m}{2}^j}{j!} \rho^{2j} I_2(\hat{\alpha}_j, \hat{c}, \beta),$$

the mass of a bivariate central F distribution and

$$s_{r_i} = \frac{e^{-\theta_i/2} (\theta_i/2)^{r_i}}{r_i!}, \quad i = 1, 2,$$

the mass of a Poisson distribution,

$$r_{t_1, t_2, t_3} = \sum_{j=t_1+1}^{\infty} \sum_{r_1=t_2+1}^{\infty} \sum_{r_2=t_3+1}^{\infty} k_j s_{r_1} s_{r_2} = 1 - \sum_{i=0}^{t_1} \sum_{r_1=0}^{t_2} \sum_{r_2=0}^{t_3} k_j s_{r_1} s_{r_2}.$$

The stopping value of t_1, t_2 and t_3 that are used depends on machine and software precision. In practice, r_t is computed successively and iteration stops when $r_{t-1} = r_t$ or when the width of r_t is less than a preselected tolerance.

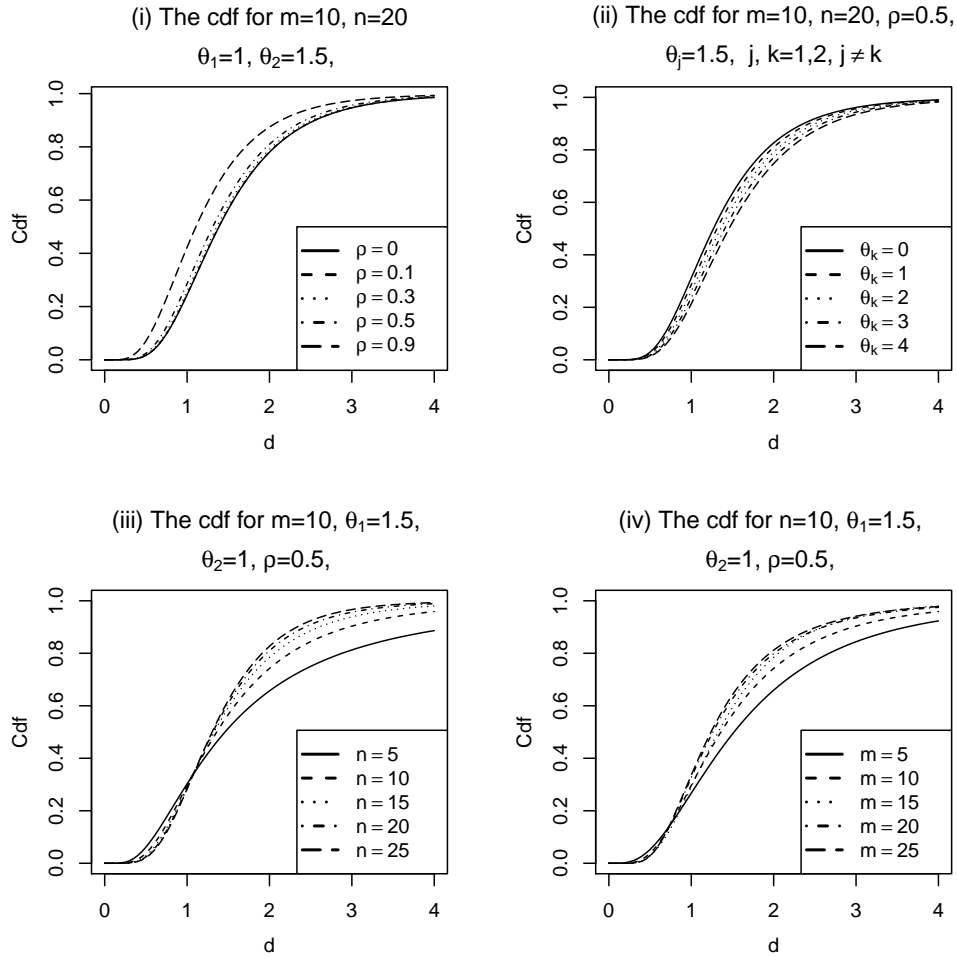


Figure 2.2: The cdf of the *doubly* bivariate noncentral F distribution.

The values of the cdf of the *doubly* BNCF distribution are computed for arbitrary degrees of freedom (m, n), noncentrality parameter (θ_1, θ_2), and correlation coefficient (ρ). The graph of the cdf of the *doubly* BNCF distribution is presented in Figure 2.2. The cdf of the BNCF distribution is computed using Equation (2.3.20). Figure 2.2 shows the cdf of the *doubly* BNCF distribution depends on the values of the noncentrality parameters (θ_1, θ_2), degrees of freedom (m, n), correlation coefficient (ρ) and upper limit (d). For the Figure 2.2, we observe that the shape of the curve of the cdf is sigmoid. Also, the value of the cdf approaches 1 quicker for a larger correlation coefficient (see Figure 2.2(i)), smaller noncentrality parameters (see Figure 2.2(ii))

and larger degrees of freedom (m, n) (see Figures 2.2(iii) and 2.2(iv)).

The R codes used to plot the cdf of the BNCF distribution in Figure 2.2 are provided in Appendix A.2.

2.4 The Methodology of Analysis

For each of the four regression models, we define the test statistic for the UT, RT and PTT, derive the sampling distribution of the test statistics and power function of the tests, present a simulation example, compare the power function and size of the tests, and recommend a test that minimizes the size and maximizes the power.

2.4.1 The UT, RT, PT and PTT

Consider the problem of testing intercept in the SRM, $H_0 : \beta_0 = \beta_{00}$ (a fixed value) against $H_a : \beta_0 > \beta_{00}$, when there is uncertain NSPI available on the value slope (β_1). In this situation, three different scenarios on β_1 are considered, namely unspecified, specified and uncertain. We then define three different statistical tests, namely, the (i) unrestricted test (UT), (ii) restricted test (RT) and (iii) pre-test test (PTT). For the UT, let ϕ^{UT} be the test function for testing H_0 . For the RT, let ϕ^{RT} be the test function for testing H_0 . Whereas for the PTT, let ϕ^{PTT} be the test function for testing H_0 following a pre-test (PT) on the slope. Then, for the PT, let ϕ^{PT} be the test function for testing $H_0^* : \beta_1 = \beta_{10}$ against $H_a^* : \beta_1 > \beta_{10}$, and it is essential for the PTT on β_0 . The PTT is a choice between the UT and RT. If H_0^* is rejected in PT, then the UT is used to test H_0 , otherwise the RT is used. The details on the UT, RT, PT and PTT are presented below.

The Unrestricted Test (UT)

If the slope (β_1) is unspecified, the test function is ϕ^{UT} . Let, T^{UT} be test statistic to test H_0 , then chose a value of α_1 ($0 < \alpha_1 < 1$) such that,

$$P [T^{UT} > \ell_{n,\alpha_1}^{UT} \mid H_0 : \beta_0 = \beta_{00}] = \alpha_1, \quad (2.4.1)$$

where ℓ_{n,α_1}^{UT} is the critical value of T^{UT} at the α_1 level of significance. If τ_{α_i} is the upper $100\alpha_i$ quantile and $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution, then

$$\Phi(\tau_{\alpha_i}) = 1 - \alpha_i, \quad (2.4.2)$$

where $0 < \alpha_i < 1$, $i = 1, 2, 3$. Thus, $1 - \alpha_1$ can be written as

$$1 - \alpha_1 = P [T^{UT} \leq \ell_{n,\alpha_1}^{UT}]. \quad (2.4.3)$$

Hence, for the test function $\phi^{UT} = I(T^{UT} > \ell_{n,\alpha_1}^{UT})$, the power function of UT becomes

$$\pi^{UT}(\beta_0) = E(\phi^{UT} \mid \beta_0) = P(T^{UT} > \ell_{n,\alpha_1}^{UT} \mid \beta_0). \quad (2.4.4)$$

The Restricted Test (RT)

If $\beta_1 = \beta_{10}$ (specified), then the test function is ϕ^{RT} . Let the proposed test statistic to test $H_0 : \beta_0 = \beta_{00}$ against $H_a : \beta_0 > \beta_{00}$, when $\beta_1 = \beta_{10}$ be T^{RT} . Then, we find

$$P [T^{RT} > \ell_{n,\alpha_2}^{RT} \mid H_0 : \beta_0 = \beta_{00}, \beta_1 = \beta_{10}] = \alpha_2, \quad (2.4.5)$$

where ℓ_{n,α_2}^{RT} is the critical value of T^{RT} at the α_2 level of significance. We then obtain

$$1 - \alpha_2 = P [T^{RT} \leq \ell_{n,\alpha_2}^{RT}]. \quad (2.4.6)$$

Thus, for the test function $\phi^{RT} = I(T^{RT} > \ell_{n,\alpha_2}^{RT})$, the power function of RT is

$$\pi^{RT}(\beta_0) = E(\phi^{RT} \mid \beta_0) = P(T^{RT} > \ell_{n,\alpha_2}^{RT} \mid \beta_0). \quad (2.4.7)$$

The Pre-test (PT)

If β_1 is uncertain, let ϕ^{PT} be the test function for testing the hypothesis $H_0^* : \beta_1 = \beta_{10}$ against $H_a^* : \beta_1 > \beta_{10}$. Let the proposed test statistic be T^{PT} . Under H_0^* , we find

$$P [T^{PT} > \ell_{n,\alpha_3}^{PT} \mid H_0^* : \beta_1 = \beta_{10}] = \alpha_3, \quad (2.4.8)$$

Similarly, we find

$$1 - \alpha_3 = P [T^{PT} \leq \ell_{n,\alpha_3}^{PT}], \quad (2.4.9)$$

where ℓ_{n,α_3}^{PT} is the critical value of the T^{PT} at the α_3 level of significance. Furthermore, for the test function $\phi^{PT} = I(T^{PT} > \ell_{n,\alpha_3}^{PT})$, the power function of the PT is given by

$$\pi^{PT}(\beta_0) = E(\phi^{PT} \mid \beta_0) = P(T^{PT} > \ell_{n,\alpha_3}^{PT} \mid \beta_0). \quad (2.4.10)$$

The Pre-test Test (PTT)

To formulate a test function of the PTT for testing the hypothesis $H_0 : \beta_0 = \beta_{00}$ against $H_a : \beta_0 > \beta_{00}$ after pre-testing (PT) on the suspected value of the slope, we write ϕ^{PTT} as

$$\phi^{PTT} = I [(T^{PT} \leq \ell_{n,\alpha_3}^{PT}, T^{RT} > \ell_{n,\alpha_2}^{RT}) \text{ or } (T^{PT} > \ell_{n,\alpha_3}^{PT}, T^{UT} > \ell_{n,\alpha_1}^{UT})]. \quad (2.4.11)$$

Then the power function of the PTT, denoted by $\pi^{PTT}(\beta_0) = E(\phi^{PTT} \mid \beta_0)$, is given as

$$\pi^{PTT}(\beta_0) = P (T^{PT} \leq \ell_{n,\alpha_3}^{PT}, T^{RT} > \ell_{n,\alpha_2}^{RT} \mid \beta_0) + P (T^{PT} > \ell_{n,\alpha_3}^{PT}, T^{UT} > \ell_{n,\alpha_1}^{UT} \mid \beta_0). \quad (2.4.12)$$

2.4.2 The Power Function and Size of the Tests

The goodness of a test is measured by the probability of the type I error (α) and the probability of the type II error (β). Whereas, to evaluate the performance of a test, we use the power of the test. The power of a test, $\pi(\theta)$, is the probability that the

test will lead to the rejection of the $H_0 : \theta = \theta_0$ against an alternative hypotheses $H_a : \theta \neq \theta_0$ when the actual parameter value is different from θ_0 . Generally, the power of a test can be written as

$$\pi(\theta) = P(W \text{ is in } RR \text{ when the parameter value is } \theta),$$

where W is the value of the test statistic and RR is the rejection region for the test. With regard to the rejection region, so

$$\pi(\theta_a) = P(\text{rejecting } H_0 \text{ when } \theta = \theta_a),$$

where θ_a is a value of θ under H_a . Note that a good test ideally has power near 1 under H_a and near 0 under H_0 (Casella and Berger, 2002, p. 383).

The following example is to describe the power function and size of a test. Let $X_i, i = 1, 2, \dots, n$ be independent and each of them follows a Bernoulli distribution with parameter θ . Then for $n = 10$ trials, $Y = \sum_{j=1}^{n=10} X_j$ follows a binomial distribution with $n = 10$ and $p = \theta$, and denoted as $Y \sim Bin(n, \theta)$. We want to test $H_0 : \theta = 0.5$ against $H_a : \theta = 0.4$, with $RR = \{(x_1, \dots, x_{10}) : Y \leq 3\}$. The power function of the test defined by RR (under H_a) is given as

$$\begin{aligned} \pi(\theta) &= P(\text{Reject } H_0 \mid \theta) = \sum_{y=0}^3 \binom{10}{y} \theta^y (1-\theta)^{10-y} \\ &= (1-\theta)^7 (84\theta^3 + 28\theta^2 + 7\theta + 1). \end{aligned} \quad (2.4.13)$$

The size of the test is the value of power function under H_0 , that is,

$$\begin{aligned} \alpha &= P(\text{Type I error}) = \pi(\theta \mid H_0 : \theta = \theta_0) = P(\text{Reject } H_0 \mid H_0 \text{ is true}) \\ &= P(Y \leq 3 \mid \theta = 0.5) = (1-\theta)^7 [84(\theta)^3 + 28(\theta)^2 + 7(\theta) + 1] \mid H_0 : \theta = 0.5 \\ &= (1-0.5)^7 (84(0.5)^3 + 28(0.5)^2 + 7(0.5) + 1) = 0.1719 \end{aligned} \quad (2.4.14)$$

Similarly, the probability of type II error or β (probability of accepting H_0 when H_a

is true) is given by

$$\begin{aligned}
 \beta &= P(\text{Type II error}) = P(\text{Accept } H_0 \mid H_a \text{ is true}) \\
 &= P(Y > 3 \mid \theta = 0.4) \\
 &= 1 - ((1 - \theta)^7(84(\theta)^3 + 28(\theta)^2 + 7(\theta) + 1)) \Big|_{H_a=\theta=0.4} = 0.618 \quad (2.4.15)
 \end{aligned}$$

Generally, the power function of the UT, RT, PT and PTT are produced using Equations (2.4.4), (2.4.7), (2.4.10) and (2.4.12). For the simulation example, the power ($\pi(\theta)$), size (α) and probability of type II error (β) are produced using Equations (2.4.13), (2.4.14) and (2.4.15). The above computation in Equations (2.4.13) and (2.4.14) proves that the size and power of the test are obtained from the power function of the test (see Pinto et al., 2003; Saleh and Sen, 1982, 1983). The size (α) is commonly chosen and set as the probability of type I error (see Wackerly et al., 2008, p. 491, and De Veaux et al., 2009, p. 544). The size of the test as the nominal value under the null hypothesis is constant and the power of the test depends on the parameter θ . Furthermore, a test that maximizes the power function and minimizes the size is preferred over any other tests.

2.4.3 The R Package, Data and Comparison of Tests

The R package was introduced by Ross Ihaka and Robert Gentleman. It is a sophisticated language for data analysis and graphics. Most of the R functions work on the vectors and also it has many built-in functions for statistical analysis such as mean, variance, etc. The current version of the R (R 2.14.1) package that is used in the study is the result of a collaborative effort from all over the world (Crawley, 2007, p. i-ii).

In simulation examples, we generate random data using the R package (data can be obtained from secondary/published sources). For consistency in results of random data, we use `set.seed` command for simulations. Simulated data is used for

the computation of the power function and size of the tests. The power function of the UT, RT and PTT are graphically compared and analysed using the R package.

Chapter 3

The Simple Regression Model

3.1 Introduction

In a recent study by Kent (2009) challenged the commonly held view that energy use in plastics processing plant is fixed and uncontrollable. He argued that energy use varies directly with production volume and is easily controlled. It is observed that the energy usage at a production plant consists of a *base load* and a *process load*. The *base load* is the energy used in non-production related use such as the lighting, heating, office equipments, compressed-air leakage, etc. However, the process load is directly related to the production volume.

The relationship between the electricity consumption (y as dependent variable) and production output (x as independent variable) is modeled by a simple regression model (SRM). For n pairs of observations on the independent variable (x) and dependent variable (y), (x_i, y_i) , for $i = 1, 2, \dots, n$, the model is given by

$$y_i = \beta_0 + \beta_1 x_i + e_i, \quad (3.1.1)$$

where e_i 's are assumed to be independent and normally distributed with mean 0 and variance σ^2 , x_i 's are known real values of the independent variable, and β_0 and β_1 are the unknown intercept and slope parameters, respectively. The functional relationship between the true mean of y_i and x_i is straight line $E(y_i) = \beta_0 + \beta_1 x_i$. In this case,

the estimation of the intercept parameter obviously depends on the slope parameter.

Here the intercept parameter (β_0) represents the *base load*. Production industry experience may provide a reliable value of the slope (β_1) as a NSPI, which can be used to improve the inference on the value of β_0 . We consider the problem of testing $H_0 : \beta_0 = \beta_{00}$ (a fixed value of base load) when NSPI is available on the value of β_1 .

In the study of the energy usage in the production plants the base load β_0 is not zero even when there is no production. To test the *base load* to be a particular value when there is NSPI on the value of the slope from the experience in the industry, we consider the following three different scenarios, and define the UT, RT and PTT as follows: (i) for the UT, let ϕ^{UT} be the test function and T^{UT} be the test statistic for testing $H_0 : \beta_0 = \beta_{00}$ (known constant) against $H_a : \beta_0 > \beta_{00}$ when β_1 is unspecified, (ii) for the RT, let ϕ^{RT} be the test function and T^{RT} be the test statistic for testing $H_0 : \beta_0 = \beta_{00}$ against $H_a : \beta_0 > \beta_{00}$ when β_1 is β_{10} (specified) and (iii) for the PTT, let ϕ^{PTT} be the test function and T^{PTT} be the test statistic for testing $H_0 : \beta_0 = \beta_{00}$ against $H_a : \beta_0 > \beta_{00}$ following a pre-test (PT) on the slope. For the PT, let ϕ^{PT} be the test function for testing $H_0^* : \beta_1 = \beta_{10}$ (a suspected constant) against $H_a^* : \beta_1 > \beta_{10}$. If H_0^* is rejected in the PT, then the UT is used to test the intercept, otherwise the RT is used to test H_0 . Thus, the PTT depends on the outcome of the PT and is a choice between the UT and RT.

The unrestricted maximum likelihood estimator (MLE) of the slope β_1 and intercept β_0 are given by

$$\tilde{\beta}_1 = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^n (X_i - \bar{X})^2} = \frac{S_{xy}}{S_{xx}} \text{ and } \tilde{\beta}_0 = \bar{Y} - \tilde{\beta}_1 \bar{X}, \quad (3.1.2)$$

where $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ and $\bar{Y} = \frac{1}{n} \sum_{i=1}^n Y_i$. Sampling distributions of the estimators $\tilde{\beta}_1$ and $\tilde{\beta}_0$ follow normal distributions with expected values $E(\tilde{\beta}_1) = \beta_1$ and $E(\tilde{\beta}_0) = \beta_0$, and variances $Var(\tilde{\beta}_1) = \frac{\sigma^2}{S_{xx}}$ and $Var(\tilde{\beta}_0) = \frac{\sigma^2}{n} \left(1 + \frac{n\bar{X}^2}{S_{xx}}\right)$, respectively.

Let NSPI on the value of the slope vector of the SRM be β_{10} . Then it can be

expressed by the null hypothesis, $H_0^* : \beta_1 = \beta_{10}$. Thus the restricted maximum likelihood estimator (under H_0^*) of the slope and intercept are given by $\hat{\beta}_1 = \beta_{10}$ and $\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$, respectively.

Our aim is to define the UT, RT and PTT for testing the null hypothesis H_0 , derive the sampling distribution and power function of the tests. Moreover, we compare the power function of the tests for known and unknown variance (σ^2) analytically and graphically.

The following Section 3.2 provides testing of the intercept for known variance. Section 3.3 covers testing of the intercept for unknown variance. In each Section we define the tests, derive sampling distribution of the test statistics and power function of the tests, and provide graphical and analytic comparison of the power of the tests and conclusions.

3.2 Testing of the Intercept for Known σ^2

3.2.1 The Proposed Tests

The test statistics of the UT, RT and PTT for testing $H_0 : \beta_0 = \beta_{00}$ against $H_a : \beta_0 > \beta_{00}$ for known σ^2 are given as follows.

- (i) If β_1 is unspecified, we then replace it by its MLE, $\tilde{\beta}_1$ and write $\tilde{\beta}_0 = \bar{Y} - \tilde{\beta}_1 \bar{X}$.

The test statistic for testing $H_0 : \beta_0 = \beta_{00}$ is then defined as

$$T_z^{UT} = \frac{\sqrt{n}(\tilde{\beta}_0 - \beta_{00})}{SE(\tilde{\beta}_0)} = \frac{\tilde{\beta}_0 - \beta_{00}}{\frac{\sigma}{\sqrt{n}} \left[1 + \frac{n\bar{X}^2}{S_{xx}}\right]^{1/2}} = \frac{\sqrt{n}(\bar{Y} - \tilde{\beta}_1 \bar{X} - \beta_{00})}{\sigma \left[1 + \frac{n\bar{X}^2}{S_{xx}}\right]^{1/2}}, \quad (3.2.1)$$

where standard error (SE) of $\tilde{\beta}_0$ is $\frac{\sigma}{\sqrt{n}} \left[1 + \frac{n\bar{X}^2}{S_{xx}}\right]^{1/2}$. Under H_0 , T_z^{UT} follows the standard normal distribution $N(0, 1)$, and under H_a the distribution of the T_z^{UT} is given as $N\left(\frac{\sqrt{n}(\beta_0 - \beta_{00})}{\sigma \left[1 + \frac{n\bar{X}^2}{S_{xx}}\right]^{1/2}}, 1\right)$, where $\beta_0 - \beta_{00} > 0$ and β_{00} is the value of β_0 under H_0 .

- (ii) When $\beta_1 = \beta_{10}$ (specified or fixed value), the restricted estimator of β_1 is written as $\hat{\beta}_1 = \beta_{10}$. Hence $\hat{\beta}_0$ is given by $\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} = \bar{Y} - \beta_{10} \bar{X}$. The test statistic for testing $H_0 : \beta_0 = \beta_{00}$ is then given by

$$T_z^{RT} = \frac{\hat{\beta}_0 - \beta_{00}}{SE(\hat{\beta}_0)} = \frac{\hat{\beta}_0 - \beta_{00}}{\sigma/\sqrt{n}} = \frac{\sqrt{n}(\hat{\beta}_0 - \beta_{00})}{\sigma} \quad (3.2.2)$$

where $SE(\hat{\beta}_0) = \sqrt{Var(\hat{\beta}_0)} = \sqrt{Var(\bar{Y})} = \frac{\sigma}{\sqrt{n}}$. Under H_0 , $T_z^{RT} \sim N(0, 1)$, and under H_a the T_z^{RT} follows a normal distribution with mean $\frac{(\beta_0 - \beta_{00}) + (\beta_1 - \beta_{10})\bar{X}}{\sigma/\sqrt{n}}$ and variance 1.

- (iii) For the preliminary test (PT) $H_0^* : \beta_1 = \beta_{10}$, the test statistic of the PT under H_a is given by

$$T_z^{PT} = \frac{\tilde{\beta}_1 - \beta_{10}}{SE(\tilde{\beta}_1)} = \frac{\tilde{\beta}_1 - \beta_{10}}{\sigma/\sqrt{S_{xx}}} \sim N\left(\frac{\beta_1 - \beta_{10}}{\sigma/\sqrt{S_{xx}}}, 1\right), \quad (3.2.3)$$

where $SE(\tilde{\beta}_1) = \sigma/\sqrt{S_{xx}}$, $\beta_1 - \beta_{10} > 0$ and β_{10} is the value of β_1 under H_0 . Under the null hypothesis the above test statistic follows the standard normal distribution.

Furthermore, we define the PTT for testing $H_0 : \beta_0 = \beta_{00}$, following the preliminary test on the value of β_1 . We first consider the case where all the tests are one sided. Let us choose a positive number α_j ($0 < \alpha_j < 1$) and real values z_{α_j} for $j = 1, 2, 3$, such that

$$P(T_z^{UT} > z_{\alpha_1} \mid \beta_0 = \beta_{00}) = \alpha_1, \quad (3.2.4)$$

$$P(T_z^{RT} > z_{\alpha_2} \mid \beta_0 = \beta_{00}) = \alpha_2, \quad (3.2.5)$$

$$P(T_z^{PT} > z_{\alpha_3} \mid \beta_1 = \beta_{10}) = \alpha_3. \quad (3.2.6)$$

Then, the PTT for testing $H_0 : \beta_0 = \beta_{00}$ when β_1 is uncertain is given by the test function

$$\Phi_z = \begin{cases} 1, & \text{if } [T_z^{PT} \leq z_{\alpha_3}, T_z^{RT} > z_{\alpha_2}] \text{ or } [T_z^{PT} > z_{\alpha_3}, T_z^{UT} > z_{\alpha_1}]; \\ 0, & \text{otherwise.} \end{cases} \quad (3.2.7)$$

The size of the PTT is then given as

$$\alpha_z = P \{T_z^{PT} \leq z_{\alpha_3}, T_z^{RT} > z_{\alpha_2}\} + \{T_z^{PT} > z_{\alpha_3}, T_z^{UT} > z_{\alpha_1}\}. \quad (3.2.8)$$

3.2.2 Sampling Distribution of Test Statistics

For the power function and size of the PTT the joint distribution of (T^{UT}, T^{PT}) and (T^{RT}, T^{PT}) is essential. Let $\{K_n\}$ be a sequence of alternative hypotheses defined as

$$K_n : (\beta_0 - \beta_{00}, \beta_1 - \beta_{10}) = \left(\frac{\lambda_1}{\sqrt{n}}, \frac{\lambda_2}{\sqrt{n}} \right) = n^{-1/2} \boldsymbol{\lambda}, \quad (3.2.9)$$

where $\boldsymbol{\lambda} = (\lambda_1, \lambda_2)$ are fixed real numbers, β_0 is true value and β_{10} is NSPI. Under K_n the value of $(\beta_0 - \beta_{00})$ is greater than zero, and under H_0 the value of $(\beta_0 - \beta_{00})$ is equal zero.

From Equation (3.2.1) under alternative hypothesis, $T_z^{UT} \longrightarrow N \left(\frac{\sqrt{n}(\beta_0 - \beta_{00})}{\sigma \left[1 + \frac{n\bar{X}^2}{S_{xx}} \right]^{1/2}}, 1 \right)$, where $\beta_{00} \neq \beta_0$ is the value of β_0 under H_a so $\beta_0 - \beta_{00} > 0$ (one-side hypothesis).

Under alternative hypothesis we then derive $Z_i, i = 1, 2, 3$ as follows,

$$\begin{aligned} Z_1 &= T_z^{UT} - \frac{\sqrt{n}(\beta_0 - \beta_{00})}{\sigma \left[1 + n \frac{\bar{X}^2}{S_{xx}} \right]^{1/2}} \\ &= T_z^{UT} - \frac{\lambda_1}{k_1} \sim N(0, 1), \end{aligned} \quad (3.2.10)$$

where $k_1 = \sigma \left[1 + \frac{n\bar{X}^2}{S_{xx}} \right]^{1/2}$. Similarly, from Equation (3.2.2) and (3.2.3) we obtain

$$Z_2 = T_z^{RT} - \frac{(\beta_0 - \beta_{00}) + (\beta_1 - \beta_{10})\bar{X}}{\sigma/\sqrt{n}} \sim N(0, 1), \text{ and} \quad (3.2.11)$$

$$Z_3 = T_z^{PT} - \frac{\beta_1 - \beta_{10}}{\sigma/\sqrt{S_{xx}}} \sim N(0, 1), \quad (3.2.12)$$

where β_{00} and β_{10} is the value of β_0 and β_1 under H_0 . Since T_z^{RT} and T_z^{PT} are independent, the joint distribution under H_a is

$$\begin{pmatrix} T_z^{RT} \\ T_z^{PT} \end{pmatrix} \sim N_b \left[\begin{pmatrix} \frac{(\beta_0 - \beta_{00}) + (\beta_1 - \beta_{10})\bar{X}}{\sigma/\sqrt{n}} \\ \frac{\beta_1 - \beta_{10}}{\sigma/\sqrt{S_{xx}}} \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right]$$

$$= N_b \left[\begin{array}{c} \left(\frac{\lambda_1 + \lambda_2 \bar{X}}{\sigma} \right) \\ \frac{\lambda_2 \sqrt{S_{xx}}}{\sigma \sqrt{n}} \end{array} \right], \quad (3.2.13)$$

In the same manner, we have

$$\begin{aligned} \begin{pmatrix} T_z^{UT} \\ T_z^{PT} \end{pmatrix} &\sim N_b \left[\begin{array}{c} \left(\frac{\frac{\sqrt{n}(\beta_0 - \beta_{00})}{\sigma \left[1 + \frac{n\bar{X}^2}{S_{xx}}\right]^{1/2}}}{\frac{\beta_1 - \beta_{10}}{\sigma \sqrt{S_{xx}}}} \right) \\ \left(\frac{\lambda_1}{\sigma \left[1 + \frac{n\bar{X}^2}{S_{xx}}\right]^{1/2}} \right) \\ \frac{\lambda_2 \sqrt{S_{xx}}}{\sigma \sqrt{n}} \end{array} \right], \\ &= N_b \left[\begin{array}{c} \left(\frac{\lambda_1}{\sigma \left[1 + \frac{n\bar{X}^2}{S_{xx}}\right]^{1/2}} \right) \\ \left(\frac{\lambda_2 \sqrt{S_{xx}}}{\sigma \sqrt{n}} \right) \end{array} \right], \end{aligned} \quad (3.2.14)$$

where ρ is correlation coefficient between T_z^{UT} and T_z^{PT} .

3.2.3 Power Function and Size of Tests

From Equations (3.2.1) and (3.2.2), the power function of the UT and RT are given, respectively, as

$$\begin{aligned} \pi_z^{UT}(\boldsymbol{\lambda}) &= P(T_z^{UT} > z_{\alpha_1} | K_n) \\ &= P\left(Z_1 > z_{\alpha_1} - \frac{\lambda_1}{k_1}\right) = 1 - G\left(z_{\alpha_1} - \frac{\lambda_1}{k_1}\right), \end{aligned} \quad (3.2.15)$$

$$\begin{aligned} \pi_z^{RT}(\boldsymbol{\lambda}) &= P(T_z^{RT} > z_{\alpha_2} | K_n) \\ &= P\left(Z_2 > z_{\alpha_2} - \frac{\lambda_1 + \lambda_2 \bar{X}}{\sigma}\right) = 1 - G\left(z_{\alpha_2} - \frac{\lambda_1 + \lambda_2 \bar{X}}{\sigma}\right). \end{aligned} \quad (3.2.16)$$

When λ_1 grows larger the power of the UT becomes higher. Similarly, the power function of the PTT is given as

$$\begin{aligned} \pi_z^{PTT}(\boldsymbol{\lambda}) &= P(\text{rejecting } H_0) \\ &= P(T_z^{PT} \leq z_{\alpha_3}, T_z^{RT} > z_{\alpha_2}) + P(T_z^{PT} > z_{\alpha_3}, T_z^{UT} > z_{\alpha_1}) \\ &= P(T_z^{PT} \leq z_{\alpha_3}) P(T_z^{RT} > z_{\alpha_2}) + P(T_z^{PT} > z_{\alpha_3}, T_z^{UT} > z_{\alpha_1}) \\ &= G\left(z_{\alpha_3} - \frac{\lambda_2 \sqrt{S_{xx}}}{\sigma \sqrt{n}}\right) \left(1 - G\left(z_{\alpha_2} - \frac{\lambda_1 + \lambda_2 \bar{X}}{\sigma}\right)\right) \\ &\quad + d_{1\rho}\left(z_{\alpha_3} - \frac{\lambda_2 \sqrt{S_{xx}}}{\sigma \sqrt{n}}, z_{\alpha_1} - \frac{\lambda_1}{k_1}, \rho \neq 0\right), \end{aligned} \quad (3.2.17)$$

where $d_{1\rho}$ is the bivariate normal probability integral. Here $d_{1\rho}$ is defined for every real p, q and $-1 < \rho < 1$ as

$$d_{1\rho}(p, q, \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_p^\infty \int_q^\infty \exp\left[-\frac{1}{2(1-\rho^2)}(x^2 + y^2 - 2\rho xy)\right] dx dy, \quad (3.2.18)$$

where $p = z_{\alpha_3} - \frac{\lambda_2\sqrt{S_{xx}}}{\sigma\sqrt{n}}$, $q = z_{\alpha_1} - \frac{\lambda_1}{k_1}$ and $G(x)$ is a cdf of the standard normal distribution.

Furthermore, the size of the UT, RT and PTT are given as

$$\begin{aligned} \alpha_z^{UT} &= P(T_z^{UT} > z_{\alpha_1} | H_0) \\ &= 1 - G\left(z_{\alpha_1} - \frac{\sqrt{n}(\beta_0 - \beta_{00})}{\sigma\sqrt{\left[1 + \frac{n\bar{X}^2}{S_{xx}}\right]}} \mid H_0 : \beta_0 = \beta_{00}\right) \\ &= 1 - G\left(z_{\alpha_1} - \frac{\sqrt{n}(\beta_{00} - \beta_{00})}{k_1}\right) = 1 - G(z_{\alpha_1}), \end{aligned} \quad (3.2.19)$$

$$\alpha_z^{RT} = 1 - G\left(z_{\alpha_2} - \frac{\lambda_2\bar{X}}{\sigma}\right), \text{ and} \quad (3.2.20)$$

$$\alpha_z^{PTT} = G(z_{\alpha_3})\left(1 - G\left(z_{\alpha_2} - \frac{\lambda_2\bar{X}}{\sigma}\right)\right) + d_{1\rho}(z_{\alpha_3}, z_{\alpha_1}, \rho \neq 0). \quad (3.2.21)$$

3.2.4 Analytical Comparison of the Tests

An analytical comparison of the tests (UT, RT and PTT) are provided in this Section.

The Power of the Tests

From Equations (3.2.15), (3.2.16) and (3.2.17), if we consider $\bar{X} = 0$ and $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$, the power of the UT, RT and PTT are given as

$$\pi_z^{UT}(\boldsymbol{\lambda}) = 1 - G\left(z_\alpha - \frac{\lambda_1}{k_1}\right), \quad (3.2.22)$$

$$\pi_z^{RT}(\boldsymbol{\lambda}) = 1 - G\left(z_\alpha - \frac{\lambda_1}{\sigma}\right). \quad (3.2.23)$$

$$\begin{aligned} \pi_z^{PTT}(\boldsymbol{\lambda}) &= G\left(z_\alpha - \frac{\lambda_2\sqrt{S_{xx}}}{\sigma\sqrt{n}}\right)\left(1 - G\left(z_\alpha - \frac{\lambda_1}{\sigma}\right)\right) \\ &\quad + d_{1\rho}\left(z_\alpha - \frac{\lambda_2\sqrt{S_{xx}}}{\sigma\sqrt{n}}, z_\alpha - \frac{\lambda_1}{k_1}, \rho \neq 0\right). \end{aligned} \quad (3.2.24)$$

The power of the UT is not the same as that of the RT and PTT. The power of the UT and RT depend on λ_1 , whereas that of the PTT depends on both λ_1 and λ_2 , and also ρ . If $\bar{X} > 0$, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$ and $\lambda_2 > 0$, the power of the RT is greater than that of the PTT. This is due to the fact that the value of $z_\alpha - \frac{\lambda_1 + \lambda_2}{\sigma}$ becomes small, so the cdf is also small, as a result the power of the RT grows larger. On the contrary, for $\bar{X} < 0$, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$ and $\lambda_2 > 0$ the power of the RT could be the same as the power of the PTT, where the power of the PTT involves the power of the PT and RT. For $\lambda_2 < 0$ is not considered here, this is because we test a one-sided hypothesis for $H_0 : \beta_0 = \beta_{00}$ following a pre-test on H_0^* .

The Size of the Tests

From Equations (3.2.19), (3.2.20) and (3.2.21), if we consider $\bar{X} = 0$ and $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$, the size of the UT, RT and PTT are given as

$$\alpha_z^{UT} = 1 - G(z_\alpha), \quad (3.2.25)$$

$$\alpha_z^{RT} = 1 - G(z_\alpha), \text{ and} \quad (3.2.26)$$

$$\alpha_z^{PTT} = G(z_\alpha)(1 - G(z_\alpha)) + d_{1\rho}(z_\alpha, z_\alpha, \rho \neq 0). \quad (3.2.27)$$

The size of the UT is the same as that of the RT, but they are not equal to the size of the PTT. The size of the PTT involves the size of the PT and RT as well as the probability integral of the bivariate normal distribution. For two situations: (1) $\bar{X} > 0$, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$ and $\lambda_2 > 0$, and (2) $\bar{X} < 0$, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$ and $\lambda_2 > 0$, the size of the UT does not change any of the two cases. The size of the RT decreases for both the case (1) and case (2). The size of the PTT is less than of the RT because the value of the multiplication of the size of the PT and RT is smaller than the size of any one of them, as well as the value of the correlated probability integral is less than 1.

3.2.5 A Simulation Example

To study the properties of the three tests we conduct a simulation study. The main aim is to compute the power function of the tests and compare them graphically. In this simulated example we generate random data using the R package. The independent variable (x) is generated from the uniform distribution between 0 and 1. The error (e) is generated from normal distribution with $\mu = 0$ and $\sigma^2 = 1$. In each case $n = 20$ random variates were generated. The dependent variable (y) is then determined by $y = \beta_0 + \beta_1 x + e$ for $\beta_0 = 5$ and $\beta_1 = 2.5$. For the computation of the power function of the tests we set $\alpha_1 = \alpha_2 = \alpha_3 = \alpha = 0.05$. The graphs for the power function and size of the three tests for known variance are produced using the formulas in Equations (3.2.15), (3.2.16) (3.2.17), (3.2.19), (3.2.20) and (3.2.21). Identical graphs for the power and size curves are observed when the slope is negative. The graphs of the power and size curves of the tests are presented in the Figures 3.1 to 3.9. The R codes for producing the Figures 3.1, 3.2 and 3.3 are provided in Appendix A.3, A.4 and A.5.

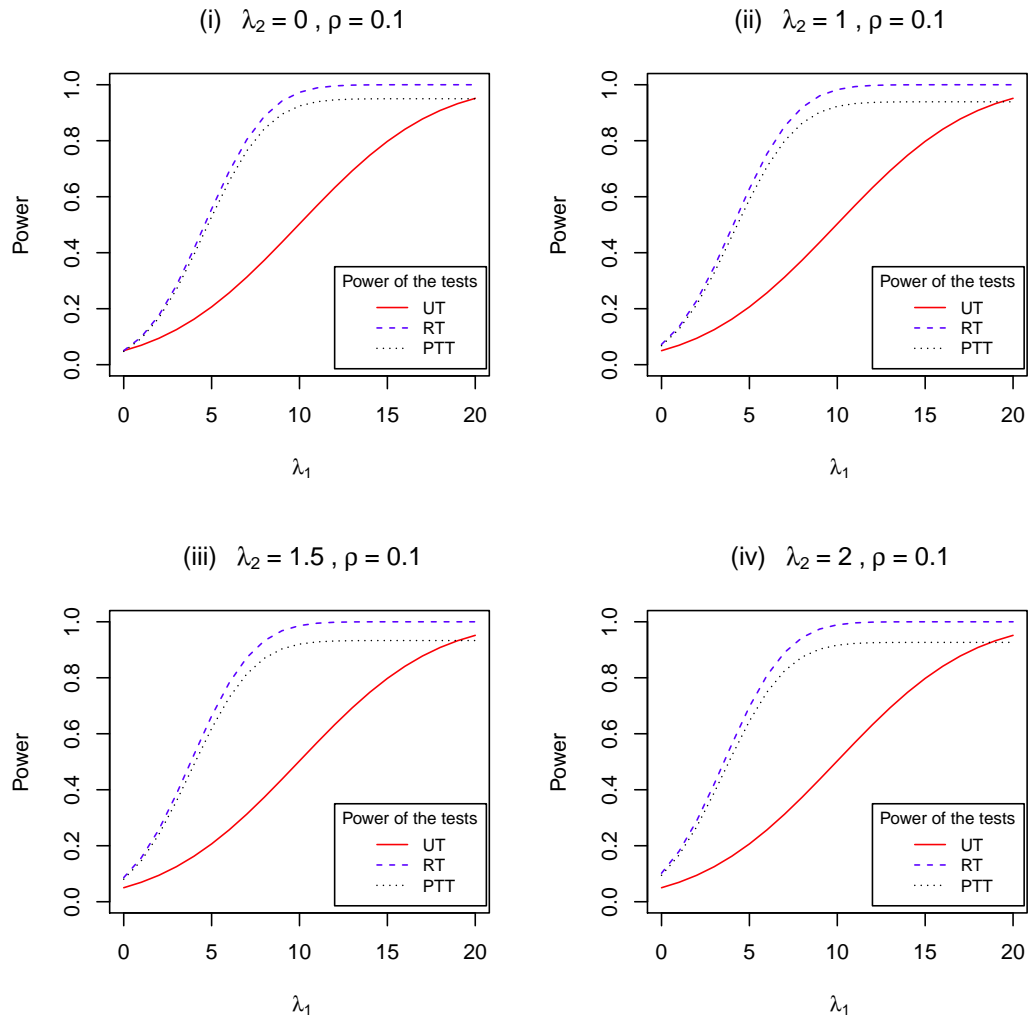


Figure 3.1: Power of the UT, RT and PTT against λ_1 with $\rho = 0.1$ and $\lambda_2 = 0, 1, 1.5, 2$.

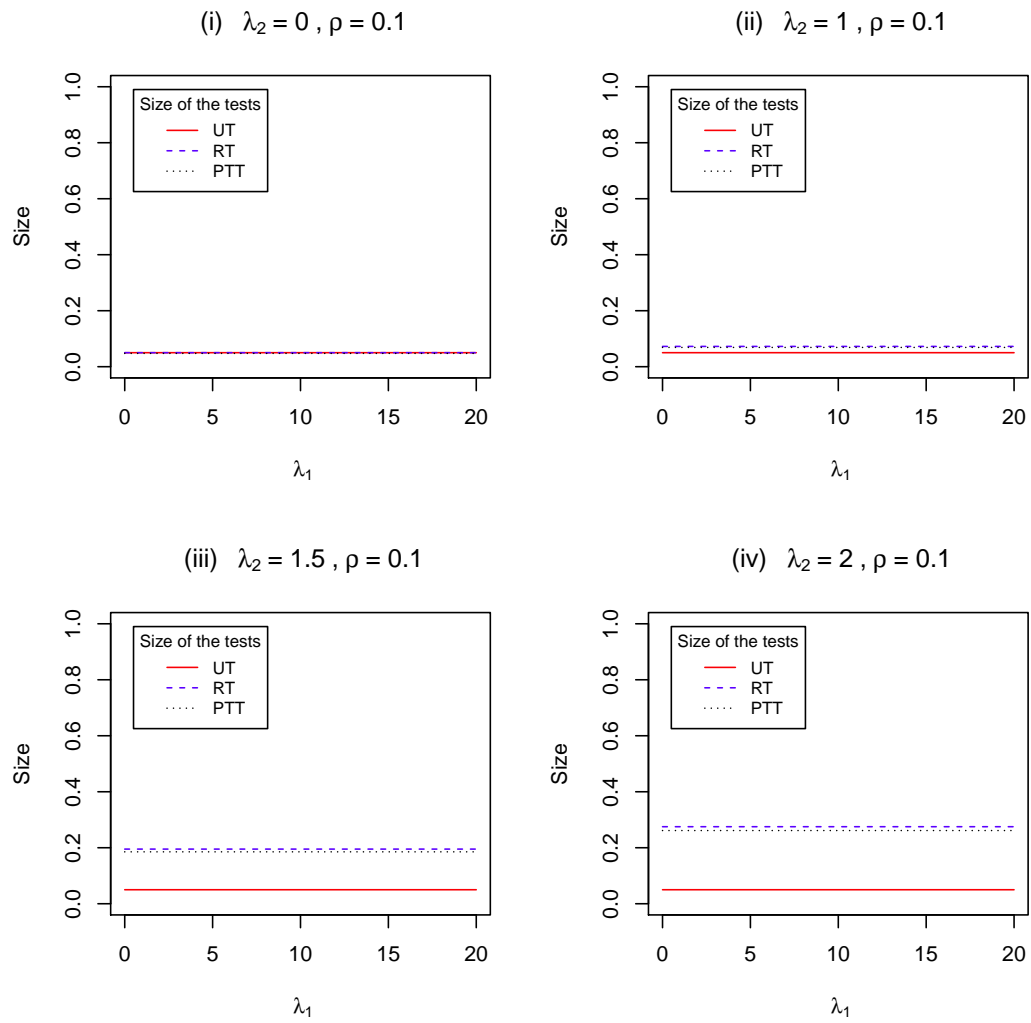


Figure 3.2: Size of the UT, RT and PTT against λ_1 with $\lambda_2 = 0, 1, 1.5, 2$.

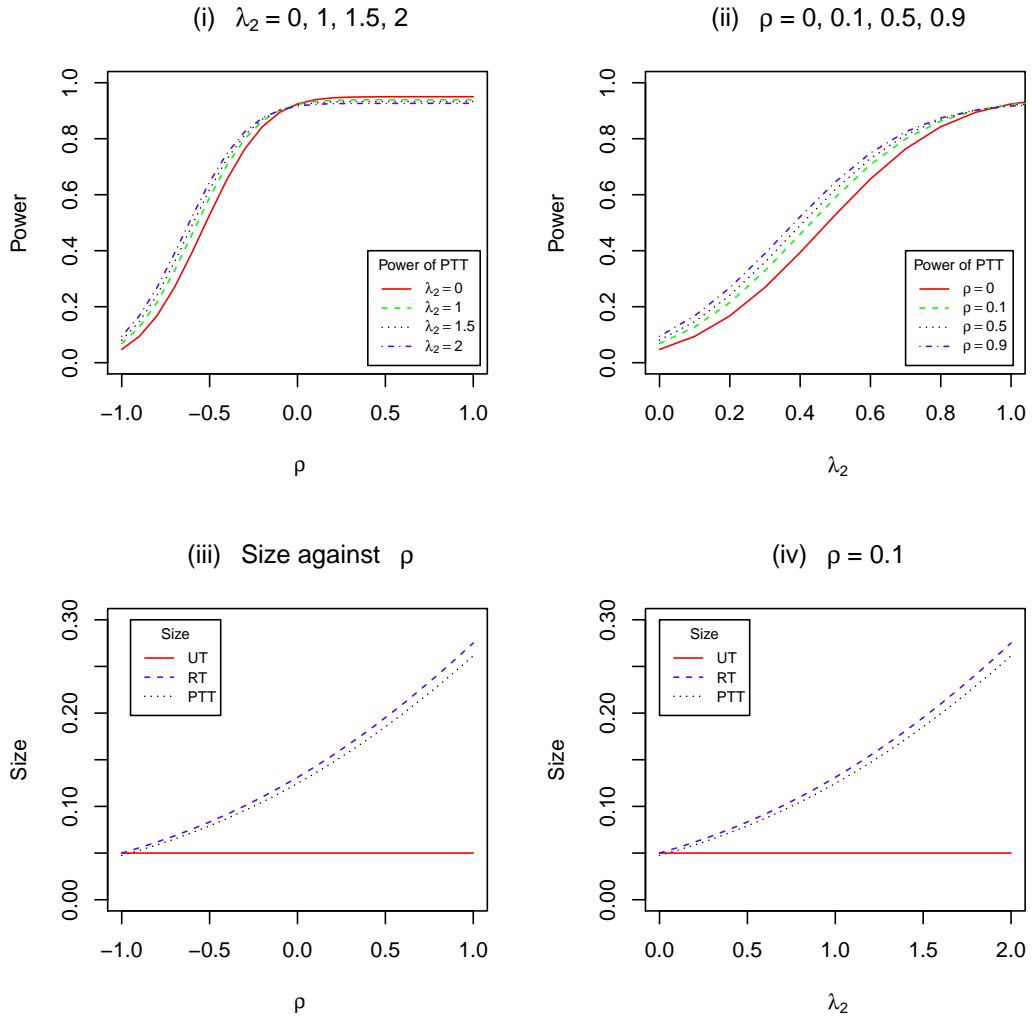


Figure 3.3: Power of the PTT and size against ρ and λ_2 .

3.2.6 Comparison of the Tests

From Figure 3.1, as well as from Equation (3.2.15) we see that the power of the UT does not depend on λ_2 and ρ but it increases as the value of λ_1 increases. Its form is sigmoid, starting from a very small value of near zero at $\lambda_1 = 0$, it approaches 1 when λ_1 is large (about 20). Thus the power of the UT changed significantly for any value of λ_1 from 0 to 20. The minimum power of the UT is around 0.05 for $\lambda_1 = 0$.

The power curve of the RT is also sigmoid for all values of λ_1 and λ_2 . The power of the RT increases as the values of λ_1 and/or λ_2 increase. Moreover, the power of the RT is always larger than that of the UT and PTT for all values of λ_1 and/or λ_2 . The minimum power of the RT is around 0.05 for $\lambda_2 = 0$ (as well as for $\lambda_1 = 0$) and increases to be around 0.1 for $\lambda_2 = 2$. The maximum power the RT is around 1 for λ_1 around 10 or above.

The power of the PTT also depends on the values of λ_1 and λ_2 . Like the power of the RT, the power of the PTT increases for large value of λ_1 and tends to decrease as λ_2 grows larger. Moreover, the power of the PTT tend to be larger than that of the UT. The minimum power of the PTT is around 0.05 for $\lambda_2 = 0$ and $\lambda_1 = 0$, and it increases to be around 0.1 for $\lambda_2 = 2$. The gap between the power curves of the RT and PTT is obviously clear for all values of λ_1 and λ_2 . Like the power of the RT, the power of the PTT depends on the values of λ_1 and λ_2 .

Figures 3.2 or Equation (3.2.19) shows the size of the UT does not depend on λ_2 . It is constant and remains unchanged for all values of λ_1 and λ_2 . The size of the RT is also constant for all values of λ_1 and λ_2 . However, the size of the RT increases as the value of λ_2 increases. Moreover, the size of the RT is always larger than that of the UT for all values of λ_2 except for $\lambda_2 = 0$ when both tests have the same size. Like the size of the RT, the size of the PTT increase as λ_2 grows larger. The difference (or gap) between the size of the RT and PTT is not obviously significantly different (too

low) as the value of λ_2 increases. The size of the UT is $\alpha^{UT} = 0.05$ for all values of λ_1 and λ_2 . The size of the RT is $\alpha^{RT} > \alpha^{UT}$ for all values of λ_2 except when $\lambda_2 = 0$. The size of the PTT is $\alpha^{PTT} \leq \alpha^{RT}$ for all values of λ_2 . The size of the RT and PTT increase as the value of λ_2 increases. Here the size of the RT is larger than that of the UT and PTT.

Figure 3.3 shows the power of the PTT depend on λ_2 and ρ . It increases significantly as the value of ρ increases from $\rho = -1$ to $\rho = 0$ and stays the same from zero to 1. The gap of the power of the PTT for $\lambda_2 = 0$ and $\lambda_2 = 2$ is significantly different as the value of ρ increases. From this figure or Equations (3.4.6) and (3.4.7) the size of the RT and PTT depend on the value of λ_2 . They increase as the value of λ_2 increases. Unlike the size of the RT and PTT, the size of the UT does not depend on the value of λ_2 and it remains constant for all the values of λ_2 . The size of the RT is always greater than the size of the UT and PTT.

3.2.7 Conclusion

Based on of the above analyses, the PTT protects against the maximum size of the RT and minimum power of the UT. It means that the power of the PTT attains a reasonable dominance over the UT and RT. The details of the conclusion are described as below.

The power of the RT is always higher than that of the UT and PTT for all values of λ_1 , and the power of the PTT lies between the power of the RT and UT for all values of λ_1 , λ_2 and ρ . The size of the UT is smaller than that of the RT and PTT. The RT has the maximum power and size, and the UT has the minimum power and size. The PTT has smaller size than the RT and the RT has larger power than the UT. The PTT protects against the maximum size of the RT and minimum power of the UT.

As $\lambda_2 \rightarrow 0$ the difference between the power of the PTT and RT diminishes for

all values of λ_1 . That is, if the NSPI is accurate the power of the PTT is about the same as that of the RT. Moreover, the power of the PTT gets closer to that of the RT as $\rho \rightarrow 1$.

The size of the PTT becomes smaller as $\lambda_2 \rightarrow 0$. Once again if the NSPI is near accurate the size of the PTT approaches that of the UT. Therefore, we recommend PTT when the quality of the NSPI is good (i.e. $\lambda_2 \rightarrow 0$) and it performs even better than the UT and RT when $\rho \rightarrow 1$.

3.3 Testing of the Intercept for Unknown σ^2

3.3.1 Proposed Tests

For testing the intercept parameter under three different scenarios of the value of the slope, the test statistic of the UT, RT and PTT for unknown variance are given as follows.

- (i) In the case of an unspecified β_1 , the test statistic of the UT for testing $H_0 : \beta_0 = \beta_{00}$ against $H_a : \beta_0 > \beta_{00}$, under H_0 , is given by

$$T^{UT} = \frac{\tilde{\beta}_0 - \beta_{00}}{SE(\tilde{\beta}_0)} = \frac{\tilde{\beta}_0 - \beta_{00}}{\frac{s_*}{\sqrt{n}} \left[1 + \frac{n\bar{X}^2}{S_{xx}}\right]^{1/2}} = \frac{\sqrt{n}(\bar{Y} - \tilde{\beta}_1\bar{X} - \beta_{00})}{s_* \left[1 + \frac{n\bar{X}^2}{S_{xx}}\right]^{1/2}}, \quad (3.3.1)$$

which follows a Student's t distribution with $(n - 2)$ degrees of freedom (d.f.), where the standard error, $SE(\tilde{\beta}_0) = \frac{s_*}{\sqrt{n}} \left[1 + \frac{n\bar{X}^2}{S_{xx}}\right]^{1/2}$ in which

$$s_*^2 = \frac{1}{n - 2} \sum_{i=1}^n (Y_i - \hat{Y})^2$$

is the mean square error (MSE), an estimator of σ^2 .

- (ii) When the slope is specified to be $\beta_1 = \beta_{10}$, the test statistic of the RT for testing $H_0 : \beta_0 = \beta_{00}$ against $H_a : \beta_0 > \beta_{00}$, under H_0 , is given by

$$T^{RT} = \frac{\hat{\beta}_0 - \beta_{00}}{SE(\hat{\beta}_0)} = \frac{\hat{\beta}_0 - \beta_{00}}{s_y/\sqrt{n}} = \frac{\sqrt{n}(\hat{\beta}_0 - \beta_{00})}{s_y}, \quad (3.3.2)$$

where $\hat{\beta}_0 = \bar{Y} - \beta_{10}\bar{X}$ and $Var(\hat{\beta}_0) = Var(\bar{Y}) = s_y^2/n$ in which

$$s_y^2 = \frac{1}{n-1} \sum_{i=1}^n (Y_i - \bar{Y})^2$$

is an unbiased estimator of σ^2 . The T^{RT} follows a Student's t distribution with $(n-1)$ degrees of freedom.

- (iii) When the value of the slope is suspected to be $\beta_1 = \beta_{10}$ but unsure, a pre-test on the slope is required before testing the intercept. For the preliminary test (PT) of $H_0^* : \beta_1 = \beta_{10}$ against $H_a^* : \beta_1 > \beta_{10}$, the test statistic under the null hypothesis is defined as

$$T^{PT} = \frac{\tilde{\beta}_1 - \beta_{10}}{SE(\tilde{\beta}_1)} = \frac{\tilde{\beta}_1 - \beta_{10}}{s_*/\sqrt{S_{xx}}} = \frac{(\tilde{\beta}_1 - \beta_{10})\sqrt{S_{xx}}}{s_*}, \quad (3.3.3)$$

which follows a Student's t distribution with $(n-2)$ degrees of freedom. The above PT depends on a preselected level of significance.

The PTT for testing $H_0 : \beta_0 = \beta_{00}$ against $H_a : \beta_0 > \beta_{00}$ when $\beta_1 = \beta_{10}$ is uncertain is then given by the test function

$$\Phi^{PTT} = \begin{cases} 1, & \text{if } (T^{PT} \leq t_{\nu_3}, T^{RT} > t_{\nu_2}) \text{ or } (T^{PT} > t_{\nu_3}, T^{UT} > t_{\nu_1}); \\ 0, & \text{otherwise,} \end{cases} \quad (3.3.4)$$

where $t_{\nu_1, \alpha_1} = t_{n-2, \alpha_1} = t_{\nu_1}$, $t_{\nu_2, \alpha_2} = t_{n-1, \alpha_2} = t_{\nu_2}$ and $t_{\nu_3, \alpha_3} = t_{n-2, \alpha_3} = t_{\nu_3}$.

3.3.2 Sampling Distribution of Test Statistics

To derive the power function and size of the UT, RT and PTT, the test statistics proposed in Section 3.3.1 are used. For the power function and size of the PTT the definition of the $\{K_n\}$ is given in Equation (3.2.9).

Following Yunus and Khan (2011b) and Equation (3.3.1), we define the test statistic of the UT, when β_1 is unspecified, under K_n , as

$$\begin{aligned} T_1^{UT} &= T^{UT} - \left(\sqrt{n}(\beta_0 - \beta_{00}) \left[s_* \sqrt{1 + \frac{n\bar{X}^2}{S_{xx}}} \right]^{-1} \right) \\ &= T^{UT} - \lambda_1 k_2^{-1}, \end{aligned} \quad (3.3.5)$$

where $s_* = \sqrt{\frac{\sum_{i=1}^n (Y_i - \hat{Y}_i)^2}{n-2}}$ and $k_2 = s_* \sqrt{\left[1 + \frac{n\bar{X}^2}{S_{xx}}\right]}$. The T_1^{UT} follows a non central t distribution with non centrality parameter, a function of $(\beta_0 - \beta_{00})$, under K_n .

From Equation (3.3.2) under K_n , when $(\beta_0 - \beta_{00}) > 0$ and $(\beta_1 - \beta_{10}) > 0$, the test statistic of the RT becomes

$$T_2^{RT} = T^{RT} - \frac{(\beta_0 - \beta_{00}) + (\beta_1 - \beta_{10})\bar{X}}{s_y/\sqrt{n}} = T^{RT} - \frac{\lambda_1 + \lambda_2\bar{X}}{s_y}. \quad (3.3.6)$$

The T_2^{RT} also follows a noncentral t distribution with a noncentrality parameter which is a function of $(\beta_0 - \beta_{00})$ and $(\beta_1 - \beta_{10})\bar{X}$ under K_n . Similarly, from Equation (3.3.3) the test statistic of the PT is given by

$$T_3^{PT} = T^{PT} - \frac{\sqrt{n}(\beta_1 - \beta_{10})\sqrt{S_{xx}}}{s_*\sqrt{n}} = T^{PT} - \frac{\lambda_2\sqrt{S_{xx}}}{s_*\sqrt{n}} \sim t_{n-2}, \quad (3.3.7)$$

where the T_3^{PT} has a noncentral t distribution with a noncentrality parameter, a function of $(\beta_1 - \beta_{10})$, under the alternative hypothesis.

From Equations (3.3.1), (3.3.2) and (3.3.3), we observe that T^{UT} and T^{PT} are correlated but T^{RT} and T^{PT} are uncorrelated (but not independent). The joint distribution of the T^{UT} and T^{PT} by

$$\begin{pmatrix} T^{UT} \\ T^{PT} \end{pmatrix} \sim t_{n-2}, \quad (3.3.8)$$

a bivariate Student- t distribution with $(n - 2)$ degrees of freedom. Thus the joint distribution of T^{UT} and T^{PT} follows a bivariate t distribution with $(n - 2)$ degrees of freedom with location vector $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and correlation matrix $\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$. Therefore, covariance of T^{PT} and T^{UT} is $\text{Cov}(T^{UT}, T^{PT}) = \frac{(n-2)}{(n-4)}\Sigma$ (Kotz and Nadarajah, 2004).

3.3.3 Power Function and Size of Tests

From Equations (3.3.1), (3.3.2), and (3.3.3) and (3.3.4), the power function of the UT, RT and PTT for unknown σ^2 are derived, respectively, as below.

$$\begin{aligned} \pi_t^{UT}(\boldsymbol{\lambda}) &= P(T^{UT} > t_{\alpha_1, n-2} \mid K_n) \\ &= 1 - P(T_1^{UT} \leq t_{\alpha_1, n-2} - \lambda_1 k_2^{-1}). \end{aligned} \quad (3.3.9)$$

When λ_1 grows larger the power of the UT becomes higher. This is obvious for any good test. The power function of the RT is

$$\begin{aligned}\pi_t^{RT}(\boldsymbol{\lambda}) &= P(T^{RT} > t_{\alpha_1, n-1} | K_n) \\ &= P(T_2^{RT} > t_{\alpha_2, n-1} - \sqrt{n}((\beta_0 - \beta_{00}) + (\beta_1 - \beta_{10})\bar{X}) s_y^{-1}) \\ &= 1 - P\left(T_2^{RT} \leq t_{\alpha_2, n-1} - \frac{\lambda_1 + \lambda_2 \bar{X}}{s_y}\right).\end{aligned}\quad (3.3.10)$$

The power of the RT grows higher as λ_1 becomes larger. The power of the PTT is then given as

$$\begin{aligned}\pi_t^{PTT}(\boldsymbol{\lambda}) &= P(T^{PT} \leq t_{n-2, \alpha_3}, T^{RT} > t_{n-1, \alpha_2}) + P(T^{PT} > t_{\alpha_3}, T^{UT} > t_{n-2, \alpha_1}) \\ &= d_{10} \left\{ t_{n-2, \alpha_3} - \lambda_2 \frac{\sqrt{S_{xx}}}{s_* \sqrt{n}}, t_{\alpha_2, n-1} - \frac{(\lambda_1 + \lambda_2 \bar{X})}{s_y}, \rho = 0 \right\} \\ &\quad + d_{2\rho} \left\{ t_{n-2, \alpha_3} - \lambda_2 \frac{\sqrt{S_{xx}}}{s_* \sqrt{n}}, t_{\alpha_1, n-2} - \lambda_1 k_2^{-1}, \rho \neq 0 \right\} \\ &= d_{10} \left\{ t_{n-2, \alpha_3} - \lambda_2 \frac{\sqrt{S_{xx}}}{s_* \sqrt{n}}, t_{\alpha_2, n-1} - \frac{(\lambda_1 + \lambda_2 \bar{X})}{s_y}, \rho = 0 \right\} \\ &\quad + d_{2\rho} \left\{ t_{n-2, \alpha_3} - \lambda_2 \frac{\sqrt{S_{xx}}}{s_* \sqrt{n}}, t_{\alpha_1, n-2} - \lambda_1 k_2^{-1}, \rho \neq 0 \right\},\end{aligned}\quad (3.3.11)$$

where d_{10} and $d_{2\rho}$ are bivariate Student's t probability integrals. Here d_{10} is defined as

$$d_{10} = \int_{-\infty}^a \int_c^{\infty} f(t^{PT}, t^{RT}) dt^{PT} dt^{RT}, \quad (3.3.12)$$

where $a = \left[t_{n-2, \alpha_3} - \lambda_2 \frac{\sqrt{S_{xx}}}{s_* \sqrt{n}} \right]$ and $c = \left[t_{n-1, \alpha_2} - \frac{\lambda_1 + \lambda_2 \bar{X}}{s_y} \right]$, and $d_{2\rho}$ is defined as

$$d_{2\rho}(a, b, \rho) = \frac{\Gamma\left(\frac{\nu+2}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right) n\pi \sqrt{1-\rho^2}} \int_a^{\infty} \int_b^{\infty} \left[1 + \frac{1}{\nu(1-\rho^2)}(x^2 + y^2 - 2\rho xy) \right]^{-\frac{\nu+2}{2}} dx dy, \quad (3.3.13)$$

in which $-1 < \rho < 1$ is the correlation coefficient between the T^{UT} and T^{PT} , and $b = \left[t_{\alpha_1, n-2} - \frac{\lambda_1}{k_2} \right]$.

Furthermore, the size of the UT, RT and PTT are given, respectively, by

$$\begin{aligned}
\alpha_t^{UT} &= P(T^{UT} > t_{n-2, \alpha_1} \mid H_0 : \beta_0 = \beta_{00}) \\
&= 1 - P(T^{UT} \leq t_{n-2, \alpha_1} \mid H_0 : \beta_0 = \beta_{00}) \\
&= 1 - P(T_1^{UT} \leq t_{\alpha_1, n-2}), \tag{3.3.14}
\end{aligned}$$

$$\begin{aligned}
\alpha_t^{RT} &= P(T^{RT} > t_{n-1, \alpha_2} \mid H_0 : \beta_0 = \beta_{00}) \\
&= 1 - P(T^{RT} \leq t_{n-1, \alpha_2} \mid H_0 : \beta_0 = \beta_{00}) \\
&= 1 - P\left(T_2^{RT} \leq t_{\alpha_2, n-1} - \frac{\lambda_2 \bar{X}}{s_y}\right), \text{ and} \tag{3.3.15}
\end{aligned}$$

$$\begin{aligned}
\alpha_t^{PTT} &= P(T^{PT} \leq a, T^{RT} > c \mid H_0) + P(T^{PT} > a, T^{UT} > b \mid H_0) \\
&= d_{10} \left\{ t_{\alpha_3, n-2}, t_{\alpha_2, n-1} - \frac{\lambda_2 \bar{X}}{s_y}, \rho = 0 \right\} + d_{2\rho} \{t_{\alpha_3, n-2}, t_{\alpha_1, n-2}, \rho\}. \tag{3.3.16}
\end{aligned}$$

3.3.4 Analytical Comparison of the Tests

An analytical comparison of the tests are provided in this Section.

The Power of the Tests

From Equations (3.3.9), (3.3.10) and (3.3.11), if we consider $\bar{X} = 0$ and $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$, the power of the UT, RT and PTT are given as

$$\pi^{UT}(\boldsymbol{\lambda}) = 1 - P(T_1^{UT} \leq t_{\alpha, n-2} - \lambda_1 k_2^{-1}). \tag{3.3.17}$$

$$\pi^{RT}(\boldsymbol{\lambda}) = 1 - P\left(T_2^{RT} \leq t_{\alpha, n-1} - \frac{\lambda_1}{s_y}\right). \tag{3.3.18}$$

$$\begin{aligned}
\pi^{PTT}(\boldsymbol{\lambda}) &= d_{10} \left\{ t_{n-2, \alpha_3} - \lambda_2 \frac{\sqrt{S_{xx}}}{s_* \sqrt{n}}, t_{\alpha_2, n-1} - \frac{\lambda_1}{s_y}, \rho = 0 \right\} \\
&\quad + d_{2\rho} \left\{ t_{n-2, \alpha_3} - \lambda_2 \frac{\sqrt{S_{xx}}}{s_* \sqrt{n}}, t_{\alpha_1, n-2} - \lambda_1 k_2^{-1}, \rho \neq 0 \right\}, \tag{3.3.19}
\end{aligned}$$

The power of the UT is not the same as that of RT and PTT. The power of the UT and RT depend on λ_1 , whereas the power of the PTT depends on λ_1 , λ_2 and ρ . If $\bar{X} > 0$, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$ and $\lambda_2 > 0$, the power of RT tends to be greater than that

of the PTT. This is due to the fact that the value of $t_\alpha - \frac{\lambda_1 + \lambda_2}{s_y}$ becomes smaller, as well as the cdf is less than 1, as a result the power of the RT goes larger. On the contrary, for $\bar{X} < 0$, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$ and $\lambda_2 > 0$ the power of the RT could be the same as that of the PTT. For $\lambda_2 < 0$ is not consider, this is because we test a one-side hypothesis for $H_0 : \beta_0 = \beta_{00}$ following a pre-test on H_0^* .

The Size of the Tests

From Equations (3.3.14), (3.3.15) and (3.3.16), if we consider $\bar{X} = 0$ and set $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$, the size of the UT, RT and PTT are given as

$$\alpha^{UT} = 1 - P(T_1^{UT} \leq t_{\alpha, n-2}), \quad (3.3.20)$$

$$\alpha^{RT} = 1 - P(T_2^{RT} \leq t_{\alpha, n-1}), \text{ and} \quad (3.3.21)$$

$$\alpha^{PTT} = d_{10} \{t_{\alpha, n-2}, t_{\alpha, n-1}, \rho = 0\} + d_{2\rho} \{t_{\alpha, n-2}, t_{\alpha, n-2}, \rho\} \quad (3.3.22)$$

Obviously, the size of the UT, RT and PTT depend on the value of the number of degrees of freedom. The size of the PTT involves the size of the PT and RT as well as the probability integral of the bivariate t distribution. For two conditions: (1) $\bar{X} > 0$, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$ and $\lambda_2 > 0$, and (2) $\bar{X} < 0$, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$ and $\lambda_2 > 0$, the size of the UT does not change for any of the two cases. The size of the RT decreases for the case (1) and it increases for the case (2). The size of the PTT is less than of the RT because the value of the multiplication of the size of the PT and RT is smaller than the size of any one of them, as well as the value of the correlated probability integral is no more than 1.

3.3.5 A Simulation Example

To study the properties of the three tests, the simulation study in Section 3.2.5 is used. The graphs for the power function and size of the three tests for unknown variance are produced using formulas in Equations (3.3.9), (3.3.10), (3.3.11), (3.3.14), (3.3.15)

and (3.3.16). Identical graphs for the power and size curves are observed when the slope has the same magnitude but the sign is negative. The graphs of the power and size curves of the tests are presented in Figures 3.4 to 3.9. The R codes that produce Figures 3.4, 3.5 and 3.6 are provided in Appendix A.6, whereas the R codes that produce Figures 3.7, 3.8 and 3.9 are given in Appendix A.7, A.8 and A.9.

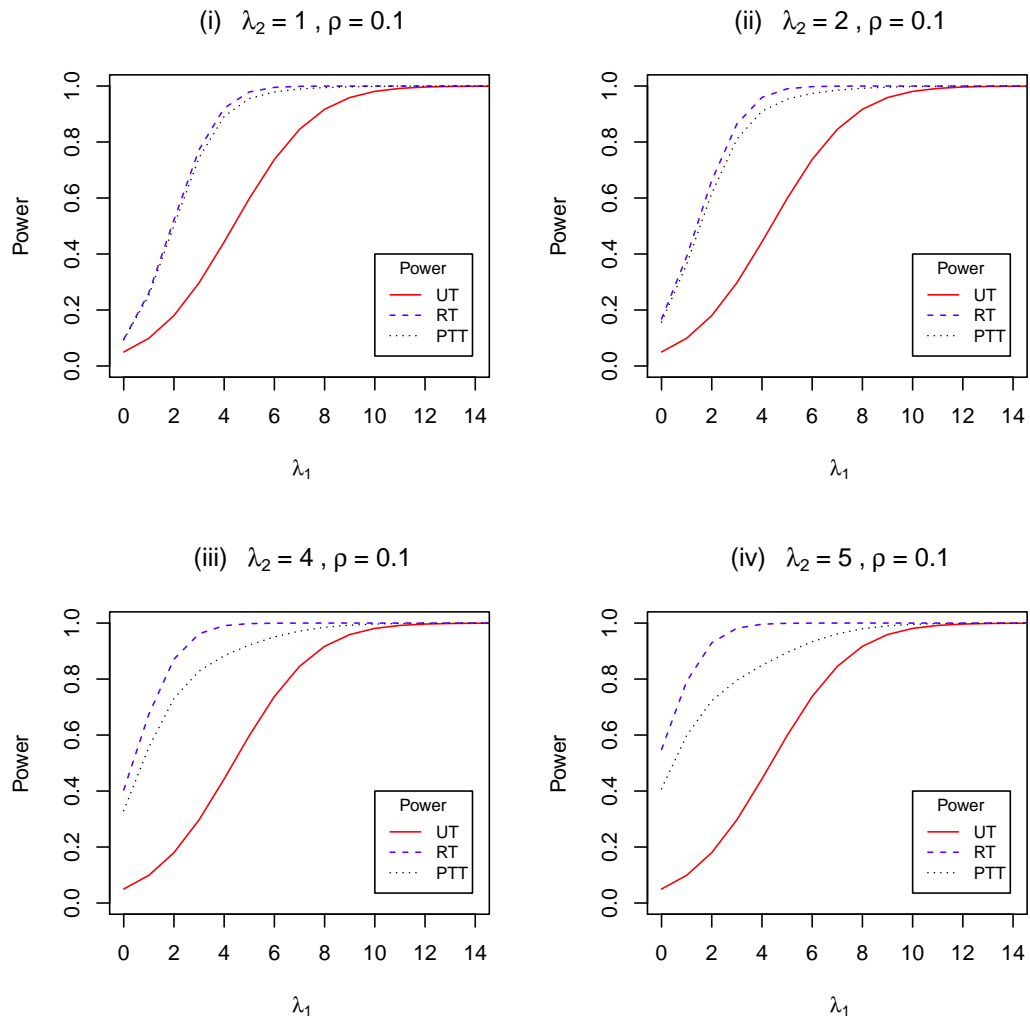


Figure 3.4: The power of the UT, RT and PTT against λ_1 with $\rho = 0.1$ and $\lambda_2 = 1, 2, 4, 5$.

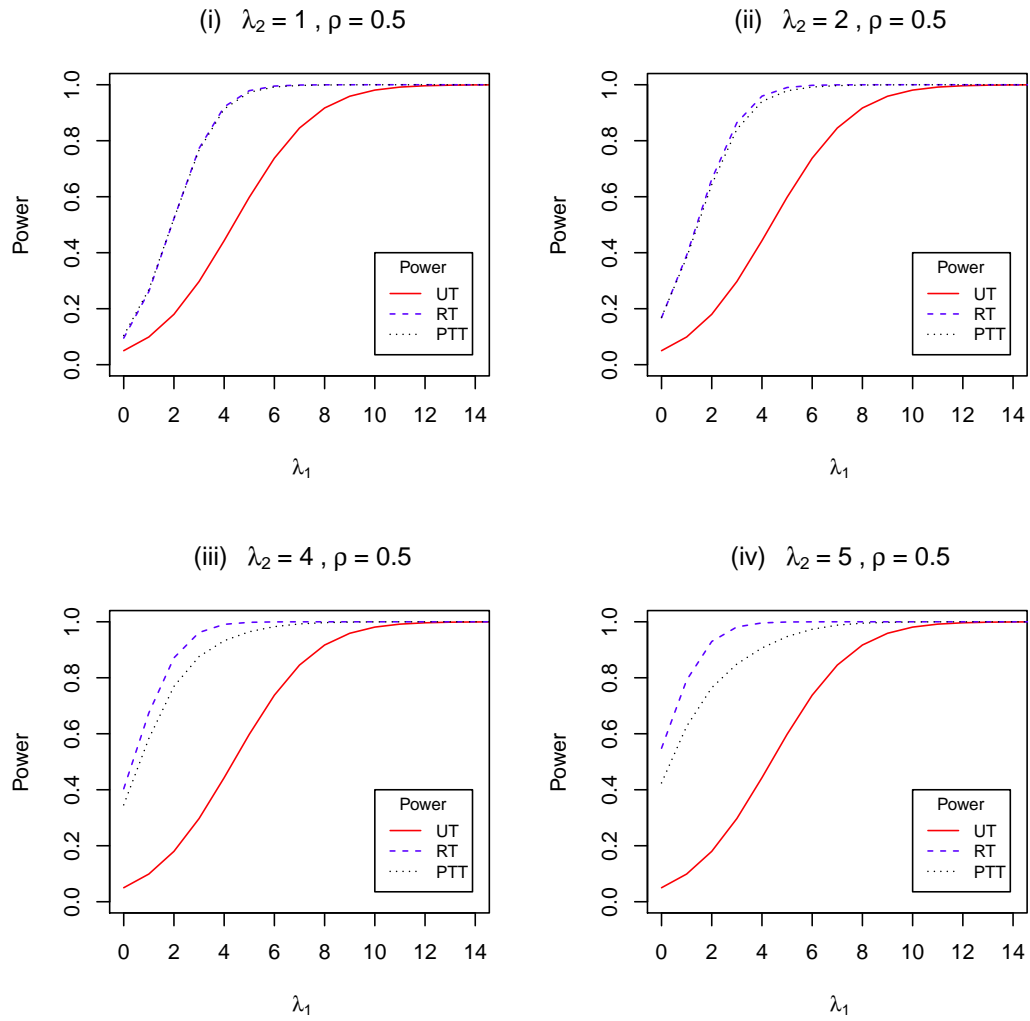


Figure 3.5: Power of the UT, RT and PTT against λ_1 with $\rho = 0.5$ and $\lambda_2 = 1, 2, 4, 5$.

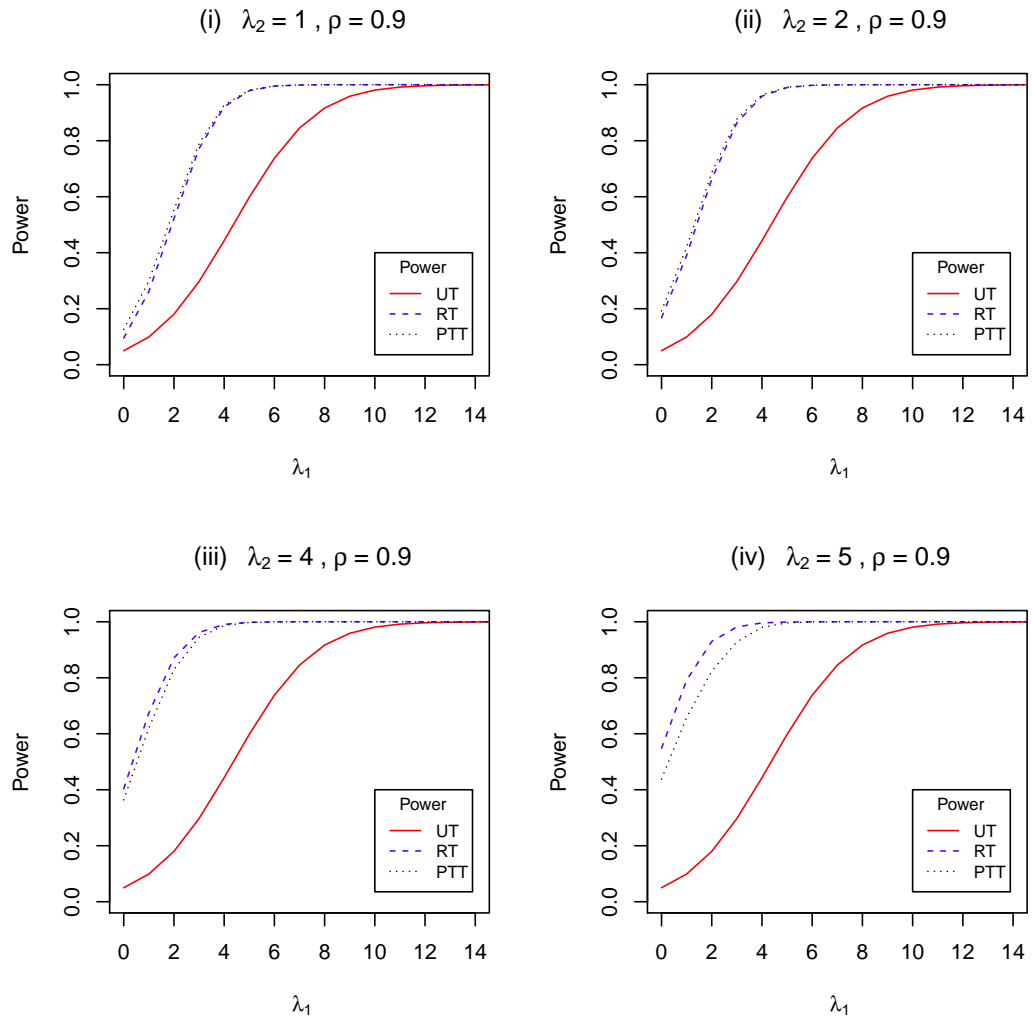


Figure 3.6: Power of the UT, RT and PTT against λ_1 with $\rho = 0.9$ and $\lambda_2 = 1, 2, 4, 5$.

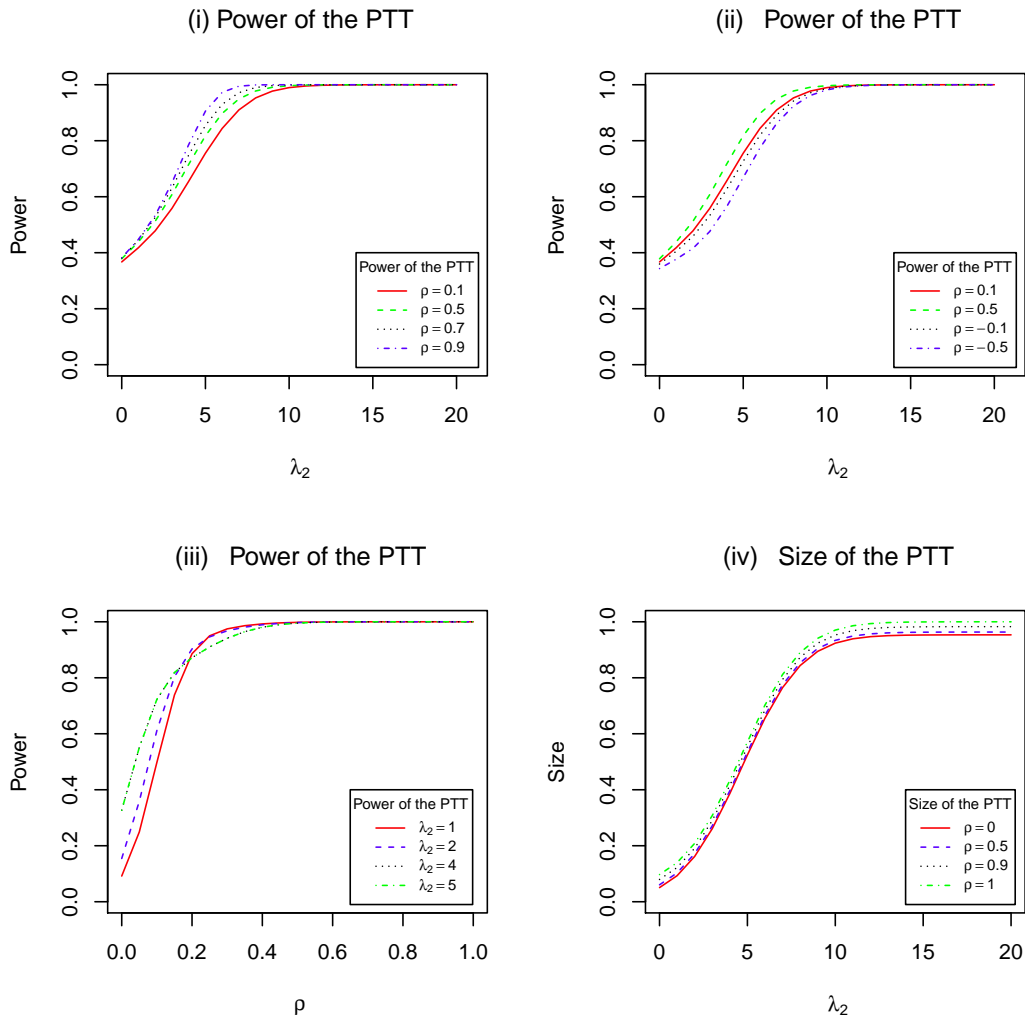


Figure 3.7: Power and size of the PTT against λ_2 , and the power of the PTT against ρ .

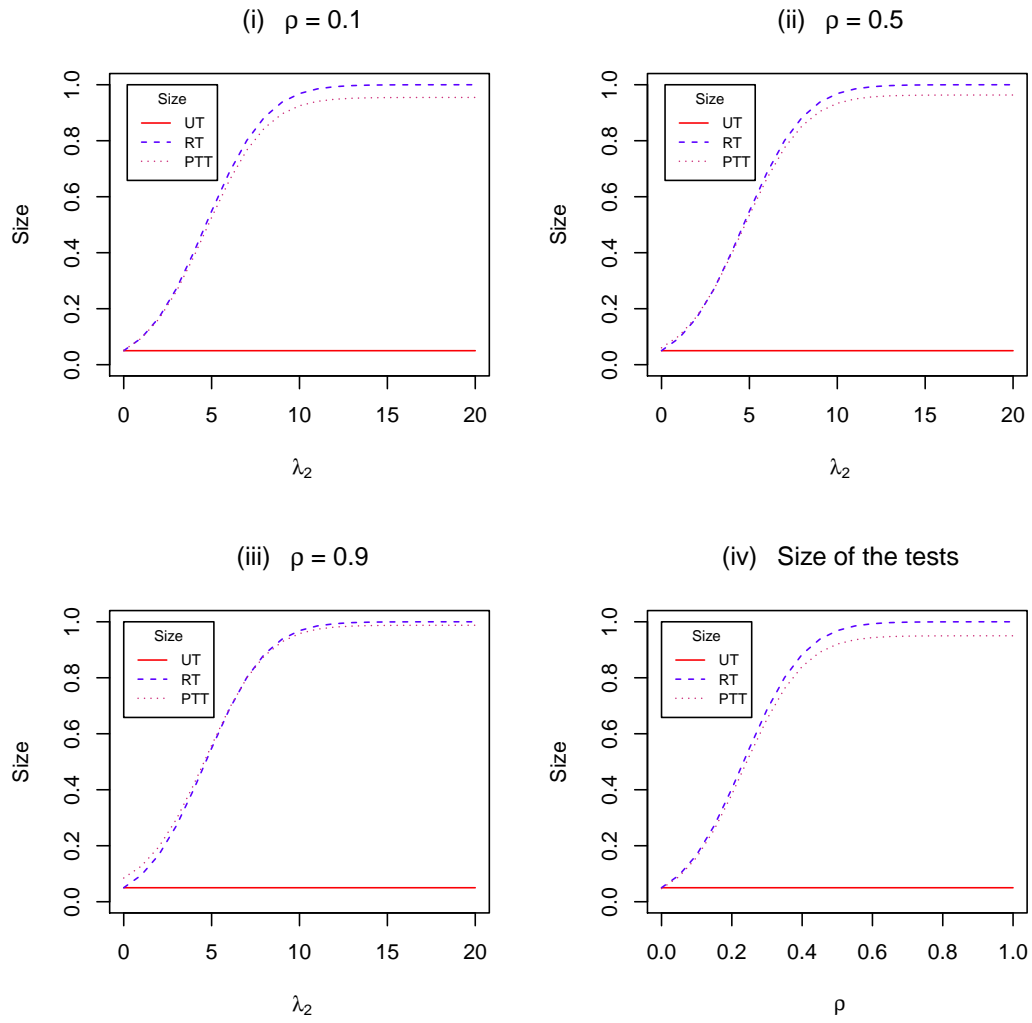


Figure 3.8: Size of the UT, RT and PTT against λ_2 for selected $\rho = 0.1, 0.5, 0.9$, and size against ρ .

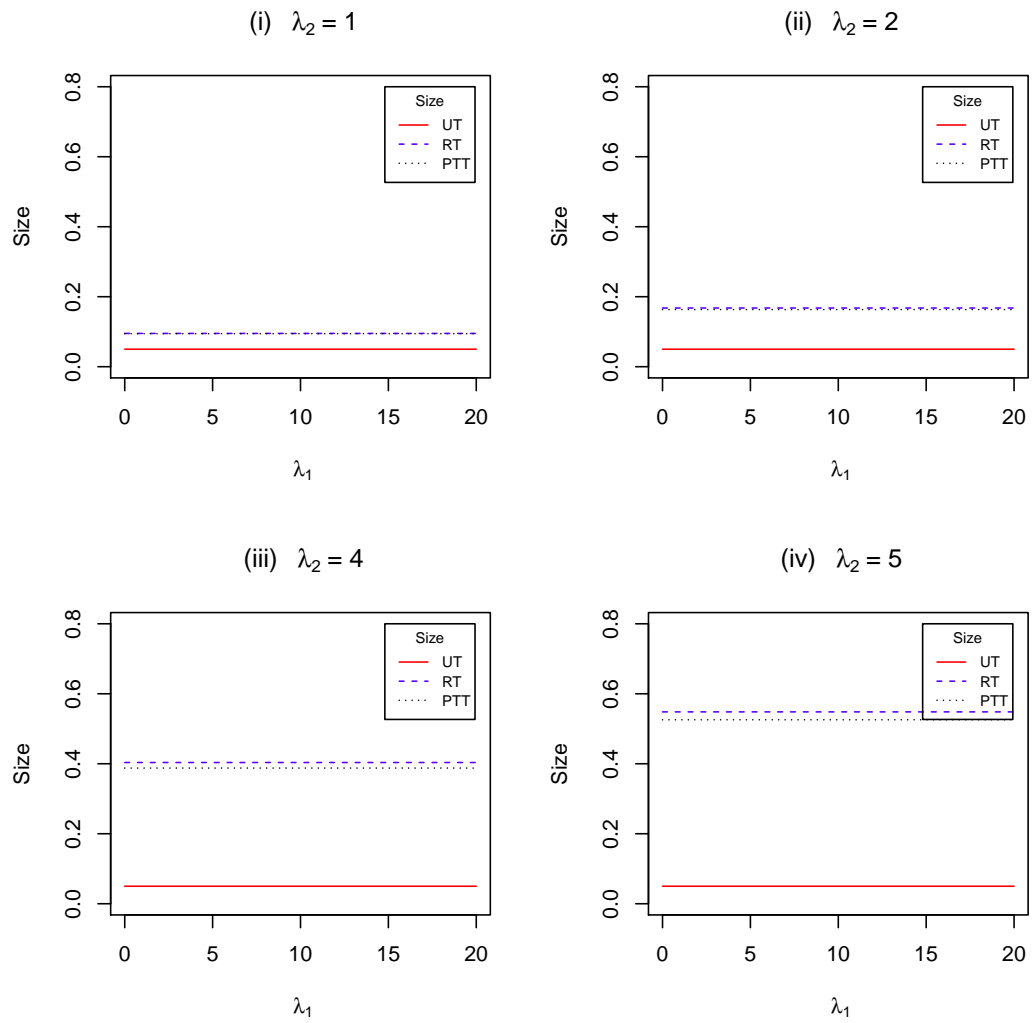


Figure 3.9: Size of the UT, RT and PTT against λ_1 for selected values $\lambda_2 = 1, 2, 4, 5$.

3.3.6 Comparison of the Tests

From Figures 3.4, 3.5 and 3.6, we see that the power of the UT does not depend on λ_2 but it increases as the value of λ_1 increases. Its form is sigmoid as the values of λ_1 goes larger and then attains one (constant) for λ_1 around 10. Thus the power of the UT changes significantly for changes in the value of λ_1 from 0 to 10, and it remains unchanged for all values of λ_2 . The minimum power of the UT is around 0.05 for all values λ_1 and λ_2 and the maximum is 1 for λ_1 around 10 and all values of λ_2 .

The power of the RT is also sigmoid for all values of λ_1 . It depends on the value of λ_2 . The power of the RT increases as the value of λ_2 increases. Moreover, the power of the RT is always larger than that of the UT for all values of λ_1 and λ_2 . The minimum power the RT is around 0.1 for $\lambda_2 = 1$ and increases to be 0.4 for $\lambda_2 = 4$ or 5. The maximum power of the RT is 1 for λ_1 around 5.

Figure 3.7 shows the power of the PTT depends on the value of λ_2 and ρ . Like the power of the RT, the power of the PTT increases as either λ_2 or ρ or both grow larger. Moreover, the power of the PTT is always larger than that of the UT but it less than that of the RT for all values of λ_1 and λ_2 . The minimum power the PTT is around 0.1 for $\lambda_2 = 1$ and increases to be around 0.35 for $\lambda_2 = 5$. The maximum power of the PTT is 1 for λ_1 around 10 and for all values of λ_2 . The gap between the power of the RT and PTT is obviously clear for the values of $\lambda_2 = 4, 5$ and $\rho = 0.1, 0.5$. Like the power of RT, the power of PTT depends on the values of λ_1 and λ_2 , also on ρ .

Figures 3.8 and 3.9 show the size of the UT does not depend on λ_2 . It is constant for all values of λ_1 . Thus the size of the UT remains unchanged for all values of λ_1 and, or λ_2 . The size of the RT increases as the value of λ_2 increases. Moreover, the size of the RT is always larger than that of the UT for all values of λ_2 . The size of the PTT depends on the value of λ_2 . Like the size of the RT, the size of the PTT increase as λ_2 grows larger. The difference between the size of the RT and PTT diminishes

as the value of λ_2 increases. The size of the RT is larger than that of the UT and PTT. The size of the UT is $\alpha^{UT} = 0.05$ for all values of λ_2 . The size of the RT, $\alpha^{RT} \geq \alpha^{UT}$ for all values of λ_2 . The size of the PTT, $\alpha^{PTT} \geq \alpha^{UT}$ for all values of λ_2 . But $\alpha^{PTT} \rightarrow \alpha^{RT}$ as $\lambda_2 \rightarrow \infty$. Like the size of the RT, Figure 3.8 also shows that the size of the PTT increases as the value of λ_2 increases. Furthermore the size of RT tends to be larger than that of the UT and PTT.

For the PTT, both the power and size depend on the value of the correlation coefficient between the UT and PT. As in Figures 3.7 and 3.8, the positive and negative values of ρ impact on the power and size of the PTT. Interestingly, the power and size of the PTT changes as the magnitude of ρ changes. In general, both the power and size increase as the value of ρ increases regardless of its sign.

3.3.7 Conclusion

Based on the analyses in this section the power of the RT is always higher than that of the UT and PTT for all λ_1 , and the power of PTT tends to lie between the power of the RT and UT for all values of λ_1 , λ_2 and ρ . The size of the UT is smaller than those of RT and PTT.

Of the three tests, the RT has the maximum power and size, and the UT has the minimum power and size. The PTT has smaller size than the RT and the RT has larger power than the UT. The PTT protects against the maximum size of the RT and the minimum power of the UT. As $\lambda_2 \rightarrow 0$ the difference between the power of the PTT and RT diminishes for all values of λ_1 . That is, if the NSPI is accurate the power of the PTT is about the same as that of the RT. Moreover, the power of the PTT gets closer to that of the RT as $\rho \rightarrow 1$. If $\rho = 1$ then the power of the PTT matches with that of RT. Thus, if there is a high (near 1) correlation between the UT and PT the power of the PTT is very close to that of the RT.

The size of the PTT becomes smaller as $\lambda_2 \rightarrow 0$. Once again if the NSPI is near

accurate the size of the PTT approaches that of the UT. Therefore, we recommend PTT when the quality of the NSPI is good (i.e. $\lambda_2 \rightarrow 0$) and it performs even better than the UT and RT when $\rho \rightarrow 1$.

Thus, for both known and unknown variance, the power of the PTT of the simple regression model attains a reasonable dominance over the UT and RT.

Chapter 4

The Multivariate Simple Regression Model

4.1 Introduction

When several responses ($p \geq 2$) or dependent variables are associated with one single value of an explanatory variable (covariate or prognostic variable), a multivariate simple regression model (MSRM) may describe the relationship. So the MSRM is a generalization of the simple regression model to analyse data from studies where there is a set of response variables for one single predictor.

In a study of energy usage in a production plant, let the usage of solar, electrical and gas energies be the level of responses corresponding to the volume of production. Obviously the usage of each kind of energy will increase as the production volume goes up. Thus, the production volume is the explanatory variable that affects the usage of different types of energies. The usage of different types of energies can be modeled by a multivariate simple regression model. Here, there is a set of response variables, for example the solar, electrical and gas energies, corresponding to a non-zero single value of the explanatory variable, e.g., the production volume. See Khan (2005, 2006a) for details on multivariate regression models and analyses.

The level of each type of energy (response variable) when no effective production is taking place, is known as the *base load*. The *base load* is related to the energy used in

lighting, heating, cooling, office equipment, machine repairs and maintenance. Since the *base load* is unaffected by the production volume, reduction in the base load is profitable to the manufacturer (cf. Kent, 2009). In the statistical context the problem is to test if the *base load* is no more than some pre-specified optimal value. The testing of the *base load* under the framework of the multivariate simple regression model with various scenarios is formally addressed in this chapter.

The linear relationship between multiple responses and a single predictor could be modeled by the MSRM. Generally, the MSRM is a generalization of the simple regression model with several responses ($p \geq 2$) or dependent variables (e.g., different energies), \mathbf{y}_i , related to a single predictor, x_i (e.g., production volume). Then the data on the single predictor and vector responses (x_i, \mathbf{y}_i) , for $i = 1, 2, \dots, n$, can be modeled by the set of linear equations. This model is given by

$$\mathbf{y}_i = \boldsymbol{\beta}_0 + \boldsymbol{\beta}_1 x_i + \mathbf{e}_i, \quad (4.1.1)$$

where $\mathbf{y}_i = (y_{i1}, \dots, y_{ip})'$ is the p dimensional i^{th} response vector, x_i is a non-zero scalar value of the explanatory variable, $\boldsymbol{\beta}_0 = (\beta_{01}, \dots, \beta_{0p})'$ and $\boldsymbol{\beta}_1 = (\beta_{11}, \dots, \beta_{1p})'$ are $p \times 1$ vectors of unknown intercept and slope parameters, and $\mathbf{e}_i = (e_{i1}, \dots, e_{ip})'$ is the $1 \times p$ dimensional i^{th} vector of errors. Assume $\mathbf{e}_i \sim N_p(\mathbf{0}, \boldsymbol{\Sigma})$, for $i = 1, \dots, n$, are independent (see Noorossana et al., 2010).

Other example of the MSRM is a group of patients treated by the same dose of a medicine (single explanatory variable) to observe changes/responses on $p \geq 2$ characteristics (response variables) on the health of the patients. For $p = 1$, there is only one response variable associated with one explanatory variable, the MSRM becomes the SRM. Thus the SRM is a special case of the model studied in this chapter.

In the context of testing the *base load* to be at most a specific value (determined by plant management), we test the intercept to be a particular value (as a specified vector, say $\boldsymbol{\beta}_{00}$). However, like estimation, the test on the intercept parameter de-

depends on the value of the slope parameter of the regression model. In the absence of no knowledge on the slope parameter commonly used maximum likelihood or least squares estimator of the slope is used. However, reliable value of the slope parameter may be available from the past experience or knowledge of the experts on the phenomenon under study. Good quality of NSPI on the value of the slope can be used to improve the quality of the statistical test on the intercept vector. The idea has been extensively used in the estimation context, but it has not been explored for the testing of hypothesis regime as such. To test the above intercept (β_{00}) when there is NSPI on the value of the slope vector, we define three different tests: (i) for the unrestricted test (UT), let ϕ^{UT} be the test function and T^{UT} be the test statistic for testing $H_0 : \beta_0 = \beta_{00}$ (unspecified vector) against $H_a : \beta_0 > \beta_{00}$ when β_1 is unspecified, (ii) for the restricted test (RT), let ϕ^{RT} be the test function and T^{RT} be the test statistic for testing $H_0 : \beta_0 = \beta_{00}$ against $H_a : \beta_0 > \beta_{00}$ when β_1 is $\beta_1 = \beta_{10}$ (specified vector) and (iii) for the pre-test test (PTT), let ϕ^{PTT} be the test function and T^{PTT} be the test statistic for testing $H_0 : \beta_0 = \beta_{00}$ against $H_a : \beta_0 > \beta_{00}$ following a pre-test (PT) on the slope. For the PT, let ϕ^{PT} be the test function for testing $H_0^* : \beta_1 = \beta_{10}$ (a suspected vector) against $H_a^* : \beta_1 > \beta_{10}$. If the H_0^* is rejected by the PT, then the UT is used to test the intercept, otherwise the RT is used to test H_0 . Thus, the PTT depends on the PT which is a choice between the UT and RT.

Following Saleh (2006, p. 19), the maximum likelihood estimator of β_0 and β_1 are given by

$$\widetilde{\beta}_0 = \bar{Y} - \widetilde{\beta}_1 \bar{x} \quad \text{and} \quad \widetilde{\beta}_1 = \sum_{i=1}^n \left(\frac{(\mathbf{Y}_i - \bar{\mathbf{Y}})(x_i - \bar{x})}{\sum_{i=1}^n (x_i - \bar{x})^2} \right), \quad (4.1.2)$$

where \mathbf{Y}_i is the observed vector of responses, and x_i is a fixed known constant, and \bar{x} and $\bar{\mathbf{Y}}$ are the sample mean of the explanatory and response variables, respectively. The above MLE of the intercept and slope parameter vectors are known as the unrestricted estimator (UE).

Let the NSPI on the value of the slope vector of the MSRM is β_{10} . Then it can be expressed by the null hypothesis, $H_0^* : \beta_1 = \beta_{10}$. Thus the restricted estimator (RE) of the intercept and slope parameters is given by

$$\widehat{\beta}_0 = \bar{Y} - \widehat{\beta}_1 \bar{x} \text{ and } \widehat{\beta}_1 = \beta_{10}, \quad (4.1.3)$$

where β_{10} is the suspected value of the slope parameter vector.

Following Saleh (2006, p. 19) and Anderson (2003, p. 170), the likelihood ratio (LR) test statistic for testing the null hypothesis $H_0 : \beta_0 = \beta_{00}$ against $H_a : \beta_0 > \beta_{00}$ is given as

$$\mathbf{T}^2 = \sum_{i=1}^n (x_i - \bar{x})^2 [(\widetilde{\beta}_0 - \beta_{00})' \widetilde{\Sigma}^{-1} (\widetilde{\beta}_0 - \beta_{00})], \quad (4.1.4)$$

where

$$\begin{aligned} \widetilde{\Sigma} &= \frac{1}{n-p} \sum_{i=1}^n (\mathbf{Y}_i - \widetilde{\mathbf{Y}}_i)(\mathbf{Y}_i - \widetilde{\mathbf{Y}}_i)' \\ &= \frac{1}{n-p} \sum_{i=1}^n (\mathbf{Y}_i - \widetilde{\beta}_0 - \widetilde{\beta}_1 \mathbf{x}_i)(\mathbf{Y}_i - \widetilde{\beta}_0 - \widetilde{\beta}_1 \mathbf{x}_i)' \end{aligned} \quad (4.1.5)$$

is an unbiased estimator of Σ . Furthermore, the $\widetilde{\Sigma}$ can be written as

$$\widetilde{\Sigma} = \frac{1}{n-p} \sum_{i=1}^n \left[(\mathbf{Y}_i - \bar{\mathbf{Y}}) - (\widetilde{\beta}_0 - \beta_{00})(x_i - \bar{x}) \right] \left[(\mathbf{Y}_i - \bar{\mathbf{Y}}) - (\widetilde{\beta}_0 - \beta_{00})(x_i - \bar{x}) \right]'. \quad (4.1.6)$$

Under H_a , the \mathbf{T}^2 follows a scale noncentral F distribution with $(p, n-p)$ degrees of freedom and noncentrality parameter $\frac{\Delta^2}{2}$ (cf. Saleh, 2006, p. 19), where

$$\Delta^2 = \left[\sum_{i=1}^n (x_i - \bar{x})^2 \right] \left[(\beta_0 - \beta_{00})' \Sigma^{-1} (\beta_0 - \beta_{00}) \right]. \quad (4.1.7)$$

More details on the multivariate simple regression model can be seen in Saleh (2006, p. 530).

This study aims to define the UT, RT and PTT; derive the sampling distribution of the test statistics and the power function of the tests; and compare them analytically and graphically.

The following Section 4.2 provides the proposed tests. Section 4.3 derives the sampling distribution of the test statistics. The power function and size of the tests are given in Section 4.4. An illustrative example is given in Section 4.5. The comparison of the power of the tests and conclusion are provided in Sections 4.6 and 4.7.

4.2 The Proposed Tests

For testing the intercept parameter under three different scenarios of the value of slope vector, the test statistics of the UT, RT and PTT for unknown Σ are given as below.

- In the case of an unspecified slope vector, β_1 , we replace β_1 by its MLE, $\widetilde{\beta}_1 = \sum_{i=1}^n \left(\frac{\mathbf{Y}_i - \bar{\mathbf{Y}}(x_i - \bar{x})}{\sum_{i=1}^n (x_i - \bar{x})^2} \right)$, and then define the UE of β_0 as $\widetilde{\beta}_0 = \bar{\mathbf{Y}} - \widetilde{\beta}_1 \bar{x}$. Following Equation (4.1.4), the test statistic of the UT for testing $H_0 : \beta_0 = \beta_{00}$ against $H_a : \beta_0 > \beta_{00}$ is given by

$$T^{UT} = \sum_{i=1}^n (x_i - \bar{x})^2 [(\widetilde{\beta}_0 - \beta_{00})' \Sigma^{-1} (\widetilde{\beta}_0 - \beta_{00})]. \quad (4.2.1)$$

The T^{UT} follows a central F distribution with $(p, n - p)$ degrees of freedom under H_0 . Under H_a , the T^{UT} follows a noncentral F distribution with $(p, n - p)$ degrees of freedom and noncentrality parameter $\frac{\Delta_1^2}{2}$, where

$$\Delta_1^2 = \left[\sum_{i=1}^n (x_i - \bar{x})^2 \right] \left[(\beta_0 - \beta_{00})' \Sigma^{-1} (\beta_0 - \beta_{00}) \right].$$

- When the slope is specified to be $\beta_1 = \beta_{10}$ (fixed), we replace β_1 by $\widehat{\beta}_1 = \beta_{10}$. The RE of β_0 is given by $\widehat{\beta}_0 = \bar{\mathbf{Y}} - \widehat{\beta}_1 \bar{x} = \bar{\mathbf{Y}} - \beta_{10} \bar{x}$. For testing $H_0 : \beta_0 = \beta_{00}$ against $H_a : \beta_0 > \beta_{00}$, the test statistic of the RT is given by

$$T^{RT} = \sum_{i=1}^n (x_i - \bar{x})^2 [(\bar{\mathbf{Y}} - \beta_{10} \bar{x} - \beta_{00})' \widehat{\Sigma}^{-1} (\bar{\mathbf{Y}} - \beta_{10} \bar{x} - \beta_{00})], \quad (4.2.2)$$

where

$$\begin{aligned}\widehat{\Sigma} &= \frac{1}{n-p} \sum_{i=1}^n (\mathbf{Y}_i - \widehat{\mathbf{Y}})(\mathbf{Y}_i - \widehat{\mathbf{Y}})' \\ &= \frac{1}{n-p} \sum_{i=1}^n (\mathbf{Y}_i - \widehat{\boldsymbol{\beta}}_0 - \widehat{\boldsymbol{\beta}}_1 \mathbf{x}_i)(\mathbf{Y}_i - \widehat{\boldsymbol{\beta}}_0 - \widehat{\boldsymbol{\beta}}_1 \mathbf{x}_i)'.\end{aligned}\quad (4.2.3)$$

Under H_a , the T^{RT} follows a noncentral F distribution with $(p, n-p)$ degrees of freedom and noncentrality parameter $\frac{\Delta_2^2}{2}$, where

$$\Delta_2^2 = \sum_{i=1}^n (\mathbf{x}_i - \bar{\mathbf{x}})^2 \left[(\bar{\mathbf{Y}} - \boldsymbol{\beta}_{10} \bar{x} - \boldsymbol{\beta}_{00})' \Sigma^{-1} (\bar{\mathbf{Y}} - \boldsymbol{\beta}_{10} \bar{x} - \boldsymbol{\beta}_{00}) \right].$$

- When the value of the slope is suspected to be $\boldsymbol{\beta}_1 = \boldsymbol{\beta}_{10}$ but unsure, a pre-test (PT) on the slope vector is required before testing the intercept. For the preliminary test (PT) of $H_0^* : \boldsymbol{\beta}_1 = \boldsymbol{\beta}_{10}$ against $H_a^* : \boldsymbol{\beta}_1 > \boldsymbol{\beta}_{10}$, the test statistic under the null hypothesis is defined by

$$T^{PT} = \sum_{i=1}^n (x_i - \bar{x})^2 [(\widetilde{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_{10})' \widetilde{\Sigma}^{-1} (\widetilde{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_{10})].\quad (4.2.4)$$

Under H_a , the T^{PT} follows a noncentral F distribution with $(p, n-p)$ degrees of freedom and noncentrality parameter $\frac{\Delta_3^2}{2}$, where

$$\Delta_3^2 = \sum_{i=1}^n (x_i - \bar{x})^2 \left[(\boldsymbol{\beta}_1 - \boldsymbol{\beta}_{10})' \Sigma^{-1} (\boldsymbol{\beta}_1 - \boldsymbol{\beta}_{10}) \right].$$

Although the PT on the slope vector is not the main objective of this study, it is essential to remove the uncertainty in the NSPI on the value of the slope, and hence define the PTT on the intercept vector.

Let us choose positive numbers α_j ($0 < \alpha_j < 1$) for $j = 1, 2, 3$ and real values $F_{\alpha_j, \nu_1, \nu_2}$ (ν_1 be numerator degrees of freedom and ν_2 be denominator degrees of freedom of the F statistic), such that

$$P(T^{UT} > F_{\alpha_1, p, n-p} \mid \boldsymbol{\beta}_0 = \boldsymbol{\beta}_{00}) = \alpha_1\quad (4.2.5)$$

$$P(T^{RT} > F_{\alpha_2, p, n-p} \mid \boldsymbol{\beta}_0 = \boldsymbol{\beta}_{00}) = \alpha_2 \quad (4.2.6)$$

$$P(T^{PT} > F_{\alpha_3, p, n-p} \mid \boldsymbol{\beta}_1 = \boldsymbol{\beta}_{10}) = \alpha_3. \quad (4.2.7)$$

Now the test function for testing $H_0 : \boldsymbol{\beta}_0 = \boldsymbol{\beta}_{00}$ following a PT on $H_0^* : \boldsymbol{\beta}_1 = \boldsymbol{\beta}_{10}$ against $H_a^* : \boldsymbol{\beta}_1 > \boldsymbol{\beta}_{10}$ is defined by

$$\Phi_f = \begin{cases} 1, & \text{if } [T^{PT} < F_c, T^{RT} > F_b] \text{ or } [T^{PT} \geq F_c, T^{UT} > F_a]; \\ 0, & \text{otherwise,} \end{cases} \quad (4.2.8)$$

where $F_a = F_{\alpha_1, p, n-p}$, $F_b = F_{\alpha_2, p, n-p}$ and $F_c = F_{\alpha_3, p, n-p}$. So the PTT is either a combination of the PT and UT, or that of the PT and RT. Note that the UT, RT, and PTT are tests on the intercept vector, but the PT is a pre-test on the suspected slope vector.

4.3 Sampling Distribution of Test Statistics

To derive the power function of the UT, RT and PTT, the sampling distribution of the test statistics proposed in Section 4.2 are required. For the power function of the PTT the joint distribution of (T^{UT}, T^{PT}) and (T^{RT}, T^{PT}) is essential. Let $\{K_n\}$ be a sequence of alternative hypotheses defined as

$$L_n : (\boldsymbol{\beta}_0 - \boldsymbol{\beta}_{00}, \boldsymbol{\beta}_1 - \boldsymbol{\beta}_{10}) = \left(\frac{\boldsymbol{\lambda}_1}{\sqrt{n}}, \frac{\boldsymbol{\lambda}_2}{\sqrt{n}} \right) = \boldsymbol{\lambda}, \quad (4.3.1)$$

where $\boldsymbol{\lambda}$ is a vector of fixed real numbers and $\boldsymbol{\beta}_0$ is the true value of the intercept vector. Under L_n the value of $(\boldsymbol{\beta}_0 - \boldsymbol{\beta}_{00})$ is greater than zero and under H_0 the value of $(\boldsymbol{\beta}_0 - \boldsymbol{\beta}_{00})$ is zero.

Following Yunus and Khan (2011b) and Equation (4.2.1), we define the test statistic of the UT when $\boldsymbol{\beta}_1$ is unspecified, under L_n , as

$$T_1^{UT} = T^{UT} - n(\boldsymbol{\beta}_0 - \boldsymbol{\beta}_{00})' \Sigma^{-1} (\boldsymbol{\beta}_0 - \boldsymbol{\beta}_{00}). \quad (4.3.2)$$

The T_1^{UT} follows a noncentral F distribution with noncentrality parameter which is a function of $(\boldsymbol{\beta}_0 - \boldsymbol{\beta}_{00})$ under L_n .

From Equation (4.2.2) under L_n , $(\beta_0 - \beta_{00}) > 0$ and $(\beta_1 - \beta_{10}) > 0$, the test statistic of the RT becomes

$$T_2^{RT} = T^{RT} - q_{1rt}, \quad (4.3.3)$$

where

$$\begin{aligned} q_{1rt} &= n[(\beta_0 - \beta_{00}) + (\beta_1 - \beta_{10})\bar{x}]'\Sigma^{-1}[(\beta_0 - \beta_{00}) + (\beta_1 - \beta_{10})\bar{x}] \\ &= [\lambda_1 + \lambda_2\bar{x}]'\Sigma^{-1}[\lambda_1 + \lambda_2\bar{x}]. \end{aligned} \quad (4.3.4)$$

The T_2^{RT} follows a noncentral F distribution with a noncentrality parameter which is a function of $(\beta_0 - \beta_{00})$ and $(\beta_1 - \beta_{10})\bar{x}$ under L_n . Similarly, from the Equation (4.2.4) the test statistic of the PT is given by

$$T_3^{PT} = T^{PT} - n(\beta_1 - \beta_{10})'\Sigma^{-1}(\beta_1 - \beta_{10}). \quad (4.3.5)$$

Under H_a , the T_3^{PT} follows a noncentral F distribution with a noncentrality parameter which is a function of $(\beta_1 - \beta_{10})$ under the alternative hypothesis.

From Equations (4.2.1), (4.2.2) and (4.2.4) we observe that the T^{UT} and T^{PT} are correlated, and the T^{RT} and T^{PT} are uncorrelated. Following Amos and Bulgren (1972), the joint distribution of the T^{UT} and T^{PT} , that is,

$$\begin{pmatrix} T^{UT} \\ T^{PT} \end{pmatrix}, \quad (4.3.6)$$

is a bivariate F distribution with $(p, n - p)$ degrees of freedom. The pdf and cdf of the bivariate central F distribution are found in Krishnaiah (1964), and Amos and Bulgren (1972), and El-Bassiouny and Jones (2009) (see Section 2.2). Following El-Bassiouny and Jones (2009), the covariance and correlation of the T^{UT} and T^{PT} are

given, respectively, as

$$\begin{aligned} \text{Cov}(T^{UT}, T^{PT}) &= \frac{2f_1 f_2}{(f_1 - 2)(f_2 - 2)(f_2 - 4)} \\ &= \frac{2(n-p)(n-p)}{[(n-p) - 2][(n-p) - 2][(n-p) - 4]} \\ &= \frac{2(n-p)^2}{(n-p-2)^2(n-p-4)} \text{ and} \end{aligned} \quad (4.3.7)$$

$$\begin{aligned} \rho_{T^{UT}T^{PT}}^2 &= \frac{d_1 d_2 (f_1 - 4)}{(f_1 + d_1 - 2)(f_2 + d_2 - 2)(f_2 - 4)} \\ &= \frac{nn[(n-p) - 4]}{[(n-p) + n - 2][(n-p) + n - 2][(n-p) - 4]} \\ &= \frac{n^2(n-p-4)}{(2n-p-2)^2(n-p-4)}. \end{aligned} \quad (4.3.8)$$

Note in the above expressions $d_1 = d_2 = p$ and $f_1 = f_2 = n - p$ are the appropriate degrees of freedom for the T^{UT} and T^{PT} , respectively.

4.4 Power Function and Size of Tests

In this section the power function of the UT, RT and PTT is derived.

4.4.1 The Power of the Tests

From Equation (4.2.1) and (4.2.5), (4.2.2) and (4.2.6), and (4.2.4), (4.2.7) and (4.2.8), the power function of the UT, RT and PTT are given, respectively, as:

(i) the power function of the UT

$$\begin{aligned} \pi^{UT}(\boldsymbol{\lambda}) &= P(T^{UT} > F_{\alpha_1, p, n-p} \mid L_n) \\ &= 1 - P\left(T_1^{UT} \leq F_{\alpha_1, p, n-p} - n(\boldsymbol{\beta}_0 - \boldsymbol{\beta}_{00})' \Sigma^{-1} (\boldsymbol{\beta}_0 - \boldsymbol{\beta}_{00})\right) \\ &= 1 - P\left(T_1^{UT} \leq F_{\alpha_1, p, n-p} - \boldsymbol{\lambda}'_1 \Sigma^{-1} \boldsymbol{\lambda}_1\right) \\ &= 1 - P\left(T_1^{UT} \leq F_{\alpha_1, p, n-p} - \phi_1\right), \end{aligned} \quad (4.4.1)$$

(ii) the power function of the RT

$$\begin{aligned}
\pi^{RT}(\boldsymbol{\lambda}) &= P(T^{RT} > F_{\alpha_2, p, n-p} | L_n) \\
&= P(T_2^{RT} > F_{\alpha_2, p, n-p} - q_{1rt}) \\
&= 1 - P\left(T_2^{RT} \leq F_{\alpha_2, p, n-p} - [\boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_2 \bar{x}]' \Sigma^{-1} [\boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_2 \bar{x}]\right) \\
&= 1 - P\left(T_2^{RT} \leq F_{\alpha_2, p, n-p} - [\phi_1 + \phi_2 \bar{x}] + \boldsymbol{\lambda}'_1 \Sigma^{-1} \boldsymbol{\lambda}_2 \bar{x} + \boldsymbol{\lambda}'_2 \Sigma^{-1} \boldsymbol{\lambda}_1 \bar{x}\right) \\
&= 1 - P\left(T_2^{RT} \leq F_{\alpha_2, p, n-p} - [\phi_1 + \phi_2 \bar{x}^2] + \omega_1 \bar{x} + \omega_2 \bar{x}\right), \quad (4.4.2)
\end{aligned}$$

where $\phi_1 = \boldsymbol{\lambda}'_1 \Sigma^{-1} \boldsymbol{\lambda}_1$, $\phi_2 = \boldsymbol{\lambda}'_2 \Sigma^{-1} \boldsymbol{\lambda}_2$, $\omega_1 = \boldsymbol{\lambda}'_1 \Sigma^{-1} \boldsymbol{\lambda}_2$ and $\omega_2 = \boldsymbol{\lambda}'_2 \Sigma^{-1} \boldsymbol{\lambda}_1$. The values of ω_1 and ω_2 depend on $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$. If either $\boldsymbol{\lambda}_1 = 0$ or $\boldsymbol{\lambda}_2 = 0$ then both ω_1 and ω_2 become 0.

The power function of the PT is given by

$$\begin{aligned}
\pi^{PT}(\boldsymbol{\lambda}) &= P(T^{PT} > F_{\alpha_3, p, n-p} | K_n) \\
&= 1 - P(T_3^{PT} \leq F_{\alpha_3, p, n-p} - \phi_2). \quad (4.4.3)
\end{aligned}$$

(iii) The power function of the PTT is

$$\begin{aligned}
\pi^{PTT}(\boldsymbol{\lambda}) &= P(T^{PT} < a, T^{RT} > c) + P(T^{PT} \geq a, T^{UT} > b) \\
&= P(T^{PT} < a) P(T^{RT} > c) + d_{1r}(a, b) \\
&= [1 - P(T^{PT} > a)] P(T^{RT} > c) + d_{1r}(a, b), \quad (4.4.4)
\end{aligned}$$

where $d_{1r}(a, b)$ is a bivariate F probability integral defined as

$$\begin{aligned}
d_{1r}(a, b) &= \int_b^\infty \int_a^\infty f(F^{PT}, F^{UT}) dF^{PT} dF^{UT} \\
&= 1 - \int_0^b \int_0^a f(F^{PT}, F^{UT}) dF^{PT} dF^{UT} \quad (4.4.5)
\end{aligned}$$

with $a = F_{\alpha_3, p, n-p} - \boldsymbol{\lambda}'_2 \Sigma^{-1} \boldsymbol{\lambda}_2 = F_{\alpha_3, p, n-p} - \phi_2$ and $b = F_{\alpha_1, p, n-p} - \boldsymbol{\lambda}'_1 \Sigma^{-1} \boldsymbol{\lambda}_1 = F_{\alpha_1, p, n-p} - \phi_1$ and $c = F_{\alpha_2, n, n-1} - [\phi_1 + \phi_2 \bar{x}] + \omega_1 \bar{x} + \omega_2 \bar{x}$. Here,

$$\int_0^b \int_0^a f(F^{PT}, F^{UT}) dF^{PT} dF^{UT}$$

is evaluated using the cdf of the correlated BNCF distribution with upper limits a and b in Equation (2.3.18). From Equation (4.4.4), the power of the PTT is defined in terms of the powers of the RT and PT as well as the cdf of the doubly BNCF distribution. Equation (2.3.20) is used for the computation of the cdf of the correlated BNCF distribution, and it is used in the calculation of the power function of the PTT. Schuurmann et al. (1975) presented the table of multivariate F distribution. This table gives the critical values (d) for different values of degrees of freedom (ν_1, ν_2), correlation coefficient (ρ) and level of significance (α). For example, by setting $a = b = d$, for $\nu_1 = 8, \nu_2 = 9, \alpha = 0.05$ and $\rho = 0.5$, the value of a is 3.79.

4.4.2 The Size of the Tests

Obviously, the size of a test is the value of its power under H_0 . The size of the UT, RT and PTT are then given by:

(i) the size of the UT

$$\begin{aligned}\alpha^{UT} &= P(T^{UT} > F_{\alpha_1, p, n-p} \mid H_0 : \boldsymbol{\beta}_0 = \boldsymbol{\beta}_{00}) \\ &= 1 - P(T^{UT} \leq F_{\alpha_1, p, n-p} \mid H_0 : \boldsymbol{\beta}_0 = \boldsymbol{\beta}_{00}) \\ &= 1 - P(T_1^{UT} \leq F_{\alpha_1, p, n-p}),\end{aligned}\tag{4.4.6}$$

(ii) the size of the RT

$$\begin{aligned}\alpha^{RT} &= P(T^{RT} > F_{\alpha_2, p, n-p} \mid H_0 : \boldsymbol{\beta}_0 = \boldsymbol{\beta}_{00}) \\ &= 1 - P(T^{RT} \leq F_{\alpha_2, p, n-p} \mid H_0 : \boldsymbol{\beta}_0 = \boldsymbol{\beta}_{00}) \\ &= 1 - P(T_2^{RT} \leq F_{\alpha_2, p, n-p} - q_{2rt})\end{aligned}\tag{4.4.7}$$

with

$$q_{2rt} = n[(\boldsymbol{\beta}_1 - \boldsymbol{\beta}_{10})\bar{x}]' \Sigma^{-1}[(\boldsymbol{\beta}_1 - \boldsymbol{\beta}_{10})\bar{x}] = [\boldsymbol{\lambda}_2 \bar{x}]' \Sigma^{-1}[\boldsymbol{\lambda}_2 \bar{x}] = \phi_2 \bar{x}^2,$$

the size of the PT

$$\begin{aligned}\alpha^{PT}(\boldsymbol{\lambda}) &= P(T^{PT} > F_{\alpha_3, p, n-p} | H_0^*) \\ &= 1 - P(T_3^{PT} \leq F_{\alpha_3, p, n-p}), \text{ and}\end{aligned}\quad (4.4.8)$$

(iii) the size of the PTT

$$\begin{aligned}\alpha^{PTT} &= P(T^{PT} \leq a | H_0, T^{RT} > g | H_0) + P(T^{PT} > a, T^{UT} > h | H_0) \\ &= P(T^{PT} \leq a) P(T^{RT} > g) + d_{1r}(a, h) \\ &= [1 - P(T^{PT} \geq a)] P(T^{RT} > g) + d_{1r}(a, h),\end{aligned}\quad (4.4.9)$$

where $g = F_{\alpha_2, p, n-p} - q_{2rt} = F_{\alpha_2, p, n-p} - \phi_2 \bar{x}^2$ and $h = F_{\alpha_1, p, n-p}$.

4.5 Analytical Comparison of the Tests

An analytical comparison of the UT, RT and PTT is provided in this Section.

4.5.1 The Power of the Tests

From Equations (4.4.1), (4.4.2) and (4.4.4), if we consider $\bar{x} = 0$ and $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$, the power of the UT, RT and PTT are given, respectively, as:

$$\pi^{UT}(\boldsymbol{\lambda}) = 1 - P(T_1^{UT} \leq F_{\alpha, p, n-p} - \phi_1), \quad (4.5.1)$$

$$\pi^{RT}(\boldsymbol{\lambda}) = 1 - P(T_2^{RT} \leq F_{\alpha, p, n-p} - \phi_1), \text{ and} \quad (4.5.2)$$

$$\pi^{PTT}(\boldsymbol{\lambda}) = [1 - P(T^{PT} > a)] P(T^{RT} > c) + d_{1r}(a, b), \quad (4.5.3)$$

with $a = F_{\alpha, p, n-p} - \phi_2$ and $b = F_{\alpha, p, n-p} - \phi_1$ and $c = F_{\alpha, n, n-1} - \phi_1$.

The power of the UT is the same as the power of the RT, but not for the power of the PTT. The power of the PTT depends on both ϕ_1 and ϕ_2 , and also ρ . If $\bar{x} > 0$, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$ and $\lambda_2 > 0$, the power of the RT is greater than that of the PTT. This is due to the fact that the value of $[\phi_1 + \phi_2 \bar{x}] + \omega_1 \bar{x} + \omega_2 \bar{x}$ becomes large, so

the cdf is very small, and as a result the power of the RT increases. On the contrary, for $\bar{x} < 0$, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$ and $\lambda_2 > 0$ the power of the RT could be the same as that of the PTT, where the power of the PTT involves the power of the PT and the RT. For $\lambda_2 < 0$ is not considered, this is because we test a one-side hypothesis for H_0 followed by pre-test on H_0^* .

4.5.2 The Size of the Tests

Obviously, from Equations (4.4.6), (4.4.7) and (4.4.9), the size of the tests for $\bar{x} = 0$ and $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$ are given by

$$\alpha^{UT} = 1 - P(T_1^{UT} \leq F_{\alpha,p,n-p}), \quad (4.5.4)$$

$$\alpha^{RT} = 1 - P(T_2^{RT} \leq F_{\alpha,p,n-p}), \text{ and} \quad (4.5.5)$$

$$\alpha^{PTT} = [1 - P(T^{PT} \geq a)] P(T^{RT} > g) + d_{1r}(a, h), \quad (4.5.6)$$

where $g = F_{\alpha,p,n-p}$ and $h = F_{\alpha,p,n-p}$.

The size of the UT and RT are equal, but the size of the PTT is not. The size of the PTT involves the size of the PT and RT as well as the probability integral of the bivariate F distribution. For two conditions: (1) $\bar{x} > 0$, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$ and $\lambda_2 > 0$, and (2) $\bar{x} < 0$, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$ and $\lambda_2 > 0$, the size of the UT does not change in either case. The size of the RT decreases for the case (1) as well as for the case (2). The size of the PTT tends to be less than that of the RT because the value of the multiplication of the sizes of the PT and RT is smaller than the size of any one of them, as well as the value of the correlated probability integral is not more than 1.

4.6 A Simulation Example

To study the properties of the three tests (UT, RT and PTT) we conduct a simulation study. The random data were generated using the R package. The explanatory variable (x) is generated from the uniform distribution between 0 and 1. The error

vector (\mathbf{e}) is generated from a $p = 5$ dimensional multivariate normal distribution with $\boldsymbol{\mu} = \mathbf{0}$ and covariance $\boldsymbol{\Sigma} = \sigma^2 \mathbf{I}_5$, where \mathbf{I}_5 is the identity matrix of order 5. Then, for $p = 5$, the dependent variable (y_1) is determined by $y_1 = \beta'_0 + \beta'_1 x + e_1$ for $\beta'_0 = 3$ and $\beta'_1 = 1.5$. Similarly, define $y_2 = \beta''_0 + \beta''_1 x + e_2$ for $\beta''_0 = 5$ and $\beta''_1 = 2.5$; $y_3 = \beta'''_0 + \beta'''_1 x + e_3$ for $\beta'''_0 = 6$ and $\beta'''_1 = 3$; $y_4 = \beta''''_0 + \beta''''_1 x + e_4$ for $\beta''''_0 = 2$ and $\beta''''_1 = 2$; and $y_5 = \beta'''''_0 + \beta'''''_1 x + e_5$ for $\beta'''''_0 = 4$ and $\beta'''''_1 = 1$. For each of the five cases $n = 30$ random variates were generated. For the computation of the power function of the tests we set $\alpha_1 = \alpha_2 = \alpha_3 = \alpha = 0.05$ and use R Package. The graphs for the power function of the three tests are produced using the formulas in Equations (4.4.1), (4.4.2) and (4.4.4) whereas the graphs for the size of the three tests are produced using the formulas in Equations (4.4.6), (4.4.7) and (4.4.9). The graphs of the power and size curves of the tests are presented in the Figures 4.1 and 4.2. The relevant R codes for producing Figures 4.1 and 4.2 are provided in Appendix A.10 and A.11.

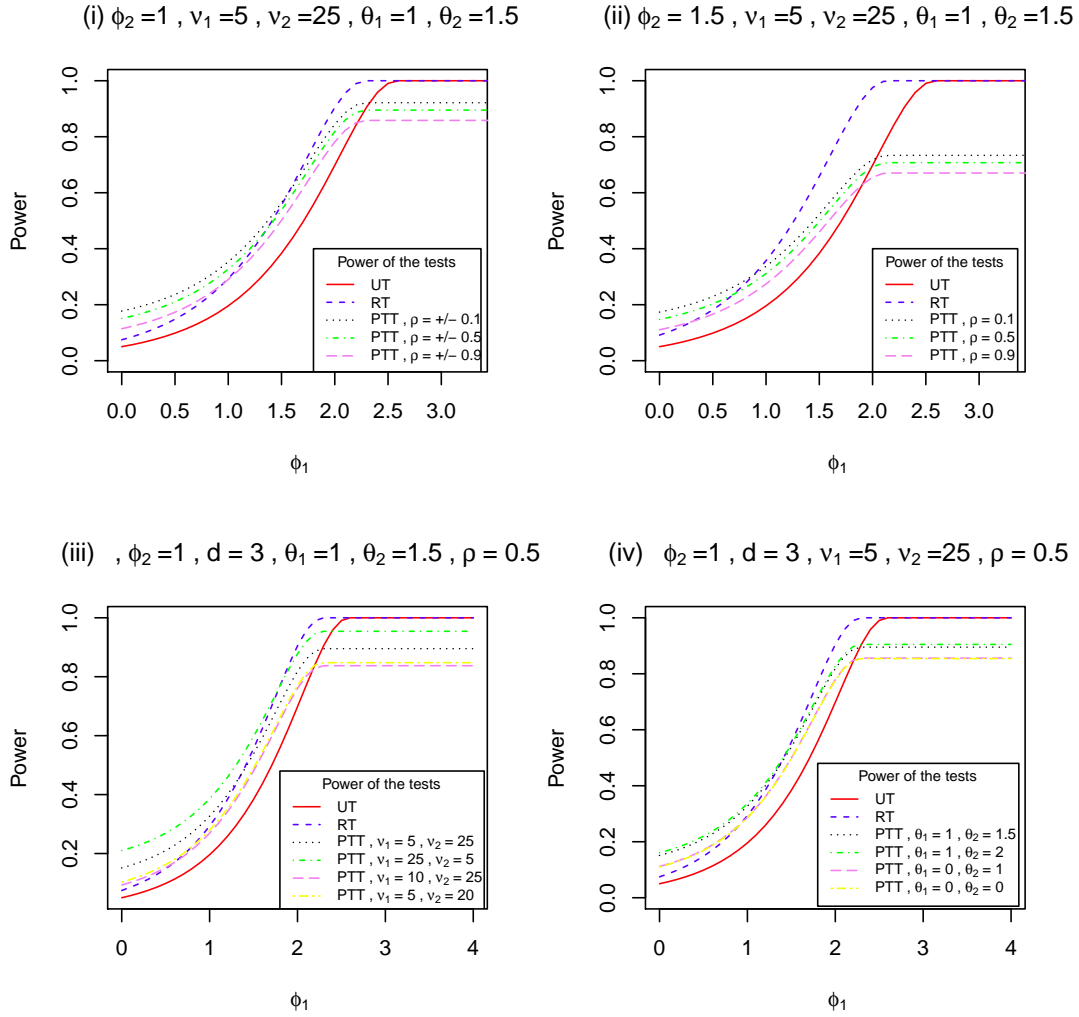


Figure 4.1: Power of the tests against ϕ_1 for selected values of ρ , degrees of freedom and noncentrality parameters.

4.7 Comparison of the Tests

From Figure 4.1 as well as from Equation (4.4.1), it is evident that the power of the UT does not depend on ϕ_2 and, or ρ , but it increases as the value of ϕ_1 increases. The form of the power curve of the UT is concave, starting from a very small value of near zero (when ϕ_1 is also near 0), it approaches 1 as ϕ_1 grows larger. The power of the UT increases rapidly as the value of ϕ_1 becomes larger. The minimum power of the UT is around 0.05 for $\phi_1 = 0$.

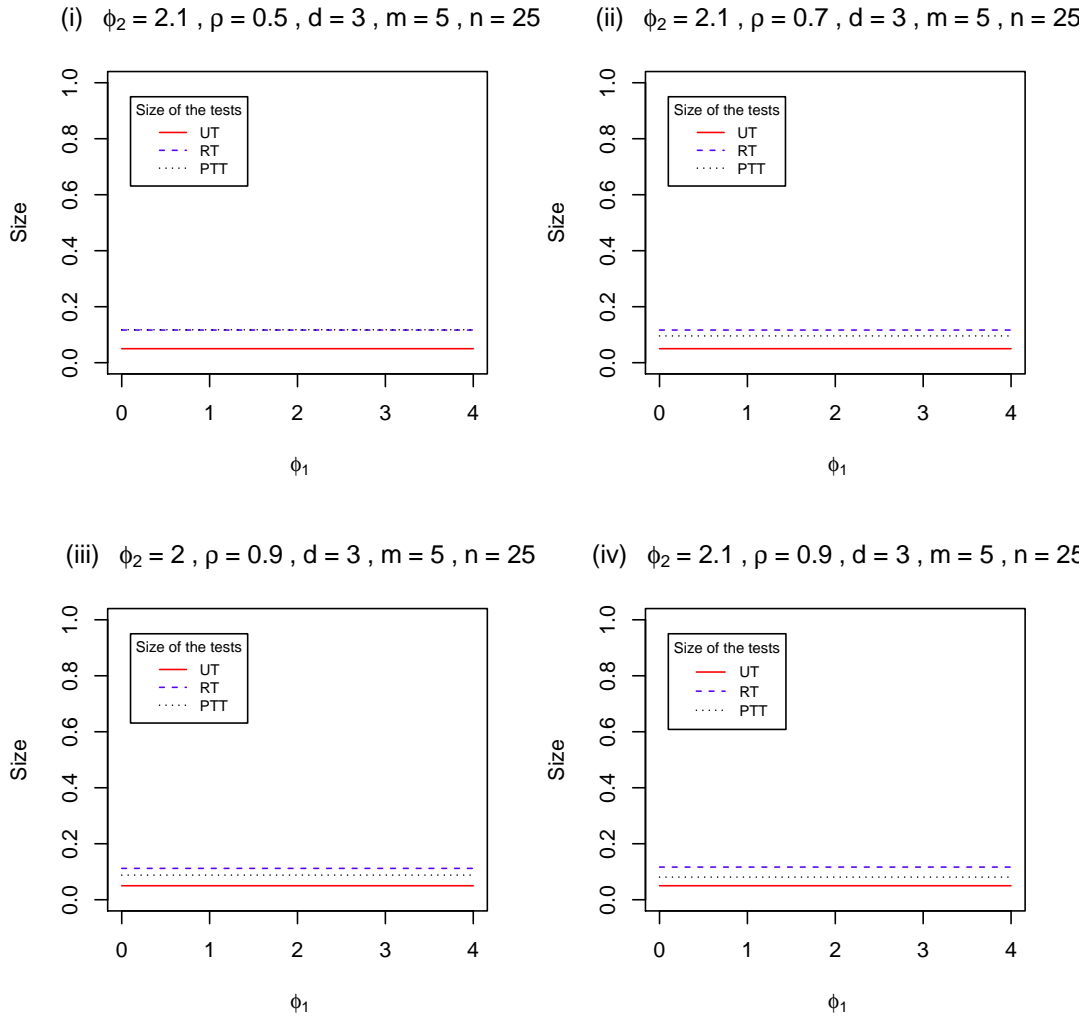


Figure 4.2: Size of the tests against ϕ_1 for selected values of ρ and ϕ_2 .

The power curve of the RT is also concave for all values of ϕ_1 and ϕ_2 (see Equation (4.4.2) and Figure 4.1). The power of the RT increases as the values of ϕ_1 and/or ϕ_2 increase. It reaches 1 much faster than that of the UT as ϕ_2 increases. Moreover, the power of the RT is always larger than that of the UT for all values of ϕ_1 and/or ϕ_2 . The minimum power of the RT is around 0.05 for $\phi_1 = 0$ and $\phi_2 = 0$. However, the minimum power of the RT becomes larger as the value of ϕ_2 increases. The maximum power the RT is 1 for reasonably larger values of ϕ_1 . The power of the RT reaches 1 much faster than that of the UT as ϕ_1 increases.

The power of the PTT depends on the values of ϕ_1 , ϕ_2 , ρ , degrees of freedom and noncentrality parameters. Like the power of the RT, the power of the PTT increases as the values of ϕ_1 , but it decreases as the values of ϕ_2 increases. Moreover, the power of the PTT is always larger than that of the UT for $0 \leq \phi_1 \leq 2$, then it becomes lower than that of the UT and RT. The power of the PTT lies between the power of the UT and RT for $1.5 \leq \phi_1 \leq 2$. The minimum power of the PTT is not too far from 0 for $\rho = 0.5$, $\nu_1 = 5$, $\nu_2 = 20$ and $\phi_1 = 0$, and it increases as the value of ϕ_2 becomes larger. The gap between the power curves of the RT and PTT is much less than that between the UT and RT. However, the power of the PTT is identical for any fixed value of ρ , regardless of its sign.

Figure 4.2 or Equation (4.4.6) shows that the size of the UT does not depend on ϕ_2 . It is constant and remains unchanged for all values of ϕ_1 and ϕ_2 . The size of the RT increases as the value of ϕ_2 increases. Moreover, the size of the RT is larger than that of the UT and PTT for the values of $\phi_2 > 1.7$ (this may change for another simulation) and of course, does not depend on ρ .

The size of the PTT is closer to that of the RT for $\phi_2 = 1.7$. It increases sharply near $\phi_2 = 0$, but reduces dramatically for larger value of ϕ_2 . The difference (gap) between the size of the RT and PTT grows significantly as the value of ϕ_2 increases. The size of the UT is $\alpha^{UT} = 0.05$ for all values of ϕ_1 and ϕ_2 . The size of the RT, $\alpha^{RT} \geq \alpha^{UT}$ for all values of ϕ_2 . The size of the PTT, $\alpha^{PTT} \geq \alpha^{RT}$ for $\phi_2 < 1.6$ (this may change for another simulation). The size of the RT is also larger than that of the UT and PTT for $\phi_2 > 1.8$ (this may change for another simulation). Overall, the size of the PTT depends on ϕ_2 , ρ , degrees of freedom, and noncentrality parameters.

4.8 Conclusion

Based on the foregoing analyses, it is clear that the power of the RT is always higher than that of the UT for all values of ϕ_1 and ϕ_2 . However, the power of the two tests are identical for $\phi_2 = 0$, and all ϕ_1 . Also, the power of the PTT is always larger than that of the UT for all values of ϕ_1 and ϕ_2 and ρ . The power of the PTT lies between that of the UT and RT for the values of $\phi_2 > 1.7$ (this may change for another simulation). The size of the UT is smaller than that of the RT and PTT for all ϕ_1 . Among the three tests, the UT has the lowest power and lowest size. In terms of power it is the worst performing test and in terms of size it is the best. Similarly, the RT has the highest size. The power of the PTT is higher than that of the UT and RT. The size of the PTT is less than that of the RT but higher than that of the UT. Thus the PTT protects against the maximum size of the RT and assures higher power than the UT and RT.

The value of ϕ_2 represents the quality of the NSPI on the rate of *process load*. If the NSPI on the rate of *process load* is accurate then $\phi_2 = 0$. The further its value away from 0 the poorer is the quality (accuracy) of the the NSPI.

Chapter 5

The Multiple Regression Model

5.1 Introduction

A multiple regression typically refers to a linear model with a single dependent (response) variable and two or more explanatory (predictor) variables. For an n pair of observations on k independent variables (X_1, \dots, X_k) and one dependent variable (Y), (X_{ij}, Y_i) , for $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, k$, the multiple regression model (MRM) is given by

$$Y_i = \beta_0 + \beta_1 X_{i1} + \dots + \beta_k X_{ik} + e_i. \quad (5.1.1)$$

The multiple regression model in Equation (5.1.1) can be written in matrix form as

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}, \quad (5.1.2)$$

where $\boldsymbol{\beta} = (\beta_0, \beta_1, \dots, \beta_{k-1}, \beta_k)'$ is a $(k+1)$ -dimensional column vector of unknown regression parameters, $\mathbf{Y} = (y_1, \dots, y_n)'$ is $n \times 1$ vector of response variables, \mathbf{X} is a $n \times (k+1)$ matrix of known fixed values of the independent variables, and \mathbf{e} is the error term which is assumed to be distributed as $N_n(\mathbf{0}, \sigma^2 \mathbf{I}_n)$. Here, \mathbf{I}_n is the identity matrix of order n and σ^2 is the common variance of the error variables.

Ostrom (1990, p. 14) presented six basic regression model assumptions, namely: (1) linearity; (2) nonstochastic \mathbf{X} : $E(e_i X_{ki}) = 0$; (3) zero mean: $E[e_i] = 0$; (4)

constant variance: $E[e_i^2] = \sigma^2$; (5) non-autoregression \mathbf{X} : $E(e_i X_{ki}) = 0$, $k \neq 0$; and (6) normality. Unfortunately, there are some assumptions of multiple regression that are not robust to violation, such as assumptions of linearity, reliability of measurement and homoscedasticity (Osborne and Waters, 2002).

In this chapter, a subset of regression parameters are tested when there is non-sample prior information (NSPI) on another subset of the regression parameters. Let $\boldsymbol{\beta}_1 = (\beta_0, \dots, \beta_{r-1})$ be a subset of r regression parameters and $\boldsymbol{\beta}_2 = (\beta_r, \dots, \beta_k)$ as another subset of mutually exclusive $(k+1-r)$ regression parameters. The regression vector $\boldsymbol{\beta}$ is then written as $\boldsymbol{\beta}' = (\boldsymbol{\beta}'_1, \boldsymbol{\beta}'_2)$, where $\boldsymbol{\beta}'_1$ is a sub-vector of dimension r and $\boldsymbol{\beta}'_2$ is a sub-vector of dimension $s = k+1-r$. In a similar way, matrix $\mathbf{X}_{(n \times (k+1))}$ is partitioned as $(\mathbf{X}_1, \mathbf{X}_2)$ with $\mathbf{X}_1 = (\mathbf{1}, \mathbf{x}_1, \dots, \mathbf{x}_{r-1})$ and $\mathbf{X}_2 = (\mathbf{x}_r, \dots, \mathbf{x}_k)$. Furthermore, for testing that \mathbf{X}_1 has no significant effect on the response, i.e. $H_0^1 : (\boldsymbol{\beta}'_1, \boldsymbol{\beta}'_2) = (\mathbf{0}, \boldsymbol{\beta}'_2)$ for $r \leq k$, we test a subset of the r regression parameters with the test statistic evaluated using comparison of the error sum of squares for the complete model (k variates) with the error sum of squares for the restricted model (r variates). Following Wackerly et al. (2008, p. 624-625), the F statistic for testing $H_0 : \beta_r = \dots = \beta_k = 0$ against $H_a : \text{not all } \beta' \text{ s are 0 for the reduced model of the form } Y = \beta_0 + \beta_1 X_1 + \dots + \beta_{r-1} X_{r-1} + e$ from the complete model $Y = \beta_0 + \beta_1 X_1 + \dots + \beta_{r-1} X_{r-1} + \beta_r x_r + \dots + \beta_k x_k + e$ is given by

$$F = \frac{\chi_r^2 / (k - (r - 1))}{\chi_c^2 / (n - (k + 1))} = \frac{(SSE_r - SSE_c) / (k - (r - 1))}{SSE_c / (n - (k + 1))} \sim F_{(k-(r-1)), n-(k+1)}. \quad (5.1.3)$$

To test a subset of the regression parameters ($\boldsymbol{\beta}_1$) to be a particular value when NSPI is available on the value of another subset ($\boldsymbol{\beta}_2$) of the regression parameters, three different tests (UT, RT and PTT) for testing $H_0 : \mathbf{H}_{1(q \times r)} \boldsymbol{\beta}_{1(r \times 1)} = \mathbf{h}_{1(q \times 1)}$ against $H_a : \mathbf{H}_{1(q \times r)} \boldsymbol{\beta}_{1(r \times 1)} > \mathbf{h}_{1(q \times 1)}$ for given $\mathbf{H}_{1(q \times r)}$ and $\mathbf{h}_{1(q \times 1)}$ are defined as follows:

- (i) for the UT, let ϕ^{UT} be the test function and T^{UT} be the test statistic for testing $H_0 : \mathbf{H}_{1(q \times r)}\boldsymbol{\beta}_{1(r \times 1)} = \mathbf{h}_{1(q \times 1)}$ against $H_a : \mathbf{H}_{1(q \times r)}\boldsymbol{\beta}_{1(r \times 1)} > \mathbf{h}_{1(q \times 1)}$ when $\boldsymbol{\beta}_2$ is unspecified,
- (ii) for the RT, let ϕ^{RT} be the test function and T^{RT} be the test statistic for testing $H_0 : \mathbf{H}_1\boldsymbol{\beta}_1 = \mathbf{h}_1$ against $H_a : \mathbf{H}_1\boldsymbol{\beta}_1 > \mathbf{h}_1$ when $\boldsymbol{\beta}_2 = \boldsymbol{\beta}_{2s}$ (specified) and
- (iii) for the PTT, let ϕ^{PTT} be the test function and T^{PTT} be the test statistic for testing $H_0 : \mathbf{H}_1\boldsymbol{\beta}_1 = \mathbf{h}_1$ against $H_a : \mathbf{H}_1\boldsymbol{\beta}_1 > \mathbf{h}_1$ when $\boldsymbol{\beta}_2$ is suspected to be $\boldsymbol{\beta}_{2s}$ following a pre-test (PT) on the $\boldsymbol{\beta}_{2s}$. For the PT, let ϕ^{PT} be the test function for testing $H_0^* : \mathbf{H}_{2(q \times s)}\boldsymbol{\beta}_{2(s \times 1)} = \mathbf{h}_{2(q \times 1)}$ (a suspected vector) against $H_a^* : \mathbf{H}_{2(q \times s)}\boldsymbol{\beta}_{2(s \times 1)} > \mathbf{h}_{2(q \times 1)}$. If H_0^* is rejected in the PT, then the UT is used to test the subset of regression parameters $\boldsymbol{\beta}_1$, otherwise the RT is used to test H_0 .

Following Saleh (2006, p. 340), the maximum likelihood unrestricted estimator (MLUE) or least square estimator (LSE) of $\boldsymbol{\beta}_1$ is

$$\tilde{\boldsymbol{\beta}}_1 = (\mathbf{X}'_1\mathbf{X}_1)^{-1}\mathbf{X}'_1\mathbf{Y} = \mathbf{C}_1^{-1}\mathbf{X}'_1\mathbf{Y}, \text{ where } \mathbf{C}_1 = \mathbf{X}'_1\mathbf{X}_1. \quad (5.1.4)$$

Let the uncertain NSPI on the value of another subset of the regression parameters of the MRM is $\boldsymbol{\beta}_2 = \boldsymbol{\beta}_{2s}$. This means that if the restriction is applied, the restricted estimator is given as

$$\hat{\boldsymbol{\beta}}_1 = \tilde{\boldsymbol{\beta}}_1 - \mathbf{C}_1^{-1}\mathbf{H}'_1(\mathbf{H}_1\mathbf{C}_1^{-1}\mathbf{H}'_1)^{-1}(\mathbf{H}_1\tilde{\boldsymbol{\beta}}_1 - \mathbf{h}_1). \quad (5.1.5)$$

Furthermore, the test statistic for testing $H_0 : \mathbf{H}_1\boldsymbol{\beta}_1 = \mathbf{h}_1$ is given by

$$F_* = \frac{1}{qs_e^2} \left((\mathbf{H}_1\tilde{\boldsymbol{\beta}}_1 - \mathbf{h}_1)' (\mathbf{H}_1[\mathbf{X}'_1\mathbf{X}_1]^{-1}\mathbf{H}'_1) (\mathbf{H}_1\tilde{\boldsymbol{\beta}}_1 - \mathbf{h}_1) \right), \quad (5.1.6)$$

where $s_e^2 = \frac{1}{n-r}(\mathbf{Y} - \mathbf{X}_1\tilde{\boldsymbol{\beta}}_1)'(\mathbf{Y} - \mathbf{X}_1\tilde{\boldsymbol{\beta}}_1)$ is an unrestricted unbiased estimator of σ^2 ($s_e^2 \rightarrow \sigma^2$). Here, $\tilde{\boldsymbol{\beta}}_1 \sim N_r(\boldsymbol{\beta}_1, \sigma^2\mathbf{C}_1^{-1})$ is independent of the distribution of

$(n-r)s_e^2/\sigma^2$ which follows a central chi-square distribution with $(n-r)$ degrees of freedom (d.f.). Since $\tilde{\boldsymbol{\beta}}_1 \sim N_r(\boldsymbol{\beta}_1, \sigma^2 \mathbf{C}_1^{-1})$, $\mathbf{H}_1 \tilde{\boldsymbol{\beta}}_1 \sim N_q[\mathbf{H}_1 \boldsymbol{\beta}_1, \sigma^2 (\mathbf{H}_1 \mathbf{C}_1^{-1} \mathbf{H}_1')]$ so that

$$(\mathbf{H}_1 \tilde{\boldsymbol{\beta}}_1 - \mathbf{h}_1) \sim N_q \left[\mathbf{H}_1 \boldsymbol{\beta}_1 - \mathbf{h}_1, \sigma^2 (\mathbf{H}_1 \mathbf{C}_1^{-1} \mathbf{H}_1') \right].$$

It is clear that $\frac{1}{\sigma^2} \left[(\mathbf{H}_1 \boldsymbol{\beta}_1 - \mathbf{h}_1)' (\mathbf{H}_1 \mathbf{C}_1^{-1} \mathbf{H}_1')^{-1} (\mathbf{H}_1 \boldsymbol{\beta}_1 - \mathbf{h}_1) \right]$ follows a noncentral chi-squared distribution with q degrees of freedom and noncentrality parameter $\Delta_1^2/2$, where

$$\Delta_1^2 = \frac{(\mathbf{H}_1 \boldsymbol{\beta}_1 - \mathbf{h}_1)' (\mathbf{H}_1 \mathbf{C}_1^{-1} \mathbf{H}_1')^{-1} (\mathbf{H}_1 \boldsymbol{\beta}_1 - \mathbf{h}_1)}{\sigma^2}. \quad (5.1.7)$$

Under H_a , F_* follows a noncentral F distribution with $(q, n-r)$ degrees of freedom and noncentrality parameter $\Delta_1^2/2$, and under H_0 , F_* follows a central F distribution with $(q, n-r)$ degrees of freedom. The assumed condition is $P(F_* \leq x) = G_{q, n-r}(x; \Delta_1^2)$ where $G_{q, n-r}(x; \Delta_1^2)$ is the cdf of a noncentral F distribution with $(q, n-r)$ degrees of freedom and noncentrality parameter $\Delta_1^2/2$. The F test statistic for testing hypothesis is also studied by Ohtani and Toyoda (1985), and Gurland and McCullough (1962).

As in the previous chapter, this study aims to define the UT, RT and PTT; derive the sampling distribution of the test statistics and the power function of the tests; and compare them graphically.

The following Section 5.2 describes the proposed tests. Section 5.3 derives the sampling distribution of the test statistics. The power function and size of the tests are obtained in Section 5.4. An illustrative example is given in Section 5.5. The comparison of the power of the tests and conclusion are provided in Sections 5.6 and 5.7.

5.2 The Proposed Tests

For testing a subset of the regression parameters $(\boldsymbol{\beta}_1)$ when NSPI is available on another subset of the regression parameters $(\boldsymbol{\beta}_2)$, we considered three different tests.

The test statistics the UT, RT and PTT for unknown Σ are given below.

- (i) If β_2 is unspecified, let ϕ^{UT} be the test function for testing $H_0 : \mathbf{H}_{1(q \times r)}\beta_{1(r \times 1)} = \mathbf{h}_{1(q \times 1)}$ against $H_a : \mathbf{H}_{1(q \times r)}\beta_{1(r \times 1)} > \mathbf{h}_{1(q \times 1)}$. From Equation (5.1.7), for complete model $\mathbf{Y} = \mathbf{X}\beta + \mathbf{e}$, the test statistic of the UT for testing $H_0 : \mathbf{H}_1\beta_1 = \mathbf{h}_1$ is given by

$$L^{UT} = \frac{(\mathbf{H}_1\widetilde{\beta}_1 - \mathbf{h}_1)'[\mathbf{H}_1(\mathbf{X}'_1\mathbf{X}_1)^{-1}\mathbf{H}'_1]^{-1}(\mathbf{H}_1\widetilde{\beta}_1 - \mathbf{h}_1)}{q s_{ut}^2}, \quad (5.2.1)$$

where

$$s_{ut}^2 = \frac{1}{n-r}(\mathbf{Y} - \mathbf{X}_1\widetilde{\beta}_1)'(\mathbf{Y} - \mathbf{X}_1\widetilde{\beta}_1)$$

and $\widetilde{\beta}_1$ is a MLUE of β_1 . We assume that $(\mathbf{H}_1\widetilde{\beta}_1 - \mathbf{h}_1) \sim N_q(\mathbf{H}_1\beta_1 - \mathbf{h}_1, \sigma^2(\mathbf{H}_1[\mathbf{X}'_1\mathbf{X}_1]^{-1}\mathbf{H}'_1))$ (cf Saleh, 2006, p. 342). From Equation (5.1.7), under H_0 the L^{UT} follows an F distribution with q and $(n-r)$ degrees of freedom whereas under H_a the L^{UT} follows a noncentral F distribution with $(q, n-r)$ degrees of freedom and noncentrality parameter $\Delta_1^2/2$.

- (ii) If $\beta_2 = \beta_{2s}$ (specified), where the order of matrix β_{2s} is $(s \times 1)$, let ϕ^{RT} be the test function for testing $H_0 : \mathbf{H}_1\beta_1 = \mathbf{h}_1$ against $H_a : \mathbf{H}_1\beta_1 > \mathbf{h}_1$. The proposed test statistic of the RT is then given by

$$L^{RT} = \frac{(\mathbf{H}_1\widehat{\beta}_1 - \mathbf{h}_1)'[(\mathbf{H}_1(\mathbf{X}'_1\mathbf{X}_1)^{-1}\mathbf{H}'_1)^{-1}(\mathbf{H}_1\widehat{\beta}_1 - \mathbf{h}_1)]}{q s_{rt}^2}, \quad (5.2.2)$$

where

$$s_{rt}^2 = \frac{1}{n-r}(\mathbf{Y} - \mathbf{X}_1\widehat{\beta}_1)'(\mathbf{Y} - \mathbf{X}_1\widehat{\beta}_1).$$

The L^{RT} follows an F distribution with q and $(n-r)$ d.f. under H_0 . The assumed conditions are $(\mathbf{H}_1\widehat{\beta}_1 - \mathbf{h}_1) \sim N_q(\mathbf{H}_1\beta_1 - \mathbf{h}_1, \sigma^2(\mathbf{H}_1[\mathbf{X}'_1\mathbf{X}_1]^{-1}\mathbf{H}'_1))$. From Equation (5.1.7), under H_a , L^{RT} follows a noncentral F distribution with $(q, n-r)$ degrees of freedom and noncentrality parameter $\Delta_2^2/2$, where

$$\Delta_2^2 = \frac{(\mathbf{H}_1\beta_1 - \mathbf{h}_1)'(\mathbf{H}_1[\mathbf{X}'_1\mathbf{X}_1]^{-1}\mathbf{H}'_1)^{-1}(\mathbf{H}_1\beta_1 - \mathbf{h}_1)}{\sigma^2}. \quad (5.2.3)$$

(iii) For the preliminary test on the subset, $\beta_2 = \beta_{2s}$, let ϕ^{PT} be the test function for testing $H_0^* : \mathbf{H}_2(q \times s)\beta_{2(s \times 1)} = \mathbf{h}_2$ against $H_a^* : \mathbf{H}_2(q \times s)\beta_{2(s \times 1)} > \mathbf{h}_2$. The proposed test statistic of the PT is

$$L^{PT} = \frac{(\mathbf{H}_2\widetilde{\beta}_2 - \mathbf{h}_2)'[\mathbf{H}_2(\mathbf{X}'_2\mathbf{X}_2)^{-1}\mathbf{H}'_2]^{-1}(\mathbf{H}_2\widetilde{\beta}_2 - \mathbf{h}_2)}{q s_{pt}^2}, \quad (5.2.4)$$

where

$$s_{pt}^2 = \frac{1}{n-s}(\mathbf{Y} - \mathbf{X}_2\widetilde{\beta}_2)'(\mathbf{Y} - \mathbf{X}_2\widetilde{\beta}_2)$$

and $\widetilde{\beta}_2$ is the MLUE of β_2 . Here, L^{PT} follows an F distribution with q and $(n-s)$ degrees of freedom. The assumed condition is $(\mathbf{H}_2\widetilde{\beta}_2 - \mathbf{h}_2) \sim N_s(\mathbf{H}_2\beta_2 - \mathbf{h}_2, \sigma^2(\mathbf{H}_2[\mathbf{X}'_2\mathbf{X}_2]^{-1}\mathbf{H}'_2))$. Similarly, under H_a , L^{PT} follows a non-central F distribution with $(q, n-s)$ degrees of freedom and noncentrality parameter $\Delta_3^2/2$, where

$$\Delta_3^2 = \frac{(\mathbf{H}_2\beta_2 - \mathbf{h}_2)'(\mathbf{H}_2[\mathbf{X}'_2\mathbf{X}_2]^{-1}\mathbf{H}'_2)^{-1}(\mathbf{H}_2\beta_2 - \mathbf{h}_2)}{\sigma^2}. \quad (5.2.5)$$

Let us choose a positive number α_j ($0 < \alpha_j < 1$, for $j = 1, 2, 3$) and real value $F_{\nu_1, \nu_2, \alpha_j}$ (ν_1 be numerator d.f. and ν_2 be denominator d.f.) such that

$$P(L^{UT} > F_{q, n-r, \alpha_1} \mid \mathbf{H}_1\beta_1 = \mathbf{h}_1) = \alpha_1, \quad (5.2.6)$$

$$P(L^{RT} > F_{q, n-r, \alpha_2} \mid \mathbf{H}_1\beta_1 = \mathbf{h}_1) = \alpha_2, \quad (5.2.7)$$

$$P(L^{PT} > F_{q, n-s, \alpha_3} \mid \mathbf{H}_2\beta_2 = \mathbf{h}_2) = \alpha_3. \quad (5.2.8)$$

Now the test function for testing $H_0 : \mathbf{H}_1\beta_1 = \mathbf{h}_1$ against $H_a : \mathbf{H}_1\beta_1 > \mathbf{h}_1$, after pre-testing on β_{2s} , is defined by

$$\Phi = \begin{cases} 1, & \text{if } (L^{PT} \leq F_c, L^{RT} > F_b) \text{ or } (L^{PT} > F_c, L^{UT} > F_a); \\ 0, & \text{otherwise,} \end{cases} \quad (5.2.9)$$

where $F_a = F_{\alpha_1, q, n-r}$, $F_b = F_{\alpha_2, q, n-r}$ and $F_c = F_{\alpha_3, q, n-s}$.

5.3 Sampling Distribution of Test Statistics

To derive the power function of the UT, RT and PTT, the sampling distribution of the test statistics proposed in Section 5.2 are required. For the power function of the PTT the joint distribution of (L^{UT}, L^{PT}) and (L^{RT}, L^{PT}) is essential. Let $\{M_n\}$ be a sequence of alternative hypotheses defined as

$$M_n : (\mathbf{H}_1\boldsymbol{\beta}_1 - \mathbf{h}_1, \mathbf{H}_2\boldsymbol{\beta}_2 - \mathbf{h}_2) = \left(\frac{\boldsymbol{\lambda}_1}{\sqrt{n}}, \frac{\boldsymbol{\lambda}_2}{\sqrt{n}} \right) = \boldsymbol{\lambda}, \quad (5.3.1)$$

where $\boldsymbol{\lambda}_{(q \times 2)}$ is a vector of fixed real numbers. Under M_n both $(\mathbf{H}_1\boldsymbol{\beta}_1 - \mathbf{h}_1)$ and $(\mathbf{H}_2\boldsymbol{\beta}_2 - \mathbf{h}_2)$ are non singular matrices and under H_0 and H_0^* , respectively, they are singular matrices.

Following Yunus and Khan (2011b) and Equation (5.2.1), we define the test statistic of the UT when $\boldsymbol{\beta}_2$ is unspecified, under M_n , as

$$L_1^{UT} = L^{UT} - \frac{n\sigma}{qs_{ut}^2} (\mathbf{H}_1\boldsymbol{\beta}_1 - \mathbf{h}_1)' [\mathbf{H}_1(\mathbf{X}'_1\mathbf{X}_1)^{-1}\mathbf{H}'_1]^{-1} (\mathbf{H}_1\boldsymbol{\beta}_1 - \mathbf{h}_1). \quad (5.3.2)$$

The L_1^{UT} follows a noncentral F distribution with $(q, n-r)$ degrees of freedom and a noncentrality parameter which is a function of $(\mathbf{H}_1\boldsymbol{\beta}_1 - \mathbf{h}_1)$ under M_n .

From Equation (5.2.2) under M_n , $(\mathbf{H}_1\boldsymbol{\beta}_1 - \mathbf{h}_1)$ is a non singular matrix, and the test statistic of the RT becomes

$$L_2^{RT} = L^{RT} - \frac{n\sigma}{qs_{rt}^2} (\mathbf{H}_1\boldsymbol{\beta}_1 - \mathbf{h}_1)' [\mathbf{H}_1(\mathbf{X}'_1\mathbf{X}_1)^{-1}\mathbf{H}'_1]^{-1} (\mathbf{H}_1\boldsymbol{\beta}_1 - \mathbf{h}_1). \quad (5.3.3)$$

The L_2^{RT} also follows a noncentral F distribution with $(q, n-r)$ degrees of freedom and a noncentrality parameter which is a function of $(\mathbf{H}_1\boldsymbol{\beta}_1 - \mathbf{h}_1)$ under M_n . Similarly, from the Equation (5.2.4) the test statistic of the PT is given by

$$L_3^{PT} = L^{PT} - \frac{n\sigma}{qs_{pt}^2} (\mathbf{H}_2\boldsymbol{\beta}_2 - \mathbf{h}_2)' [\mathbf{H}_2(\mathbf{X}'_2\mathbf{X}_2)^{-1}\mathbf{H}'_2]^{-1} (\mathbf{H}_2\boldsymbol{\beta}_2 - \mathbf{h}_2). \quad (5.3.4)$$

Under H_a^* , the L_3^{PT} follows a noncentral F distribution with $(q, n-s)$ degrees of freedom and a noncentrality parameter which is a function of $(\mathbf{H}_2\boldsymbol{\beta}_2 - \mathbf{h}_2)$ under the alternative hypothesis.

From Equations (5.2.1), (5.2.2) and (5.2.4) we observe that the L^{UT} and L^{PT} are correlated, and the L^{RT} and L^{PT} are uncorrelated. The joint distribution of the L^{UT} and L^{PT} , that is,

$$\begin{pmatrix} L^{UT} \\ L^{PT} \end{pmatrix}, \quad (5.3.5)$$

is a correlated bivariate F distribution with $(q, n-r)$ degrees of freedom for $n-r = n-s$. The details of the BCF and of BNCF distributions are available in Sections 2.2 and 2.3, respectively. The covariance and correlation coefficient of the correlated bivariate F distribution for the $L^{UT} \sim F_{1(q, n-r)}$ and $L^{PT} \sim F_{2(q, n-s)}$ are then given, respectively, as

$$\begin{aligned} \text{Cov}(L^{UT}, L^{PT}) &= \frac{2(n-r)(n-s)}{(n-r-2)(n-s-2)(n-s-4)} \\ &= \frac{2(n-r)(n-s)}{(n-r-2)(n-s-2)(n-s-4)} \quad \text{and} \end{aligned} \quad (5.3.6)$$

$$\begin{aligned} \rho_{L^{UT}L^{PT}} &= \left(\frac{q \cdot q(n-r-4)}{(n-r+q-2)(n-s+q-2)(n-s-4)} \right)^{1/2} \\ &= \left(\frac{q^2(n-r-4)}{(n-r+q-2)(n-s+q-2)(n-s-4)} \right)^{1/2}. \end{aligned} \quad (5.3.7)$$

5.4 Power Function and Size of Tests

In this section the power function and size of the UT, RT and PTT are derived.

5.4.1 The Power of the Tests

From Equations (5.2.1) and (5.3.2), (5.2.2) and (5.3.3), and (5.2.4), (5.2.9) and (5.3.4), the power function of the UT, RT and PTT are given, respectively, as:

(i) the power of the UT

$$\begin{aligned} \pi^{UT}(\boldsymbol{\lambda}) &= P(L^{UT} > F_{\alpha_1, q, n-r} \mid M_n) \\ &= 1 - P(L_1^{UT} \leq F_{\alpha_1, q, n-r} - \Omega_{ut}) \\ &= 1 - P(L_1^{UT} \leq F_{\alpha_1, q, n-r} - k_{ut} \zeta_1), \end{aligned} \quad (5.4.1)$$

where $\Omega_{ut} = \frac{\sigma}{qs_{ut}^2}(\boldsymbol{\lambda}_1)'[\boldsymbol{\gamma}_1]^{-1}(\boldsymbol{\lambda}_1)$, $\boldsymbol{\gamma}_1 = \mathbf{H}_1(\mathbf{X}'_1\mathbf{X}_1)^{-1}\mathbf{H}'_1$, $\zeta_1 = (\boldsymbol{\lambda}_1)'[\boldsymbol{\gamma}_1]^{-1}(\boldsymbol{\lambda}_1)$ and $k_{ut} = \frac{\sigma}{qs_{ut}^2}$.

(ii) the power of the RT

$$\begin{aligned}\pi^{RT}(\boldsymbol{\lambda}) &= P(L^{RT} > F_{\alpha_1, q, n-r} | M_n) \\ &= P(L_2^{RT} > F_{\alpha_2, q, n-r} - \Omega_{rt}) \\ &= 1 - P(L_2^{RT} \leq F_{\alpha_2, q, n-r} - \Omega_{rt}) \\ &= 1 - P(L_2^{RT} \leq F_{\alpha_2, q, n-r} - k_{rt} \zeta_1),\end{aligned}\quad (5.4.2)$$

where $\Omega_{rt} = \frac{\sigma}{qs_{rt}^2}(\boldsymbol{\lambda}_1)'[\boldsymbol{\gamma}_1]^{-1}(\boldsymbol{\lambda}_1)$, $\boldsymbol{\gamma}_1 = \mathbf{H}_1(\mathbf{X}'_1\mathbf{X}_1)^{-1}\mathbf{H}'_1$, $\zeta_1 = (\boldsymbol{\lambda}_1)'[\boldsymbol{\gamma}_1]^{-1}(\boldsymbol{\lambda}_1)$ and $k_{rt} = \frac{\sigma}{qs_{rt}^2}$.

The power function of the PT is given by

$$\begin{aligned}\pi^{PT}(\boldsymbol{\lambda}) &= P(T^{PT} > F_{\alpha_3, q, n-s} | M_n) \\ &= 1 - P(L_3^{PT} \leq F_{\alpha_3, q, n-s} - k_{pt}\zeta_2),\end{aligned}\quad (5.4.3)$$

where $k_{pt} = \frac{\sigma}{qs_{pt}^2}$ and $\zeta_2 = (\boldsymbol{\lambda}_2)'[\boldsymbol{\gamma}_2]^{-1}(\boldsymbol{\lambda}_2)$ with $\boldsymbol{\gamma}_2 = \mathbf{H}_2(\mathbf{X}'_2\mathbf{X}_2)^{-1}\mathbf{H}'_2$, and then

(iii) the power of the PTT

$$\begin{aligned}\pi^{PTT}(\boldsymbol{\lambda}) &= P(L^{PT} < F_{\alpha_3, q, n-s}, L^{RT} > F_{\alpha_2, q, n-r} | M_n) \\ &\quad + P(L^{PT} \geq F_{\alpha_3, q, n-s}, L^{UT} > F_{\alpha_1, q, n-r} | M_n) \\ &= P[L^{PT} < F_{\alpha_3, q, n-s}] P[L^{RT} > F_{\alpha_2, q, n-r}] + d_{1r}(a, b) \\ &= [1 - P(L^{PT} > F_{\alpha_3, q, n-s})] P(L^{RT} > F_{\alpha_2, q, n-r}) + d_{1r}(a, b),\end{aligned}\quad (5.4.4)$$

where $a = F_{\alpha_3, q, n-s} - \frac{\sigma}{qs_{pt}^2}(\boldsymbol{\lambda}_2)'[\boldsymbol{\gamma}_{pt}]^{-1}(\boldsymbol{\lambda}_2) = F_{\alpha_3, q, n-s} - k_{pt}\zeta_2$, and $d_{1r}(a, b)$ is bivariate F probability integral. The value of ζ_1 and ζ_2 depend on $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$,

respectively, and $d_{1r}(a, b)$ is defined as

$$\begin{aligned} d_{1r}(a, b) &= \int_a^\infty \int_b^\infty f(F^{PT}, F^{UT}) dF^{PT} dF^{UT} \\ &= 1 - \int_0^b \int_0^a f(F^{PT}, F^{UT}) dF^{PT} dF^{UT}, \end{aligned} \quad (5.4.5)$$

with $b = F_{\alpha_1, q, n-r} - \Omega_{ut}$. The integral $\int_0^b \int_0^a f(F^{PT}, F^{UT}) dF^{PT} dF^{UT}$ is evaluated using the cdf correlated BNCF distribution in Equation (2.3.18). Equation (2.3.20) is then used for the computation of the cdf of the correlated BNCF distribution.

5.4.2 The Size of the Tests

The size of a test is the value of its power under H_0 . The size of the UT, RT and PTT are then given by:

(i) the size of the UT

$$\begin{aligned} \alpha^{UT} &= P(L^{UT} > F_{\alpha_1, q, n-r} \mid H_0 : \mathbf{H}_1 \boldsymbol{\beta}_1 = \mathbf{h}_1) \\ &= 1 - P(L_1^{UT} \leq F_{\alpha_1, q, n-r} \mid H_0 : \mathbf{H}_1 \boldsymbol{\beta}_1 = \mathbf{h}_1) \\ &= 1 - P(L_1^{UT} \leq F_{\alpha_1, q, n-r}), \end{aligned} \quad (5.4.6)$$

(ii) the size of the RT

$$\begin{aligned} \alpha^{RT} &= P(L^{RT} > F_{\alpha_2, q, n-r} \mid H_0 : \mathbf{H}_1 \boldsymbol{\beta}_1 = \mathbf{h}_1) \\ &= 1 - P(L_2^{RT} \leq F_{\alpha_2, q, n-r} \mid H_0 : \mathbf{H}_1 \boldsymbol{\beta}_1 = \mathbf{h}_1) \\ &= 1 - P(L_2^{RT} \leq F_{\alpha_2, q, n-r} - k_2 \zeta_2), \end{aligned} \quad (5.4.7)$$

where the value of $\zeta_1 = 0$ but $\zeta_2 \neq 0$. The size of the PT is given by

$$\begin{aligned} \alpha^{PT}(\boldsymbol{\lambda}) &= P(T^{PT} > F_{\alpha_3, q, n-s} \mid H_0 : \mathbf{H}_2 \boldsymbol{\beta}_2 = \mathbf{h}_2) \\ &= 1 - P(L_3^{PT} \leq F_{\alpha_3, q, n-s}) \text{ and then} \end{aligned} \quad (5.4.8)$$

(iii) the size of the PTT

$$\begin{aligned}
 \alpha^{PTT} &= P(L^{PT} \leq a, L^{RT} > d \mid H_0) + P(L^{PT} > a, L^{UT} > h \mid H_0) \\
 &= P(L^{PT} \leq a) P(L^{RT} > d) + d_{1r}(a, h) \\
 &= [1 - P(L^{PT} \leq a)] P(L^{RT} > d) + d_{1r}(a, h), \tag{5.4.9}
 \end{aligned}$$

where $d = F_{\alpha_2, q, n-r}$, $h = F_{\alpha_1, q, n-r}$ and under H_0 the value of a is $F_{\alpha_1, q, n-s}$.

5.5 A Simulation Example

To study the properties of the three tests a simulation study was conducted. The random data were generated using the R package. For $k = 3$ number of variates, the independent variables (x_j , $j = 1, 2, 3$) were generated from the uniform distribution between 0 and 1, respectively. The error vector (\mathbf{e}) was generated from the normal distribution with $\boldsymbol{\mu} = \mathbf{0}$ and $\boldsymbol{\Sigma} = \sigma^2 \mathbf{I}_3$, where \mathbf{I}_3 is the identity matrix of order 3. In each case $n = 100$ random variates were generated. The dependent variable (y) is determined by $y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + e_i$ for $i = 1, 2, \dots, n$. For the computation of the power function of the tests we set $\boldsymbol{\beta}_1 = (\beta_0, \beta_1)$, $\boldsymbol{\beta}_2 = (\beta_2, \beta_3)$, and $\alpha_1 = \alpha_2 = \alpha_3 = \alpha = 0.05$. The graphs for the power function of the three tests are produced using the formulas in Equations (5.4.1), (5.4.2) and (5.4.4). Whereas the graphs for the size of the three tests are produced using formulas in Equations (5.4.6), (5.4.7) and (5.4.9). The graphs of the power and size curves of the tests are presented in the Figures 5.1 and 5.2. The relevant R codes for producing Figures 5.1 and 5.2 are provided in Appendix A.12 and A.13.

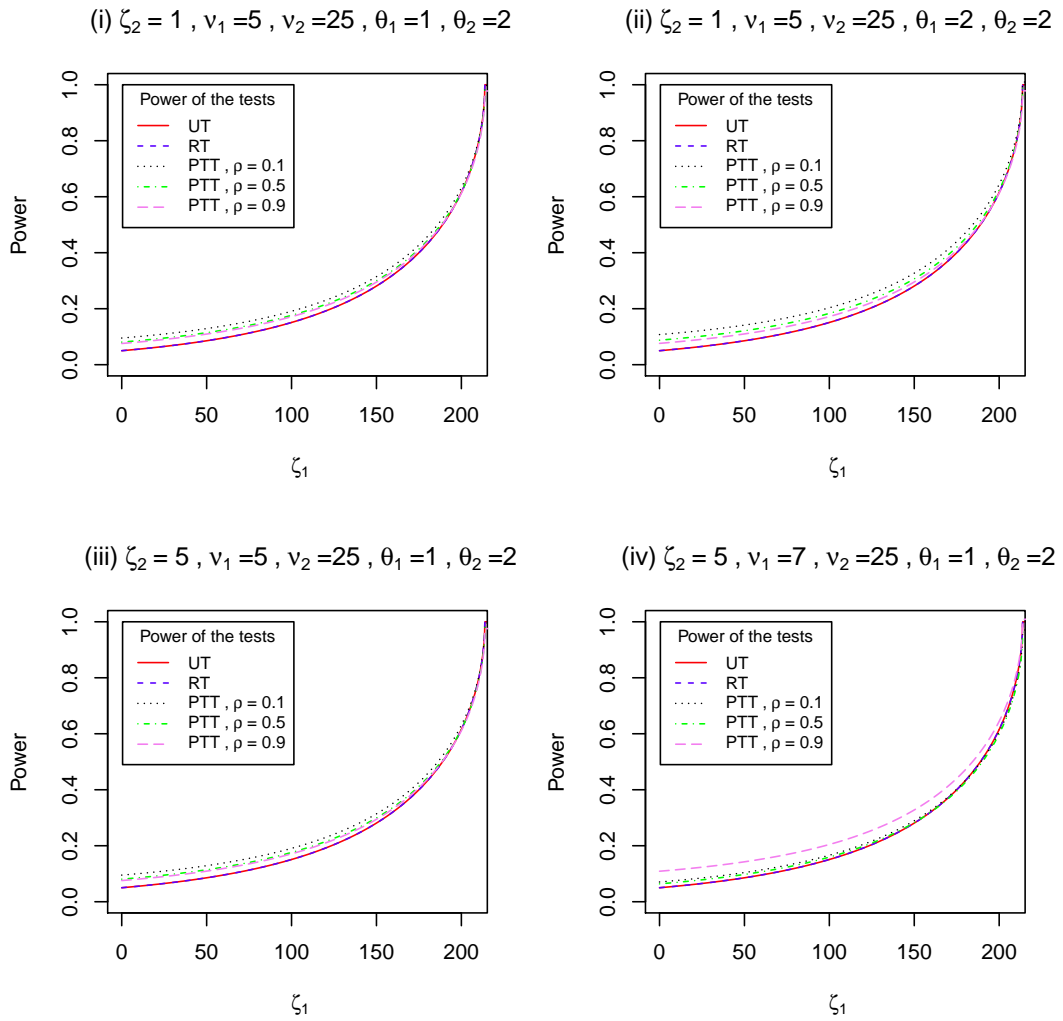
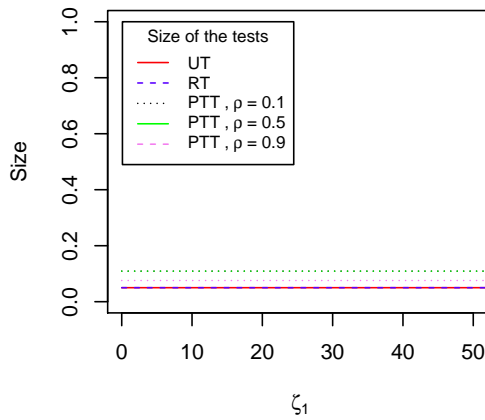
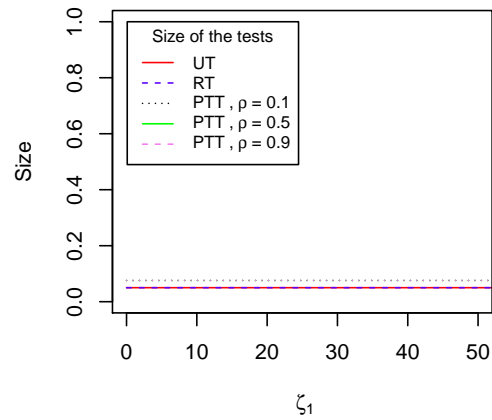


Figure 5.1: Power function of the UT, RT and PTT against ζ_1 for some selected ρ , ζ_2 , degrees of freedom and noncentrality parameters.

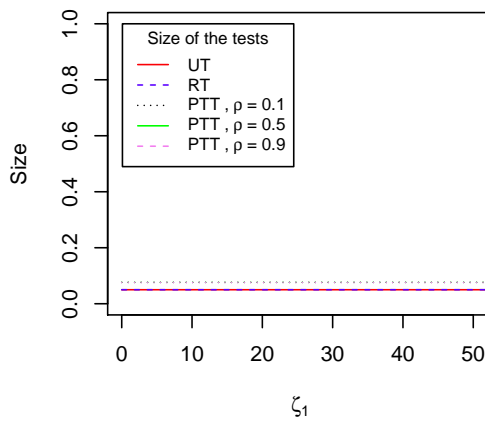
(iii) $\zeta_2 = 1, v_1 = 5, v_2 = 25, \theta_1 = 1, \theta_2 = 2$



(iii) $\zeta_2 = 1, v_1 = 5, v_2 = 25, \theta_1 = 2, \theta_2 = 2$



(iii) $\zeta_2 = 5, v_1 = 5, v_2 = 25, \theta_1 = 1, \theta_2 = 2$



(iv) $\zeta_2 = 5, v_1 = 7, v_2 = 25, \theta_1 = 1, \theta_2 = 2$

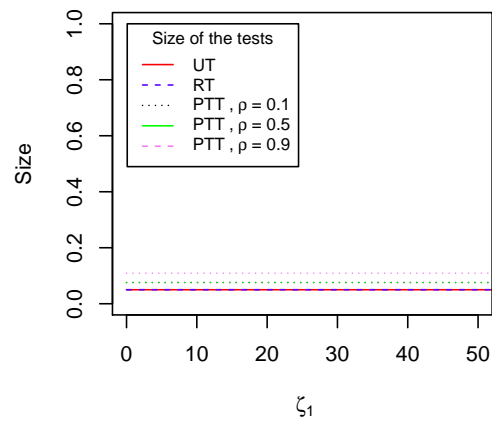


Figure 5.2: Size of the UT, RT and PTT against ζ_1 for some selected ρ, ζ_2 , degrees of freedom and noncentrality parameters.

5.6 Comparison of the Tests

From Figure 5.1, as well as from Equation (5.4.1), it is evident that the power of the UT does not depend on ζ_2 and ρ but it slowly increases as the value of ζ_1 increases. The form of the power curve of the UT is concave. Starting from a very small value near zero, it slowly increase when ζ_1 grows larger. The minimum power of the UT is around 0.05 for $\zeta_1 = 0$ and any values of ζ_2 .

Like the power of the UT, the power of the RT increases as the values of ζ_1 increase and reaches 1 for ζ_1 around 205 or above (see Figure 5.1). Moreover, the power of the RT is the same as (and/or larger than) that of the UT for all values of ζ_1 and/or ζ_2 . The minimum power of the RT is around 0.05 for $\zeta_1 = 0$ and all values of ζ_2 (see Figure 5.1). The maximum power of the RT is 1 for reasonably larger values of ζ_1 .

The power of the PTT depends on the values of ζ_1 , ζ_2 and ρ . Like the power of the RT, the power of the PTT increases as the values of ζ_1 and ζ_2 increase for $\rho = 0.9$. For $\zeta_2 = 5$ and $\rho = 0.9$, the power of the PTT increases as the ν_1 increase (see Figure 5.1(iv)), but not for $\rho = 0.1$ and $\rho = 0.5$. Moreover, the power of the PTT is always larger than that of the UT and tends to be the same as of the RT for ζ_1 around $150 < \zeta_1 < 200$ (see Figure 5.1(iv)) and/or above 200 (see Figure 5.1). The minimum power of the PTT is around 0.07 for $\zeta_1 = 0$, $\zeta_2 = 5$, and $\rho = 0.1, 0.5$ (see Figure 5.1(iv)), and it decreases (close to RT) as the value of ζ_2 and ν_1 becomes larger.

Figure 5.2 or Equation (5.4.6) shows that the size of the UT does not depend on ζ_2 . It is constant and remains unchanged for all values of ζ_1 and ζ_2 . Like the size of the UT, the size of the RT is also constant for all values of ζ_1 and ζ_2 . Moreover, the size of the RT is the same as (or larger) than that of the UT for all values of ζ_2 and does not depend on ρ .

The size of the PTT is higher than that of the UT and RT for all values of ζ_1 and

ζ_2 for $\rho = 0.9$. It attains 0.5 (the same as that the size of the UT) for $\rho_2 = 0.5$. The size of the PTT increases as the values of ν_1 and ζ_2 increase for $\rho = 0.9$ (see Figures 5.2(iii) and 5.2(iv)). But it decreases as the values of θ_1 increases (see Figures 5.2(i) and 5.2(ii)).

The size of the UT is $\alpha^{UT} = 0.05$ for all values of ζ_1 and ζ_2 . The size of the RT, $\alpha^{RT} \geq \alpha^{UT}$ for all values of ζ_2 . The size of the PTT, $\alpha^{PTT} \geq \alpha^{RT}$ for all values of ζ_1 , ζ_2 and ρ .

5.7 Conclusion

These analyses suggest that the power of the RT is the same as (and/or larger than) that of the UT. The power of the PTT is also larger than that of the UT for all values of ζ_1 and ζ_2 and ρ . The size of the UT is smaller (and/or the same as) than that of the RT and PTT for all values of ζ_1 and ζ_2 . The RT demonstrates similar behavior with the UT.

For smaller values of ζ_1 (see Figure 5.1) the PTT has higher power than the UT and RT. But for larger values of ζ_1 the RT has higher (and/or the same as) power than the PTT and UT. Thus when the NSPI is reasonably accurate (that is ζ_1 is small) the PTT over performs the UT and RT with higher power.

Since the size of the RT is larger (and/or the same as) than UT, the PTT has larger size than the UT. So in terms of the size the UT is the best among the three tests. However, the UT performs the worst in terms of the power. Thus the PTT protects against the maximum size of the RT and assures higher power than the UT and RT, and hence a better choice, especially when the NSPI is reasonably accurate on the slope parameters.

Chapter 6

The Parallel Regression Model

6.1 Introduction

A parallelism problem can be described as a special case of two related regression lines on the same dependent and independent variable that come from two different categories of the respondents. If the independent data sets come from two random samples ($p = 2$), researchers often wish to model the regression lines to see if the two lines are parallel (i.e. the slopes of the two regression lines are equal) or whether the lines have the same intercept on the vertical-axis. To test the parallelism of the two regression equations, namely

$$y_{1j} = \theta_1 + \beta_1 x_{1j} + e_{1j} \text{ and } y_{2j} = \theta_2 + \beta_2 x_{2j} + e_{2j}, j=1,2,\dots, n_i,$$

for the two data sets: $\mathbf{y} = [\mathbf{y}'_1, \mathbf{y}'_2]'$ and $\mathbf{x} = [\mathbf{x}'_1, \mathbf{x}'_2]'$ where $\mathbf{y}_1 = [y_{11}, \dots, y_{1n_1}]'$, $\mathbf{y}_2 = [y_{21}, \dots, y_{2n_2}]'$, $\mathbf{x}_1 = [x_{11}, \dots, x_{1n_1}]'$ and $\mathbf{x}_2 = [x_{21}, \dots, x_{2n_2}]'$, we use an appropriate two-sample t test (test statistic) for testing $H_0 : \beta_1 = \beta_2$ (parallelism). This t statistic is given as

$$t = \frac{\tilde{\beta}_1 - \tilde{\beta}_2}{S_{(\tilde{\beta}_1 - \tilde{\beta}_2)}},$$

where $\tilde{\beta}_1$ and $\tilde{\beta}_2$ are estimate of the slopes β_1 and β_2 , respectively, and $S_{(\tilde{\beta}_1 - \tilde{\beta}_2)}$ is estimate of the standard error of the estimated difference between slopes (Kleinbaum et al., 2008, p. 223). The parallelism of the two regression equations above can be

expressed as a single model of matrix form, that is,

$$\mathbf{y} = \mathbf{X}\Phi + \mathbf{e},$$

where $\Phi = [\theta_1, \theta_2, \beta_1, \beta_2]'$, $\mathbf{X} = [\mathbf{X}_1, \mathbf{X}_2]'$ with $\mathbf{X}_1 = [1, 0, x_1, 0]'$ and $\mathbf{X}_2 = [0, 1, 0, x_2]'$ and $\mathbf{e} = [e_1, e_2]'$ (cf. Khan, 2006b). The matrix form of the intercept and slope parameters can be written, respectively, as $\boldsymbol{\theta} = [\theta_1, \theta_2]'$ and $\boldsymbol{\beta} = [\beta_1, \beta_2]'$.

In this model, p independent bivariate samples are considered such that $y_{ij} \sim N(\theta_i + \beta_i x_{ij}, \sigma^2)$ for $i = 1, \dots, p$ and $j = 1, \dots, n_i$. The parameters $\boldsymbol{\theta} = (\theta_1, \dots, \theta_p)'$ and $\boldsymbol{\beta} = (\beta_1, \dots, \beta_p)'$ are the intercept and slope vectors of the p lines. See Khan (2003, 2006b, 2008) for details on parallel regression models and analyses.

To explain the importance of testing the equality of the intercepts (parallelism) when the equality of slopes is uncertain, we consider the general form of the PRM of a set of p ($p > 1$) simple regression models as

$$\mathbf{Y}_i = \theta_i \mathbf{1}_{n_i} + \beta_i \mathbf{x}_{ij} + \mathbf{e}_{ij}, \quad i = 1, 2, \dots, p, \quad \text{and } j=1, 2, \dots, n_i, \quad (6.1.1)$$

where $\mathbf{Y}_i = (Y_{i1}, \dots, Y_{in_i})'$ is a vector of n_i observable random variables, $\mathbf{1}_{n_i} = (1, \dots, 1)'$ is an n_i -tuple of 1's, $\mathbf{x}_{ij} = (x_{i1}, \dots, x_{in_i})'$ is a vector of n_i independent variables, θ_i and β_i are unknown intercept and slope, respectively, and $\mathbf{e}_i = (e_{i1}, \dots, e_{in_i})'$ is the vector of errors which are mutually independent and identically distributed as normal variable, that is, $\mathbf{e}_i \sim N(\mathbf{0}, \sigma^2 \mathbf{I}_{n_i})$ where \mathbf{I}_{n_i} is the identity matrix of order n_i . Equation (6.1.1) represents p linear models with different intercept and slope parameters. If $\beta_1 = \dots = \beta_p = \beta$, then there are p parallel simple linear models if θ'_i s are different. Here, the parameters $\boldsymbol{\theta} = (\theta_1, \dots, \theta_p)'$ and $\boldsymbol{\beta} = (\beta_1, \dots, \beta_p)'$ are the intercept and slope vectors of the p lines.

In this chapter, we test the intercept vector $\boldsymbol{\theta} = (\theta_1, \dots, \theta_p)'$ while it is uncertain if the p -slope parameters are equal (parallel). Under three different scenarios of the slope parameters, we define three different tests: (i) for the UT, let ϕ^{UT} be the test

function and T^{UT} be the test statistic for testing $H_0 : \boldsymbol{\theta} = \boldsymbol{\theta}_0$ against $H_a : \boldsymbol{\theta} > \boldsymbol{\theta}_0$ when $\boldsymbol{\beta} = (\beta_1, \dots, \beta_p)'$ is unspecified, (ii) for the RT, let ϕ^{RT} be the test function and T^{RT} be the test statistic for testing $H_0 : \boldsymbol{\theta} = \boldsymbol{\theta}_0$ against $H_a : \boldsymbol{\theta} > \boldsymbol{\theta}_0$ when $\boldsymbol{\beta} = \beta_0 \mathbf{1}_p$ (fixed vector), and (iii) for the PTT, let ϕ^{PTT} be the test function and T^{PTT} be the test statistic for testing $H_0 : \boldsymbol{\theta} = \boldsymbol{\theta}_0$ against $H_a : \boldsymbol{\theta} > \boldsymbol{\theta}_0$ following a pre-test (PT) on the slope parameters. For the PT, let ϕ^{PT} be the test function for testing $H_0^* : \boldsymbol{\beta} = \beta_0 \mathbf{1}_p$ (a suspected constant) against $H_a^* : \boldsymbol{\beta} > \beta_0 \mathbf{1}_p$. If H_0^* is rejected in the PT, then the UT is used to test the intercept, otherwise the RT is used. Thus, the PTT depends on the PT, and it is a choice between the UT and RT.

The unrestricted maximum likelihood estimator or least square estimator of intercept and slope vectors, $\boldsymbol{\theta} = (\theta_1, \dots, \theta_p)'$ and $\boldsymbol{\beta} = (\beta_1, \dots, \beta_p)'$, are given as

$$\tilde{\boldsymbol{\theta}} = \bar{\mathbf{Y}} - \mathbf{T}\tilde{\boldsymbol{\beta}} \text{ and } \tilde{\boldsymbol{\beta}} = \frac{(\mathbf{x}'_i \mathbf{y}_i) - (\frac{1}{n_i})[\mathbf{1}'_i \mathbf{x}_i \mathbf{1}'_i \mathbf{y}_i]}{n_i Q_i}, \quad (6.1.2)$$

where $\tilde{\boldsymbol{\theta}} = (\tilde{\theta}_1, \dots, \tilde{\theta}_p)'$, $\tilde{\boldsymbol{\beta}} = (\tilde{\beta}_1, \dots, \tilde{\beta}_p)'$, $\mathbf{T} = \text{Diag}(\bar{x}_1, \dots, \bar{x}_p)$, $n_i Q_i = \mathbf{x}'_i \mathbf{x}_i - (\frac{1}{n_i})[\mathbf{1}'_i \mathbf{x}_i]$ and $\tilde{\theta}_i = \bar{Y}_i - \tilde{\beta}_i \bar{x}_i$ for $i = 1, \dots, p$.

Furthermore, the likelihood ratio (LR) test statistics for testing $H_0 : \boldsymbol{\theta} = \boldsymbol{\theta}_0$ against $H_a : \boldsymbol{\theta} > \boldsymbol{\theta}_0$ is given by

$$F = \frac{\tilde{\boldsymbol{\theta}}' \mathbf{H}' \mathbf{D}_{22}^{-1} \mathbf{H} \tilde{\boldsymbol{\theta}}}{(p-1)s_e^2}, \quad (6.1.3)$$

where $\mathbf{H} = \mathbf{I}_p - \frac{1}{nQ} \mathbf{1}_p \mathbf{1}'_p \mathbf{D}_{22}^{-1}$, $\mathbf{D}_{22}^{-1} = \text{Diag}(n_1 Q_1, \dots, n_p Q_p)$, $nQ = \sum_{i=1}^p n_i Q_i$, $n_i Q_i = \mathbf{x}'_i \mathbf{x}_i - \frac{1}{n_i} (\mathbf{1}'_i \mathbf{x}_i)^2$ and $s_e^2 = (n-2p)^{-1} \sum_{i=1}^p (\mathbf{Y} - \tilde{\boldsymbol{\theta}}_i \mathbf{1}_{n_i} - \tilde{\boldsymbol{\beta}} \mathbf{x}_i)' (\mathbf{Y} - \tilde{\boldsymbol{\theta}}_i \mathbf{1}_{n_i} - \tilde{\boldsymbol{\beta}} \mathbf{x}_i)$ (Saleh, 2006, p. 14-15). Under H_0 , F follows a central F distribution with $(p-1, n-2p)$ degrees of freedom, and under H_a it follows a noncentral F distribution with $(p-1, n-2p)$ degrees of freedom and noncentrality parameter $\Delta^2/2$, where

$$\begin{aligned} \Delta^2 &= \frac{\boldsymbol{\theta}' \mathbf{H}' \mathbf{D}_{22}^{-1} \mathbf{H} \boldsymbol{\theta}}{\sigma^2} = \frac{(\boldsymbol{\theta} - \boldsymbol{\theta}_0)' \mathbf{H}' \mathbf{D}_{22}^{-1} \mathbf{H} (\boldsymbol{\theta} - \boldsymbol{\theta}_0)}{\sigma^2} \\ &= \frac{(\boldsymbol{\theta} - \boldsymbol{\theta}_0)' \mathbf{D}_{22} (\boldsymbol{\theta} - \boldsymbol{\theta}_0)}{\sigma^2} \end{aligned} \quad (6.1.4)$$

and $\mathbf{D}_{22} = \mathbf{H}'\mathbf{D}_{22}^{-1}\mathbf{H}$. When the slopes ($\boldsymbol{\beta}$) are equal to $\beta_0\mathbf{1}_p$ (specified), the restricted maximum likelihood estimator of intercept and slope vectors are given as

$$\hat{\boldsymbol{\theta}} = \tilde{\boldsymbol{\theta}} + \mathbf{T}\mathbf{H}\tilde{\boldsymbol{\beta}} \text{ and } \hat{\boldsymbol{\beta}} = \frac{\mathbf{1}_k\mathbf{1}_k'\mathbf{D}_{22}^{-1}\tilde{\boldsymbol{\beta}}}{n\mathbf{Q}}. \quad (6.1.5)$$

The proposed tests are obtained in Section 6.2. Section 6.3 derives the sampling distribution of the test statistics. The power function and size of the tests are obtained in Section 6.4. An illustrative example is given in Section 6.5. The comparison of the power of the tests and conclusion are provided in Sections 6.6 and 6.7.

6.2 The Proposed Tests

To test the equality of the intercepts when the equality of slopes is suspected, we consider three different scenarios of the slopes. The test statistics of the UT, RT and PTT are then defined as follows.

- (i) For $\boldsymbol{\beta}$ unspecified, the test statistic of the UT for testing $H_0 : \boldsymbol{\theta} = \boldsymbol{\theta}_0$ against $H_a : \boldsymbol{\theta} > \boldsymbol{\theta}_0$, under H_0 , is given by

$$T^{UT} = \frac{\tilde{\boldsymbol{\theta}}'\mathbf{H}'\mathbf{D}_{22}^{-1}\mathbf{H}\tilde{\boldsymbol{\theta}}}{(p-1)s_{ut}^2}, \quad (6.2.1)$$

where

$$s_{ut}^2 = (n-2p)^{-1} \sum_{i=1}^p (\mathbf{Y} - \tilde{\boldsymbol{\theta}}_i\mathbf{1}_{n_i} - \tilde{\boldsymbol{\beta}}\mathbf{x}_i)'(\mathbf{Y} - \tilde{\boldsymbol{\theta}}_i\mathbf{1}_{n_i} - \tilde{\boldsymbol{\beta}}\mathbf{x}_i).$$

The T^{UT} follows a central F distribution with $(p-1, n-2p)$ degrees of freedom. Under H_a , it follows a noncentral F distribution with $(p-1, n-2p)$ degrees of freedom and noncentrality parameter $\boldsymbol{\Delta}^2/2$. Under normal model we have

$$\begin{pmatrix} \tilde{\boldsymbol{\theta}} - \boldsymbol{\theta} \\ \tilde{\boldsymbol{\beta}} - \boldsymbol{\beta} \end{pmatrix} \sim N_{2p} \left[\begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}, \sigma^2 \begin{pmatrix} \mathbf{D}_{11} & -\mathbf{T}\mathbf{D}_{22} \\ -\mathbf{T}\mathbf{D}_{22} & \mathbf{D}_{22} \end{pmatrix} \right], \quad (6.2.2)$$

where $\mathbf{D}_{11} = \mathbf{N}^{-1} + \mathbf{T}\mathbf{D}_{22}\mathbf{T}\boldsymbol{\beta}$ and $\mathbf{N} = \text{Diag}(n_1, \dots, n_p)$.

- (ii) When the slope is specified to be $\boldsymbol{\beta} = \beta_0 \mathbf{1}_p$ (fixed vector), the test statistic of the RT for testing $H_0 : \boldsymbol{\theta} = \boldsymbol{\theta}_0$ against $H_a : \boldsymbol{\theta} > \boldsymbol{\theta}_0$ under H_0 , is given by

$$T^{RT} = \frac{(\widehat{\boldsymbol{\theta}}' \mathbf{H}' \mathbf{D}_{22}^{-1} \mathbf{H} \widehat{\boldsymbol{\theta}}) + (\widetilde{\boldsymbol{\beta}}' \mathbf{H}' \mathbf{D}_{22}^{-1} \mathbf{H} \widetilde{\boldsymbol{\beta}})}{(p-1)s_{rt}^2}, \quad (6.2.3)$$

where

$$s_{rt}^2 = (n-p)^{-1} \sum_{i=1}^p (\mathbf{Y} - \widehat{\boldsymbol{\theta}}_i \mathbf{1}_{n_i} - \widehat{\boldsymbol{\beta}} \mathbf{x}_i)' (\mathbf{Y} - \widehat{\boldsymbol{\theta}}_i \mathbf{1}_{n_i} - \widehat{\boldsymbol{\beta}} \mathbf{x}_i) \text{ and } \widehat{\boldsymbol{\beta}} = \beta_0 \mathbf{1}_p.$$

The T^{RT} follows a central F distribution with $(p-1, n-2p)$ degrees of freedom. Under H_a , it follows a noncentral F distribution with $(p-1, n-2p)$ degrees of freedom and noncentrality parameter $\boldsymbol{\Delta}^2/2$. Again, note that

$$\begin{pmatrix} \widehat{\boldsymbol{\theta}} - \boldsymbol{\theta} \\ \widehat{\boldsymbol{\beta}} - \boldsymbol{\beta} \end{pmatrix} \sim N_{2p} \left[\begin{pmatrix} \mathbf{T} \mathbf{H} \boldsymbol{\beta} \\ \mathbf{0} \end{pmatrix}, \sigma^2 \begin{pmatrix} \mathbf{D}_{11}^* & \mathbf{D}_{12}^* \\ \mathbf{D}_{12}^* & \mathbf{D}_{22}^* \end{pmatrix} \right], \quad (6.2.4)$$

where $\mathbf{D}_{11}^* = \mathbf{N}^{-1} + \frac{\mathbf{T} \mathbf{1}_p \mathbf{1}_p' \mathbf{T}}{nQ}$ and $\mathbf{D}_{12}^* = -\frac{1}{nQ} \mathbf{1}_p \mathbf{1}_p' \mathbf{T}$.

- (iii) When the value of the slope is suspected to be $\boldsymbol{\beta} = \beta_0 \mathbf{1}_p$ but unsure, a pre-test on the slope is required before testing the intercept. For the preliminary test (PT) of $H_0^* : \boldsymbol{\beta} = \beta_0 \mathbf{1}_p$ against $H_a^* : \boldsymbol{\beta} > \beta_0 \mathbf{1}_p$, the test statistic under the null hypothesis is defined as

$$T^{PT} = \frac{\widetilde{\boldsymbol{\beta}}' \mathbf{H}' \mathbf{D}_{22}^{-1} \mathbf{H} \widetilde{\boldsymbol{\beta}}}{(p-1)s_{ut}^2}, \quad (6.2.5)$$

which follows a central F distribution with $(p-1, n-2p)$ degrees of freedom. Under H_a , it follows a noncentral F distribution with $(p-1, n-2p)$ degrees of freedom and noncentrality parameter $\boldsymbol{\Delta}^2/2$. Again, note that

$$\begin{pmatrix} \widetilde{\boldsymbol{\theta}} - \beta_0 \mathbf{1}_p \\ \widetilde{\boldsymbol{\beta}} - \widehat{\boldsymbol{\beta}} \end{pmatrix} \sim N_{2p} \left[\begin{pmatrix} (\widetilde{\boldsymbol{\beta}}^* - \beta_0) \mathbf{1}_p \\ \mathbf{H} \boldsymbol{\beta} \end{pmatrix}, \sigma^2 \begin{pmatrix} \mathbf{1}_p \mathbf{1}_p' / nQ & \mathbf{0} \\ \mathbf{0} & \mathbf{H} \mathbf{D}_{22} \end{pmatrix} \right] \quad (6.2.6)$$

where $\widetilde{\boldsymbol{\beta}}^* \mathbf{1}_p = \frac{\mathbf{1}_p \mathbf{1}_p' \mathbf{D}_{22}^{-1} \boldsymbol{\beta}}{nQ}$ (Saleh, 2006, p. 273).

Let us choose a positive number α_j ($0 < \alpha_j < 1$, for $j = 1, 2, 3$) and real value $F_{\nu_1, \nu_2, \alpha_j}$ (ν_1 be numerator d.f. and ν_2 be denominator d.f.) such that

$$P(T^{UT} > F_{p-1, n-2p, \alpha_1} \mid \boldsymbol{\theta} = \boldsymbol{\theta}_0) = \alpha_1, \quad (6.2.7)$$

$$P(T^{RT} > F_{p-1, n-2p, \alpha_2} \mid \boldsymbol{\theta} = \boldsymbol{\theta}_0) = \alpha_2, \quad (6.2.8)$$

$$P(T^{PT} > F_{p-1, n-2p, \alpha_3} \mid \boldsymbol{\beta} = \beta_0 \mathbf{1}_p) = \alpha_3. \quad (6.2.9)$$

Now the test function for testing $H_0 : \boldsymbol{\theta} = \boldsymbol{\theta}_0$ against $H_a : \boldsymbol{\theta} > \boldsymbol{\theta}_0$, after pre-testing on H_0^* , is defined by

$$\Phi = \begin{cases} 1, & \text{if } (T^{PT} \leq F_c, T^{RT} > F_b) \text{ or } (T^{PT} > F_c, T^{UT} > F_a); \\ 0, & \text{otherwise,} \end{cases} \quad (6.2.10)$$

where $F_a = F_{\alpha_1, p-1, n-2p}$, $F_b = F_{\alpha_2, p-1, n-2p}$ and $F_c = F_{\alpha_3, p-1, n-2p}$.

6.3 Sampling Distribution of Test Statistics

To derive the power function of the UT, RT and PTT, the sampling distribution of the test statistics proposed in Section 6.2 are required. For the power function of the PTT the joint distribution of (T^{UT}, T^{PT}) and (T^{RT}, T^{PT}) is essential. Let $\{N_n\}$ be a sequence of alternative hypotheses defined as

$$N_n : (\boldsymbol{\theta} - \boldsymbol{\theta}_0, \boldsymbol{\beta} - \beta_0 \mathbf{1}_p) = \left(\frac{\boldsymbol{\lambda}_1}{\sqrt{n}}, \frac{\boldsymbol{\lambda}_2}{\sqrt{n}} \right) = \boldsymbol{\lambda}, \quad (6.3.1)$$

where $\boldsymbol{\lambda}$ is a vector of fixed real numbers and $\boldsymbol{\theta}$ is the true value of the intercept. Under N_n the value of $(\boldsymbol{\theta} - \boldsymbol{\theta}_0)$ is greater than zero and under H_0 the value of $(\boldsymbol{\theta} - \boldsymbol{\theta}_0)$ is equal zero.

Following Yunus and Khan (2011b) and Equation (6.2.1), we define the test statistic of the UT when $\boldsymbol{\beta}$ is unspecified, under N_n , as

$$T_1^{UT} = T^{UT} - n \left\{ \frac{(\boldsymbol{\theta} - \boldsymbol{\theta}_0)' \mathbf{H}' \mathbf{D}_{22}^{-1} \mathbf{H} (\boldsymbol{\theta} - \boldsymbol{\theta}_0)}{(p-1)s_{ut}^2} \right\}. \quad (6.3.2)$$

The T_1^{UT} follows a noncentral F distribution with noncentrality parameter which is a function of $(\boldsymbol{\theta} - \boldsymbol{\theta}_0)$ and $(p - 1, n - 2p)$ degrees of freedom, under N_n .

From Equation (6.2.3) under N_n , $(\boldsymbol{\theta} - \boldsymbol{\theta}_0) > 0$ and $(\boldsymbol{\beta} - \beta_0 \mathbf{1}_p) > 0$, the test statistic of the RT becomes

$$T_2^{RT} = T^{RT} - n \left\{ \frac{(\boldsymbol{\theta} - \boldsymbol{\theta}_0)' \mathbf{H}' \mathbf{D}_{22}^{-1} \mathbf{H} (\boldsymbol{\theta} - \boldsymbol{\theta}_0) + (\boldsymbol{\beta} - \beta_0 \mathbf{1}_p)' \mathbf{H}' \mathbf{D}_{22}^{-1} \mathbf{H} (\boldsymbol{\beta} - \beta_0 \mathbf{1}_p)}{(p - 1) s_{rt}^2} \right\}. \quad (6.3.3)$$

The T_2^{RT} also follows a noncentral F distribution with a noncentrality parameter which is a function of $(\boldsymbol{\theta} - \boldsymbol{\theta}_0)$ and $(p - 1, n - 2p)$ degrees of freedom, under N_n . Similarly, from the Equation (6.2.5) the test statistic of the PT is given by

$$T_3^{PT} = T^{PT} - n \left\{ \frac{(\boldsymbol{\beta} - \beta_0 \mathbf{1}_p)' \mathbf{H}' \mathbf{D}_{22}^{-1} \mathbf{H} (\boldsymbol{\beta} - \beta_0 \mathbf{1}_p)}{(p - 1) s_{ut}^2} \right\}. \quad (6.3.4)$$

Under H_a , the T_3^{PT} follows a noncentral F distribution with a noncentrality parameter which is a function of $(\boldsymbol{\beta} - \beta_0 \mathbf{1}_p)$ and $(p - 1, n - 2p)$ degrees of freedom.

From Equations (6.2.1), (6.2.3) and (6.2.5) the T^{UT} and T^{PT} are correlated, and the T^{RT} and T^{PT} are uncorrelated. The joint distribution of the T^{UT} and T^{PT} , that is,

$$\begin{pmatrix} T^{UT} \\ T^{PT} \end{pmatrix}, \quad (6.3.5)$$

is a correlated bivariate F distribution with $(p - 1, n - 2p)$ degrees of freedom. The covariance and correlation of the T^{UT} with $(p - 1)$ and $(n - 2p)$ degrees of freedom and T^{PT} with $(p - 1)$ and $(n - 2p)$ degrees of freedom are then given as

$$\begin{aligned} \text{Cov}(T_1^{UT}, T_3^{PT}) &= \frac{2(n - 2p)(n - 2p)}{[(n - 2p) - 2][(n - 2p) - 2][(n - 2p) - 4]} \\ &= \frac{2(n^2 - 4np + 4p^2)}{(n - 2p - 2)^2(n - 2p - 4)}, \text{ and} \end{aligned} \quad (6.3.6)$$

$$\begin{aligned} \rho_{T_1^{UT} T_3^{PT}}^2 &= \frac{(n - p)(n - p)[(n - 2p) - 4]}{[(n - 2p) + (n - p) - 2][(n - 2p) + (n - p) - 2][(n - 2p) - 4]} \\ &= \frac{(n^2 - 2np + p^2)(n - 2p - 4)}{(2n - 3p - 2)^2(n - 2p - 4)}. \end{aligned} \quad (6.3.7)$$

6.4 Power Function and Size of Tests

The power function of the UT, RT and PTT are derived below. From Equation (6.2.1) and (6.3.2), (6.2.3) and (6.3.3), and (6.2.5), (6.2.10) and (6.3.4), the power function of the UT, RT and PTT are given, respectively, as:

(i) the power of the UT

$$\begin{aligned}
 \pi^{UT}(\boldsymbol{\lambda}) &= P(T^{UT} > F_{\alpha_1, p-1, n-2p} \mid N_n) \\
 &= 1 - P\left(T_1^{UT} \leq F_{\alpha_1, p-1, n-2p} - \frac{\boldsymbol{\lambda}'_1 \mathbf{H}' \mathbf{D}_{22}^{-1} \mathbf{H} \boldsymbol{\lambda}_1}{(p-1)s_{ut}^2}\right) \\
 &= 1 - P\left(T_1^{UT} \leq F_{\alpha_1, p-1, n-2p} - \frac{\boldsymbol{\lambda}'_1 \mathbf{D}_{22} \boldsymbol{\lambda}_1}{(p-1)s_{ut}^2}\right) \\
 &= 1 - P(T_1^{UT} \leq F_{\alpha_1, p-1, n-2p} - k_{ut}\delta_1), \tag{6.4.1}
 \end{aligned}$$

where $\delta_1 = \boldsymbol{\lambda}'_1 \mathbf{D}_{22} \boldsymbol{\lambda}_1$ and $k_{ut} = \frac{1}{(p-1)s_{ut}^2}$.

(ii) the power of the RT

$$\begin{aligned}
 \pi^{RT}(\boldsymbol{\lambda}) &= P(T^{RT} > F_{\alpha_1, n-1, n-2p} \mid N_n) \\
 &= P\left(T_2^{RT} > F_{\alpha_2, n-1, n-2p} - \frac{(\boldsymbol{\theta} - \boldsymbol{\theta}_0)' \mathbf{H}' \mathbf{D}_{22}^{-1} \mathbf{H} (\boldsymbol{\theta} - \boldsymbol{\theta}_0)}{(p-1)s_{rt}^2}\right) \\
 &= 1 - P\left(T_2^{RT} \leq F_{\alpha_2, p-1, n-2p} - \frac{(\boldsymbol{\lambda}'_1 \mathbf{H}' \mathbf{D}_{22}^{-1} \mathbf{H} \boldsymbol{\lambda}_1) + (\boldsymbol{\lambda}'_2 \mathbf{H}' \mathbf{D}_{22}^{-1} \mathbf{H} \boldsymbol{\lambda}_2)}{(p-1)s_{rt}^2}\right) \\
 &= 1 - P(T_1^{RT} \leq F_{\alpha_1, p-1, n-2p} - k_{rt}(\delta_1 + \delta_2)), \tag{6.4.2}
 \end{aligned}$$

where $\delta_2 = \boldsymbol{\lambda}'_2 \mathbf{D}_{22} \boldsymbol{\lambda}_2$ and $k_{rt} = \frac{1}{(p-1)s_{rt}^2}$.

The power function of the PT is

$$\begin{aligned}
 \pi^{PT}(\boldsymbol{\lambda}) &= P(T^{PT} > F_{\alpha_3, p-1, n-2p} \mid K_n) \\
 &= 1 - P\left(T_3^{PT} \leq F_{\alpha_3, p-1, n-2p} - \frac{\boldsymbol{\lambda}'_2 \mathbf{H}' \mathbf{D}_{22}^{-1} \mathbf{H} \boldsymbol{\lambda}_2}{(p-1)s_{ut}^2}\right) \\
 &= 1 - P(T_3^{PT} \leq F_{\alpha_3, p-1, n-2p} - k_{ut}\delta_2). \tag{6.4.3}
 \end{aligned}$$

(iii) the power of the PTT

$$\begin{aligned}
\pi^{PTT}(\boldsymbol{\lambda}) &= P(T^{PT} < F_{\alpha_3, p-1, n-2p}, T^{RT} > F_{\alpha_2, p-1, n-2p}) \\
&\quad + P(T^{PT} \geq F_{\alpha_3, p-1, n-2p}, T^{UT} > F_{\alpha_1, p-1, n-2p}) \\
&= (1 - \pi^{PT})\pi^{RT} + d_{1r}(a, b), \tag{6.4.4}
\end{aligned}$$

where $d_{1r}(a, b)$ is bivariate F probability integrals, and it is defined as

$$\begin{aligned}
d_{1r}(a, b) &= \int_a^\infty \int_b^\infty f(F^{PT}, F^{UT}) dF^{PT} dF^{UT} \\
&= 1 - \int_0^a \int_0^b f(F^{PT}, F^{UT}) dF^{PT} dF^{UT}, \tag{6.4.5}
\end{aligned}$$

in which

$$\begin{aligned}
a &= F_{\alpha_3, p-1, n-2p} - \frac{\boldsymbol{\lambda}'_2 \mathbf{H}' \mathbf{D}_{22}^{-1} \mathbf{H} \boldsymbol{\lambda}_2}{(p-1)s_{ut}^2} = F_{\alpha_3, p-1, n-2p} - k_{ut}\delta_2, \text{ and} \\
b &= F_{\alpha_1, p-1, n-2p} - \frac{(\boldsymbol{\theta} - \boldsymbol{\theta}_0)' \mathbf{H}' \mathbf{D}_{22}^{-1} \mathbf{H} (\boldsymbol{\theta} - \boldsymbol{\theta}_0)}{(p-1)s_{ut}^2} = F_{\alpha_1, p-1, n-2p} - k_{ut}\delta_1.
\end{aligned}$$

The value of $\int_0^a \int_0^b f(F^{PT}, F^{UT}) dF^{PT} dF^{UT}$ is evaluated using the cdf of the BNCF distribution in Equation (2.3.18). We then use it in the calculation of the power and size of the PTT.

Furthermore, the size of the UT, RT and PTT are given, respectively, as:

(i) the size of the UT

$$\begin{aligned}
\alpha^{UT} &= P(T^{UT} > F_{\alpha_1, p-1, n-2p} \mid H_0 : \boldsymbol{\theta} = \boldsymbol{\theta}_0) \\
&= 1 - P(T^{UT} \leq F_{\alpha_1, p-1, n-2p} \mid H_0 : \boldsymbol{\theta} = \boldsymbol{\theta}_0) \\
&= 1 - P(T_1^{UT} \leq F_{\alpha_1, p-1, n-2p}), \tag{6.4.6}
\end{aligned}$$

(ii) the size of the RT

$$\begin{aligned}
\alpha^{RT} &= P(T^{RT} > F_{\alpha_2, p-1, n-2p} \mid H_0 : \boldsymbol{\theta} = \boldsymbol{\theta}_0) \\
&= 1 - P(T^{RT} \leq F_{\alpha_2, p-1, n-2p} \mid H_0 : \boldsymbol{\theta} = \boldsymbol{\theta}_0) \\
&= 1 - P(T_2^{RT} \leq F_{\alpha_2, p-1, n-2p} - k_{rt}\delta_2). \tag{6.4.7}
\end{aligned}$$

The size of the PT is given by

$$\begin{aligned}\alpha^{PT}(\boldsymbol{\lambda}) &= P(T^{PT} > F_{\alpha_3, p-1, n-2p} | H_0) \\ &= 1 - P(T_3^{PT} \leq F_{\alpha_3, p-1, n-2p}).\end{aligned}\quad (6.4.8)$$

(iii) the size of the PTT

$$\begin{aligned}\alpha^{PTT} &= P(T^{PT} \leq a, T^{RT} > d | H_0) + P(T^{PT} > a, T^{UT} > h | H_0) \\ &= P(T^{PT} < F_{\alpha_3, p-1, n-2p}) P(T^{RT} > F_{\alpha_2, p-1, n-2p}) + d_{1r}(a, h) \\ &= (1 - \alpha^{PT})\alpha^{RT} + d_{1r}(a, h),\end{aligned}\quad (6.4.9)$$

where $h = F_{\alpha_1, p-1, n-2p}$.

6.5 A Simulation Example

To study the properties of the tests we conduct a simulation study. The random data were generated using the R package. For $p = 3$, each of the three independent variables $(x_{ij}, i = 1, 2, 3, j = 1, \dots, n_i)$ are generated from the uniform distribution between 0 and 1. The errors $(e_i, i = 1, 2, 3)$ are generated from the normal distribution with $\mu = 0$ and $\sigma^2 = 1$. The dependent variable (y_{1j}) is determined by $y_{1j} = \theta_{01} + \beta_{11}x_{1j} + e_1$ for $\theta_{01} = 3$ and $\beta_{11} = 2$. Similarly, define $y_{2j} = \theta_{02} + \beta_{12}x_{2j} + e_2$ for $\theta_{02} = 3.6$ and $\beta_{12} = 2$; and $y_{3j} = \theta_{03} + \beta_{13}x_{3j} + e_3$ for $\theta_{03} = 4$ and $\beta_{13} = 2$. For each of three cases $n_i = n = 100$ random variates were generated. For the computation of the power function of the tests we set $\alpha_1 = \alpha_2 = \alpha_3 = \alpha = 0.05$. The graphs for the power function of the three tests are produced using the formulas in Equations (6.4.1), (6.4.2) and (6.4.4). The graphs for the size of the three tests are produced using Equations (6.4.6), (6.4.7) and (6.4.9). The graphs of the power and size curves of the tests are presented in Figures 6.1 and 6.2. The relevant R codes for producing Figures 6.1 and 6.2 are attached in Appendix A.14 and A.15.

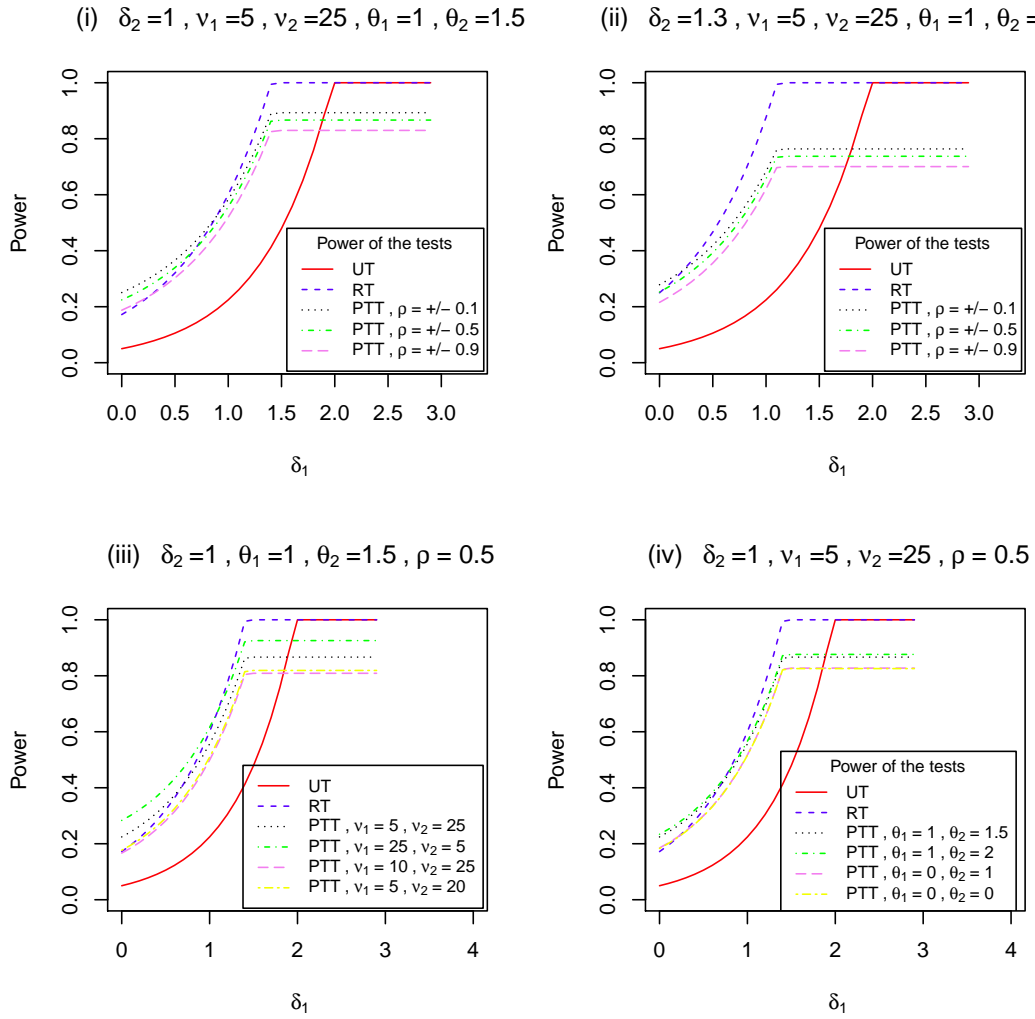


Figure 6.1: Power functions of the UT, RT and PTT against δ_1 for some selected ρ , degrees of freedom and noncentrality parameters.

6.6 Comparison of the Tests

From Figure 6.1, as well as from Equation (6.4.1), it is evident that the power of the UT does not depend on δ_2 and ρ , but it increases as the value of δ_1 increases. The form of the power curve of the UT is concave, starting from a very small value of near zero (when δ_1 is also near 0), it approaches 1 as δ_1 grows larger. The power of the UT increases rapidly as the value of δ_1 becomes larger. The minimum power of the UT is approximately 0.05 for $\delta_1 = 0$.

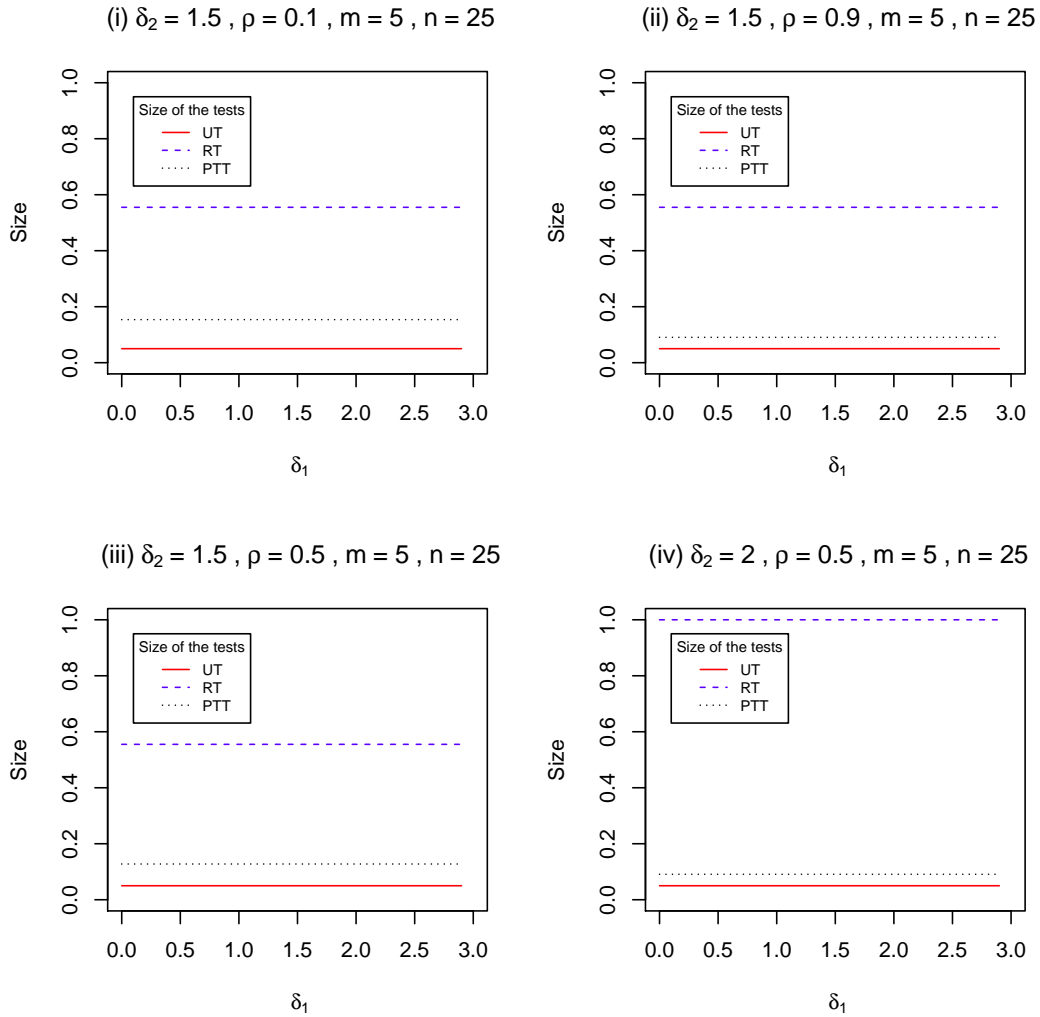


Figure 6.2: Size of the UT, RT and PTT against δ_1 for some selected ρ and δ_2 .

The shape of the power curve of the RT is also concave for all values of δ_1 and δ_2 . The power of the RT increases as the values of δ_1 and/or δ_2 increase (see Figures 6.1(i) and 6.1(ii), and Equation (6.4.2)). Moreover, the power of the RT is always larger than that of the UT for all values of δ_1 and/or δ_2 . The minimum power of the RT is approximately 0.2 for $\delta_1 = 0$ and $\delta_2 = 0$. The maximum power of the RT is 1 for reasonably larger values of δ_1 . The power of the RT reaches 1 much faster than that of the UT as δ_1 increases.

The power of the PTT depends on the values of δ_1 , δ_2 and ρ (see Figure 6.1 and

Equation (6.4.4)). Like the power of the RT, the power of the PTT increases as the values of δ_1 increase. Moreover, the power of the PTT is always larger than that of the UT and RT for the values of δ_1 around 0.7 to 1.5 (for given simulated data). The minimum power of the PTT is around 0.18 for $\delta_1 = 0$ (see Figure 6.1(i)), and it decreases as the value of δ_2 becomes larger. The gap between the power curves of the RT and PTT is much less than that between the UT and RT and/or UT and PTT. The power curve of the PTT tends to lie between the power curves of the UT and RT. However, the power of the PTT is identical for fixed value of ρ , regardless of its sign.

The powers of the UT, RT and PTT for the PRM demonstrate similar behavior to that of the MSRM.

Figure 6.2 and Equation (6.4.6) show that the size of the UT does not depend on δ_2 . It is a constant and remains unchanged for all values of δ_1 and δ_2 . The size of the RT increases as the value of δ_2 increases (see Equation (6.4.7)). Moreover, the size of the RT is always larger than that of the UT, but not for PTT for the smaller values of the δ_1 (not far from zero). The size of the UT and RT do not depend on ρ .

The size of the PTT is closer to that of the UT for $\delta_2 > 2$. The difference (or gap) between the size of the RT and PTT increases significantly as the value of δ_2 and ρ increases. The size of the UT is $\alpha^{UT} = 0.05$ for all values of δ_1 and δ_2 . For all values of δ_1 and δ_2 , the size of the RT is larger than that of the UT, $\alpha^{RT} > \alpha^{UT}$. For all the values of ρ , $\alpha^{PTT} \leq \alpha^{RT}$. Thus, the size of the RT is always larger than that of the UT and PTT.

6.7 Conclusion

Based on the analyses in this chapter the power of the RT is always higher than that of the UT for all values of δ_1 and δ_2 . Also, the power of the PTT is always larger than

that of the UT for all values δ_1 (see the curves on interval values of $0.7 < \delta_1 < 1.5$ for given simulated data), δ_2 and ρ .

For smaller values of δ_1 (see Figure 6.1) the PTT has higher power than the UT and RT. But for larger values of δ_1 the RT has higher power than the PTT and UT. Thus when the NSPI is reasonably accurate (that is δ_1 is small) the PTT overperforms the UT and RT with higher power.

Since the size of the RT is the highest, and the PTT has larger size than UT, in terms of the size the UT is the best among the three tests. However, the UT performs the worst in terms of the power. Thus the PTT ensures higher power than the UT and lower size than the RT, and hence a better choice, especially when the NSPI is reasonably accurate on the slope parameters.

Chapter 7

Discussions, Conclusions and Future Research

7.1 Discussions and Conclusions

This thesis studies the testing of parameters in the presence of uncertain NSPI in the parametric context for the SRM, MSRM, MRM and PRM. In this study, we define the test statistics of the UT, RT and PTT, derive their sampling distributions and compare graphically the power function of the tests for four different regression models. For the computation of the power function of the PTT, we required the bivariate normal and correlated bivariate Student's t distributions for the SRM, and correlated BNCF distribution for the MSRM, MRM and PRM. This is because the cdf for the bivariate normal, correlated bivariate Student's t and BNCF distributions are involved in the expression of the power function of the PTT. Unfortunately, the correlated BNCF distribution is not available in the literature, and hence we derived the pdf and cdf of the BNCF distribution as well as proposed appropriate computational formulas. The R codes are written to calculate and plot the pdf and cdf of the BNCF distribution as well as the power functions of the UT, RT and PTT.

For both known and unknown variance, under the alternative hypothesis, the sampling distribution of the test statistics of the tests of the SRM follow a normal and Student's t distributions, respectively. However, the power function of the PTT

involves the value of the probability integrals of the bivariate normal (for known variance) and correlated bivariate Student's t distributions (for unknown variance). The test statistics of the PT and UT are correlated but PT and RT are uncorrelated.

Similarly, the sampling distribution of the test statistics of the tests of the MSRM, MRM and PRM follow a F distribution. In this case, the power function of the PTT involves the cdf of the correlated BNCF distribution. The UT and PT are also correlated, but the RT and PT are not correlated, and are in fact independent.

For the four regression models covered in this thesis, the relevant R codes were written to evaluate the probability integrals of the bivariate normal and correlated bivariate t distributions as well as the cdf of the correlated BNCF distribution. We then used it in the calculation of the power function of the PTT.

Based on our analyses of the SRM for known variance, the power of the RT was always higher than that of the UT and PTT for all values of λ_1 , and the power of the PTT lies between the power of the RT and UT for all values of λ_1 , λ_2 and ρ . The size of the UT was smaller than that of the RT and PTT. The RT had the maximum power and size, and the UT had the minimum power and size. The PTT had smaller size than the RT and the RT had larger power than the UT. The PTT protected against the maximum size of the RT and the minimum power of the UT. As $\lambda_2 \rightarrow 0$ the difference between the power of the PTT and RT diminished for all values of λ_1 . That is, if the NSPI is accurate the power of the PTT was about the same as that of the RT. Moreover, the power of the PTT got closer to that of the RT as $\rho \rightarrow 1$. If $\rho = 1$ then the power of the PTT matches with that of the RT. Thus, if there is a high (near 1) correlation between the UT and PT the power of the PTT is very close to that of the RT.

For unknown variance, it was evident that the power of the RT was always higher than that of the UT and PTT for all the values of λ_1 , and the power of the PTT

tends to lie between the power of the RT and UT for all values of λ_1 , λ_2 and ρ . The size of the UT was smaller than that of the RT and PTT.

For both known and unknown variance, the size of the PTT becomes smaller as $\lambda_2 \rightarrow 0$. Once again if the NSPI is near accurate the size of the PTT approaches that of the UT. Therefore, we recommend the PTT when the quality of the NSPI is good (i.e. $\lambda_2 \rightarrow 0$) as it performs better than the UT and RT when $\rho \rightarrow 1$.

Based on the analyses of the MSRM the power of the RT is always higher than that of the UT for all values of ϕ_1 and ϕ_2 . However, the power of the two tests are identical for $\phi_2 = 0$, and all ϕ_1 . Also, the power of the PTT is always larger than that of the UT for all values of ϕ_1 and ϕ_2 and ρ . The size of the UT is smaller than that of the RT and PTT for all ϕ_1 . Among the three tests, the UT had the lowest power and lowest size. In terms of power it performed the worst and in terms of size it performed the best. Similarly, the RT had the highest size. The power of the PTT was higher than that of the UT and RT. The size of the PTT was less than that of the RT but higher than that of the UT. Thus, the PTT appears to have protected against the maximum size of the RT and assured higher power than the UT and RT.

The power of the RT of the MRM tended to be similar to that of the PTT for the larger values of ζ_1 and ζ_2 . It was also the same as (and/or larger than) that of the UT. The gap between the power of the RT and PTT is relatively close for all values of ζ_1 and ζ_2 and ρ . The gap between the power curves of the RT and PTT was much greater than that between the UT and RT. However, the powers of the UT, RT and PTT will be identical for larger ζ_2 .

The powers of the UT, RT and PTT for the PRM demonstrate similar behavior as those of the MSRM. The power of the PTT is not always larger than or between that of the UT and RT. It is identical for any fixed value of ρ , regardless of its sign.

For all four regression models, the PTT shows a reasonable domination over the

other tests when the suspected NSPI value of the parameter is not too far away from its true value (under H_0). Similar to that of the SRM, the PTTs for the MSRM, MRM and PRM have higher power than those of the UTs and lower than that of the RTs.

7.2 Limitations and Future Directions

This thesis considers a one-side alternative hypothesis, that is, $H_0 : \beta = \beta_{00}$ against $H_a : \beta > \beta_{00}$. This means that if the null hypothesis is rejected, then the true intercept is significantly larger than that its suspected value. Under the alternative hypothesis, the true intercept is significantly less than its suspected value if the null hypothesis is rejected. So, it is recommended to consider a test for a two-side alternative hypothesis. It is likely to lead to a more complicated power function of the PTT.

The complicated formula of the power function of the PTT limits the theoretical analysis difficult. As an alternative, computational and graphical analyses may be pursued. R codes are written to compute the power of the UT, RT and PTT and represent them graphically. The theoretical analysis of the joint distribution (correlated bivariate t and F distributions) of the PTT have not been discussed in detail due to the complex form of the power function.

The PTT should be tried for the semiparametric regression model and/or other regression models. This is because the PTT has currently been studied in both nonparametric and parametric models, but not semiparametric model. To date this has not been done due to the complicated form of the sampling distribution and power function of the PTT as well as lack of computational tools.

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Appendices

Appendix A

R codes

A.1 Figure 2.1. The cdf of the singly BNCF distribution

- The following R codes are used for producing Figure 2.1.

```
library(cubature); par(mfrow=c(2,2))

jj <- seq(0,10,1); rr <- seq(0,10,1);
nu1 <- v1 <- 5; nu2 <- v2 <- 20; rho <- 0.5;
d <- c(0,0.5,1,1.5,2,2.5,3); lambda <- nct <- 1
Lrj <- numeric(length(jj)); Hrj <- numeric(length(jj));
LHr <- numeric(length(rr)); Tr <- numeric(length(jj));
Pr <- numeric(length(jj)); cdf1 <- numeric(length(d));
for (d in 0:length(d)){
for (r in 0:length(rr)){
nur <- nu1+2*r;
for (j in 0:length(jj)){
qrj <- (nur/2 + j-1); srj <- (nur+(nu2/2)+2*j);
f <- function(x){((x[1] * x[2])qrj)/((nu2 * (1 - rho2) + nur * (x[1] + x[2]))srj)}
INT <- adaptIntegrate(f, lowerLimit = c(0, 0), upperLimit = c( d, d ))
Lrj[j] <- INT$integral
Hcal <- ((rho(2*j)) * (nur(nur+2*j)) * factorial(srj - 1))/
(factorial(j)*factorial(nur/2+j-1))
Hrj[j] <- Hcal
LHr[r] <- sum(Lrj*Hrj)
}
}
```

```

Tcal <- ((exp(-lambda/2)) * ((lambda/2)^r) * ((1 - rho^2)^((nur+nu2)/2)) *
(nu2^(nu2/2))) / (factorial(r)*factorial(nur/2-1)*factorial(nu2/2-1))
Tr[r] <- Tcal
Pr <- LHR*Tr
}
cdf1[d] <- sum(Pr)
}
cdf1

# 2nd Graph
d <- c(0,0.5,1,1.5,2,2.5,3); lambda <- nct <- 2
Lrj <- numeric(length(jj)); Hrj <- numeric(length(jj));
LHR <- numeric(length(rr)); Tr <- numeric(length(jj));
Pr <- numeric(length(jj)); cdf1 <- numeric(length(d));
for (d in 0:length(d)){
for (r in 0:length(rr)){
nur <- nu1+2*r;
for (j in 0:length(jj)){
qrj <- (nur/2 + j-1); srj <- (nur+(nu2/2)+2*j);
f <- function(x){((x[1] * x[2])^qrj)/((nu2 * (1 - rho^2) + nur * (x[1] + x[2]))^srj)}
INT <- adaptIntegrate(f, lowerLimit = c(0, 0), upperLimit = c( d, d ))
Lrj[j] <- INT$integral
Hcal <- ((rho^(2*j)) * (nur^(nur+2*j)) * factorial(srj - 1))/
(factorial(j)*factorial(nur/2+j-1))
Hrj[j] <- Hcal
LHR[r] <- sum(Lrj*Hrj)
}
Tcal <- ((exp(-lambda/2)) * ((lambda/2)^r) * ((1 - rho^2)^((nur+nu2)/2)) *
(nu2^(nu2/2))) / (factorial(r)*factorial(nur/2-1)*factorial(nu2/2-1))
Tr[r] <- Tcal
Pr <- LHR*Tr
}
cdf2[d] <- sum(Pr)
}
cdf2

```

```

# 3rd Graph
d <- c(0,0.5,1,1.5,2,2.5,3); lambda <- nct <- - 4
Lrj <- numeric(length(jj)); Hrj <- numeric(length(jj));
LHr <- numeric(length(rr)); Tr <- numeric(length(jj));
Pr <- numeric(length(jj)); cdf1 <- numeric(length(d));
for (d in 0:length(d)){
for (r in 0:length(rr)){
nur <- nu1+2*r;
for (j in 0:length(jj)){
qrj <- (nur/2 + j-1); srj <- (nur+(nu2/2)+2*j);
f <- function(x){((x[1] * x[2])qrj)/((nu2 * (1 - rho2) + nur * (x[1] + x[2]))srj)}
INT <- adaptIntegrate(f, lowerLimit = c(0, 0), upperLimit = c( d, d ))
Lrj[j] <- INT$integral
Hcal <- -((rho(2*j)) * (nur(nur+2*j)) * factorial(srj - 1))/
(factorial(j)*factorial(nur/2+j-1))
Hrj[j] <- Hcal
LHr[r] <- sum(Lrj*Hrj)
}
Tcal <- -((exp(-lambda/2)) * ((lambda/2)r) * ((1 - rho(2))((nur+nu2)/2))*
(nu2(nu2/2)) / (factorial(r)*factorial(nur/2-1)*factorial(nu2/2-1))
Tr[r] <- Tcal
Pr <- LHr*Tr
}
cdf3[d] <- sum(Pr)
}
cdf3

# 4th graph
d <- c(0,0.5,1,1.5,2,2.5,3); lambda <- nct <- - 6
Lrj <- numeric(length(jj)); Hrj <- numeric(length(jj));
LHr <- numeric(length(rr)); Tr <- numeric(length(jj));
Pr <- numeric(length(jj)); cdf1 <- numeric(length(d));
for (d in 0:length(d)){
for (r in 0:length(rr)){
nur <- nu1+2*r;

```

```

for (j in 0:length(jj)){
qrj <- (nur/2 + j-1); srj <- (nur+(nu2/2)+2*j);
f <- -function(x){((x[1] * x[2])qrj)/((nu2 * (1 - rho2) + nur * (x[1] + x[2]))srj)}
INT <- - adaptIntegrate(f, lowerLimit = c(0, 0), upperLimit = c( d, d ))
Lrj[j] <- - INT$integral
Hcal <- -((rho(2*j)) * (nur(nur+2*j)) * factorial(srj - 1))/
(factorial(j)*factorial(nur/2+j-1))
Hrj[j] <- - Hcal
LHr[r] <- - sum(Lrj*Hrj)
}
Tcal <- -((exp(-lambda/2)) * ((lambda/2)r) * ((1 - rho(2))((nur+nu2)/2)) *
(nu2(nu2/2))) / (factorial(r)*factorial(nur/2-1)*factorial(nu2/2-1))
Tr[r] <- - Tcal
Pr <- - LHr*Tr
}
cdf4[d] <- - sum(Pr) }
cdf4
d <- - c(0,0.5,1,1.5,2,2.5,3)

# Plot
plot(d,cdf1,type="l",lwd=1,col='red',cex=0.01,xlim=c(0,3),
xlab= paste(d), ylab='cdf', main= paste('(i)') paste(' ') paste(nu[1])
paste('= 5') paste(', ') paste(nu[2]) paste('= 20') paste(', ')
paste(rho) paste('= 0.5'),ylim=c(0,1))
lines(d,cdf2,lty=2,lwd=1,col='blue',cex=0.01)
lines(d,cdf3, lty=3,lwd=1,col='black',cex=0.01)
lines(d,cdf4, lty=4,lwd=1,col='violet',cex=0.01)
legend(0,1,col=c('red','blue','black','violet'),
legend=c(expression( paste(lambda) paste('= 1')),expression( paste(lambda)
paste('= 2')), expression( paste(lambda) paste('= 4')),
expression( paste(lambda) paste('= 6'))),
lty=1:4,cex=0.7,lwd=c(1,1,1,1))

```

Figure 2.1(ii) is plotted for d.f. (20, 5) with $\rho = 0.5$, Figure 2.1(iii) is plotted for d.f. (5, 20) with $\rho = -0.5$ and Figure 2.1(iv) is plotted for d.f. (20, 5) with $\rho = -0.5$.

A.2 Figure 2.2. The cdf of the doubly BNCF distribution

- The following R codes are used for producing the cdf of the *doubly* BNCF only in Figure 2.2.

```

library(zipfR)

bI2 <- function(alpha,cc,beta)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2]); v <- cc[2]/(1+cc[2])
sum.k <- 0; h <- alpha[1]-1;
for (k in 0:h) {
sum.k <- sum.k+(Cgamma(beta+k)/(Cgamma(beta)*factorial(k)))*
(uk)*Rbeta(y,alpha[2], k+beta,lower=TRUE) }
ans <- Rbeta(v,alpha[2],beta,lower=TRUE)-((1 - u)beta)*sum.k
ans
}

bI2 <- function(alpha,cc,beta,len.k)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2])
sum.k <- 0
for (k in 0:len.k)
{ sum.k <- sum.k+Cgamma(beta+alpha[1]+k)/Cgamma(beta+alpha[1])/
(Cgamma(1+alpha[1]+k)/Cgamma(1+alpha[1]))* u(k+alpha[1]) *
Rbeta(1-y,k+beta+alpha[1],alpha[2],lower=TRUE)
}
ans <- Rbeta(u,alpha[1],beta,lower=TRUE)-((1 - u)beta)/alpha[1]*
Cgamma(beta+alpha[1])/ (Cgamma(beta)*Cgamma(alpha[1]))*sum.k
ans }

sim_ncbivF <- function(rho,m,n,f1,f2,th1,th2,len.w1,len.w2,len.i,len.k)
{
beta <- n/2; cc <- 0; alpha <- 0
cc[1] <- f1*m/(n*(1 - rho2)); cc[2] <- f2*m/(n*(1 - rho2))
sum.I <- 0
for(w1 in 0:len.w1)
{
for(w2 in 0:len.w2)

```

```

{
for(i in 0:len.i)
{ alpha[1] <- m/2 + i + w1; alpha[2] <- m/2 + i + w2
sum.I <- sum.I + (exp(-th1/2)*(th1/2)w1)/factorial(w1)*
(exp(-th2/2)*(th2/2)w2)/factorial(w2)*
(Cgamma(m/2+i)*rho(2*i))/factorial(i))*bI2(alpha,cc,beta,len.k)
} } }
prob <- ((1 - rho2)(m/2))/Cgamma(m/2)*sum.I
list(prob=prob)
}
d <- 0 cdf <- sim_ncbivF(rho=0.5,m=10,n=20,f1=d,f2=d,th1=1,th2=1.5,
len.w1=10, len.w2=10,len.i=30,len.k=40)
cdfv <- cdf$prob (# the value of the cdf of the BNCF distribution)

# The values of cdf BNCF for d = 0, 1, 2, 3, 4 with ρ = 0 are given below
d <- 0
cdf <- sim_ncbivF(rho=0,m=10,n=20,f1=d,f2=d,th1=1,th2=1.5,
len.w1=10,len.w2=10,len.i=30,len.k=40)
cdf0 <- cdf $ prob; cdf0
d <- 1
cdf <- sim_ncbivF(rho=0,m=10,n=20,f1=d,f2=d,th1=1,th2=1.5,
len.w1=10,len.w2=10,len.i=30,len.k=40)
cdf1 <- cdf $ prob; cdf1
d <- 2
cdf <- sim_ncbivF(rho=0,m=10,n=20,f1=d,f2=d,th1=1,th2=1.5,
len.w1=10,len.w2=10,len.i=30,len.k=40)
cdf2 <- cdf $ prob; cdf2
d <- 3
cdf <- sim_ncbivF(rho=0,m=10,n=20,f1=d,f2=d,th1=1,th2=1.5,
len.w1=10,len.w2=10,len.i=30,len.k=40)
cdf3 <- cdf $ prob; cdf3
d <- 4
cdf <- sim_ncbivF(rho=0,m=10,n=20,f1=d,f2=d,th1=1,th2=1.5,
len.w1=10,len.w2=10,len.i=30,len.k=40)
cdf4 <- cdf $ prob; cdf4;

```

Plot of Figure 2.2(i) for $d = seq(0, 4, 0.25)$ and $\rho = 0, 0.3$ and 0.5 .

```
d <- seq(0,4,0.25);
cdf0 <- c(0,0.0002346121,0.01627232,0.09684624,0.243807,0.4123603,
0.5654642,0.68816,0.7798577,0.845757,0.8921182,0.9243944,
0.9467815,0.9623181,0.9731364,0.9807075,0.9860386)
cdf3 <- c(0,0.0002980258, 0.01867541,0.1055504,0.2580473,0.4286718,
0.5810194,0.7015529,0.7907202,0.8542579,0.8986304,0.929322,
0.9504858,0.9650954, 0.9752186,0.9822712, 0.9872166)
cdf5 <- c(0,0.0004706181, 0.02413454,0.1236418,0.2861159,0.4596505,
0.6097067,0.7256446, 0.809835,0.8689235,0.9096642,0.9375339,
0.9565658,0.9695905 ,0.9785452, 0.9847396,0.9890551)
plot(d,cdf0,type='l',lwd=1,col='black',cex=1,xlim=c(0,4),
xlab= paste(d), ylab='cdf', main= paste('(i)') paste(' ')
paste('The cdf of for') paste(' ') paste('m = 10') paste(',')
paste('n = 20') paste(',') paste(theta[1]) paste('= 1')
paste(',') paste(theta[2]) paste('= 1.5'),ylim=c(0,1))
lines(d,cdf3,lty=2,lwd=1,col='blue',cex=1)
lines(d,cdf5,lty=3,lwd=1,col='red',cex=1)
legend(0,1,col=c('black','blue','red'),legend=c(expression( paste(rho)
paste('= 0')),expression( paste(rho) paste('= 0.3')),expression(
paste(rho) paste('= 0.5'))),cex=0.8,lty=c(1,2,3),lwd=c(1,1,1))
```

Similarly, we can do for $\rho = 0.1$ and 0.9 on Figure 2.2(i). For Figure 2.2(ii), 2.2(iii), and 2.2(iv), we select some arbitrary θ_2 , m and n .

A.3 Figure 3.1. The power against λ_1 of the SRM for known σ^2

- The following R codes are used for producing Figure 3.1.

```
library(mvtnorm); par(mfrow=c(2,2))
n <- 20; beta0 <- 5; beta1 <- 1.7; sigm <- -1
set.seed(555555); x <- runif(n,0,1); set.seed(555555); e <- rnorm(n,0,1)
y <- beta0+beta1*x+e; ybar <- mean(y); xbar <- mean(x)
sy <- sqrt(sum ((y - mean(y))^2)/(n - 1)); ssx <- sum ((x - mean(x))^2)
```



```

h <- -lm(y ~ x); res <- residuals(h); se <- sqrt(sum ((res2)/(n - 2))) ;
k1 <- - 1/(sigm*sqrt ((1 + (n * xbar2)/ssx)))
lambda1 <- seq(0,20,1); lambda2 <- seq(0,2,0.1); alpha <- 0.05;
df <- n-2; zUT <- qnorm(1-alpha); zRT <- qnorm(1-alpha)

# Power calculation for  $\lambda_2 = 0$ 

pwrUT <- pnorm (zUT - k1 * lambda1)
for (i in lambda1) {pwrUT1 <- 1-pnorm(zUT-k1*lambda1)}
pwrRT <- 1-pnorm(zRT-(lambda1+lambda2[1]*xbar)/sigm)
for (i in lambda1) {
pwrRT1 <- 1-pnorm(zRT-(lambda1+lambda2[1]*xbar)/sigm)}
delta <- rep(0,2); dft <- rep(df,2); rho <- 0.50
cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((zUT-lambda2[1]*sqrt(ssx)/(sqrt(n)*sigm)), length(lambda1))
b <- zUT-k1*lambda1; dd <- dmvnorm(a,b)
c1 <- zRT-(lambda1+lambda2[1]*xbar)/sigm
pwrPTT <- pnorm(a)*(1-pnorm(c1))+ dd

# Plot

plot(lambda1,pwrUT,type='l',col='red',cex=0.01,lwd=1,xlim=c(0,20),
xlab= paste(lambda1[1]), ylab='Power', main= paste('i') paste(' ')
paste(lambda2[2]) paste('=0') paste(',') paste(rho) paste('=0.1'),ylim=c(0,1))
lines(lambda1,pwrRT,lty=2,lwd=1,col='blue',cex=0.01)
lines(lambda1,pwrPTT,lty=3,lwd=1,col='black',cex=0.01)
legend(13,0.35,col=c('red','blue','black'),legend=c('UT','RT','PTT'),
lty=1:3,cex=0.75,lwd=c(1,1,1),title='Power of the tests')

# Power calculation for  $\lambda_2 = 1$ 

pwrUT <- pnorm (zUT - k1 * lambda1)
for (i in lambda1) {pwrUT1 <- 1-pnorm(zUT-k1*lambda1)}
pwrRT <- 1-pnorm(zRT-(lambda1+lambda2[11]*xbar)/sigm)
for (i in lambda1) {
pwrRT1 <- 1-pnorm(zRT-(lambda1+lambda2[11]*xbar)/sigm)}
delta <- rep(0,2); dft <- rep(df,2); rho <- 0.50
cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((zUT-lambda2[11]*sqrt(ssx)/(sqrt(n)*sigm)), length(lambda1))

```

```

b <- zUT-k1*lambda1; dd <- dmvnorm(a,b)
c1 <- zRT-(lambda1+lambda2[11]*xbar)/sigm
pwrPTT <- pnorm(a)*(1-pnorm(c1))+ dd

# Plot
plot(lambda1,pwrUT,type='l',col='red',cex=0.01,lwd=1,xlim=c(0,20),
xlab= paste(lambda[1]), ylab='Power', main= paste('(ii)') paste(' ')
paste(lambda[2]) paste('=1') paste(',') paste(rho) paste('=0.1'),ylim=c(0,1))
lines(lambda1,pwrRT,lty=2,lwd=1,col='blue',cex=0.01)
lines(lambda1,pwrPTT,lty=3,lwd=1,col='black',cex=0.01)
legend(13,0.35,col=c('red','blue','black'),legend=c('UT','RT','PTT'),
lty=1:3,cex=0.75,lwd=c(1,1,1),title='Power of the tests')

# Power calculation for  $\lambda_2 = 1.5$ 
pwrUT <- pnorm (zUT - k1 * lambda1)
for (i in lambda1) {pwrUT1 <- 1-pnorm(zUT-k1*lambda1)}
pwrRT <- 1-pnorm(zRT-(lambda1+lambda2[16]*xbar)/sigm)
for (i in lambda1) {
pwrRT1 <- 1-pnorm(zRT-(lambda1+lambda2[16]*xbar)/sigm)}
delta <- rep(0,2); dft <- rep(df,2); rho <- 0.50
cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((zUT-lambda2[16]*sqrt(ssx)/(sqrt(n)*sigm)), length(lambda1))
b <- zUT-k1*lambda1; dd <- dmvnorm(a,b)
c1 <- zRT-(lambda1+lambda2[11]*xbar)/sigm
pwrPTT <- pnorm(a)*(1-pnorm(c1))+ dd

# Plot
plot(lambda1,pwrUT,type='l',col='red',cex=0.01,lwd=1,xlim=c(0,20),
xlab= paste(lambda[1]), ylab='Power', main= paste('(iii)') paste(' ')
paste(lambda[2]) paste('=1.5') paste(',') paste(rho) paste('=0.1'),ylim=c(0,1))
lines(lambda1,pwrRT,lty=2,lwd=1,col='blue',cex=0.01)
lines(lambda1,pwrPTT,lty=3,lwd=1,col='black',cex=0.01)
legend(13,0.35,col=c('red','blue','black'),legend=c('UT','RT','PTT'),
lty=1:3,cex=0.75,lwd=c(1,1,1),title='Power of the tests')

# Power calculation for  $\lambda_2 = 2$ 

```

```

pwrUT <- pnorm (zUT - k1 * lambda1)
for (i in lambda1) {pwrUT1 <- 1-pnorm(zUT-k1*lambda1)}
pwrRT <- 1-pnorm(zRT-(lambda1+lambda2[21]*xbar)/sigm)
for (i in lambda1) {
pwrRT1 <- 1-pnorm(zRT-(lambda1+lambda2[21]*xbar)/sigm)}
delta <- rep(0,2); dft <- rep(df,2); rho <- 0.50
cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((zUT-lambda2[21]*sqrt(ssx)/(sqrt(n)*sigm)), length(lambda1))
b <- zUT-k1*lambda1; dd <- dmvnorm(a,b)
c1 <- zRT-(lambda1+lambda2[11]*xbar)/sigm
pwrPTT <- pnorm(a)*(1-pnorm(c1))+ dd

# Plot

plot(lambda1,pwrUT,type='l',col='red',cex=0.01,lwd=1,xlim=c(0,20),
xlab= paste(lambda[1]), ylab='Power', main= paste('(iv)') paste(' ')
paste(lambda[2]) paste('=2') paste(',') paste(rho) paste('=0.1'),ylim=c(0,1))
lines(lambda1,pwrRT,lty=2,lwd=1,col='blue',cex=0.01)
lines(lambda1,pwrPTT,lty=3,lwd=1,col='black',cex=0.01)
legend(13,0.35,col=c('red','blue','black'),legend=c('UT','RT','PTT'),
lty=1:3,cex=0.75,lwd=c(1,1,1),title='Power of the tests')

```

A.4 Figure 3.2. The size against λ_1 of the SRM for known σ^2

- The following R codes are used for producing Figure 3.2.

```

library(mvtnorm); par(mfrow=c(2,2))

n <- 20; beta0 <- 5; beta1 <- 1.7; sigm <- -1
set.seed(555555); x <- runif(n,0,1); set.seed(555555); e <- rnorm(n,0,1)
y <- beta0+beta1*x+e; ybar <- mean(y); xbar <- mean(x)
sy <- sqrt(sum ((y - mean(y))^2)/(n - 1)); ssx <- sum ((x - mean(x))^2)
h <- -lm(y ~ x) ; res <- residuals(h); se <- sqrt(sum ((res^2)/(n - 2)))
k1 <- - 1/(sigm*sqrt ((1 + (n * xbar^2)/ssx)))
lambda1 <- seq(0,20,1); lambda2 <- seq(0,2,0.1); alpha <- 0.05;
df <- n-2; zUT <- qnorm(1-alpha); zRT <- qnorm(1-alpha)

```

```
# Size calculation for  $\lambda_2 = 0$ 
```

```
szUT <- 1-pnorm(zUT) for (i in lambda1) {szUT1;-1-pnorm(zUT)}
sizeUT <- rep(szUT, length(lambda1))
szRT <- 1-pnorm(zRT-(lambda2[1]*xbar)/sigm)
for (i in lambda1) {szRT1 <- 1-pnorm(zRT-(lambda2[1]*xbar)/sigm)}
sizeRT <- rep(szRT, length(lambda1));
rho <- 0.1; cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep(zUT, length(lambda1)); b <- rep(zUT, length(lambda1))
dd <- dmvnorm(a,b);
c <- rep(zRT-(lambda2[1]*xbar)/sigm, length(lambda1))
szPTT <- pnorm(a)*(1-pnorm(c)) + dd
```

```
# Plot
```

```
plot(lambda1,sizeUT,type='l',col='red',cex=0.01,lwd=1,xlim=c(0,20),
xlab= paste(lambda[1]),ylab='Size', main= paste('(i)') paste(' ')
paste(lambda[2]) paste('= 0') paste(',') paste(rho) paste('= 0.1'),ylim=c(0,1))
lines(lambda1,sizeRT,lty=2,lwd=1,col='blue',cex=0.01)
lines(lambda1,szPTT,lty=3,lwd=1,col='black',cex=0.01)
legend(0.5,0.99,col=c('red','blue','black'),legend=c('UT','RT','PTT'),
lty=1:3,cex=0.75,lwd=c(1,1,1),title='Size of the tests')
```

```
# Size calculation for  $\lambda_2 = 1$ 
```

```
szUT <- 1-pnorm(zUT) for (i in lambda1) {szUT1;-1-pnorm(zUT)}
sizeUT <- rep(szUT, length(lambda1))
szRT <- 1-pnorm(zRT-(lambda2[11]*xbar)/sigm)
for (i in lambda1) {szRT1 <- 1-pnorm(zRT-(lambda2[11]*xbar)/sigm)}
sizeRT <- rep(szRT, length(lambda1))
rho <- 0.1; cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep(zUT, length(lambda1)); b <- rep(zUT, length(lambda1))
dd <- dmvnorm(a,b);
c <- rep(zRT-(lambda2[11]*xbar)/sigm, length(lambda1))
szPTT <- pnorm(a)*(1-pnorm(c)) + dd
```

```
# Plot
```

```
plot(lambda1,sizeUT,type='l',col='red',cex=0.01,lwd=1,xlim=c(0,20),
xlab= paste(lambda[1]),ylab='Size', main= paste('(ii)') paste(' '))
```

```

paste(lambda[2]) paste('= 1') paste(',') paste(rho) paste('= 0.1'),ylim=c(0,1))
lines(lambda1,sizeRT,lty=2,lwd=1,col='blue',cex=0.01)
lines(lambda1,szPTT,lty=3,lwd=1,col='black',cex=0.01)
legend(0.5,0.99,col=c('red','blue','black'),legend=c('UT','RT','PTT'),
lty=1:3,cex=0.75,lwd=c(1,1,1),title='Size of the tests')

```

Size calculation $\lambda_2 = 1.5$

```

szUT <- 1-pnorm(zUT) for (i in lambda1) {szUT1;-1-pnorm(zUT)}
sizeUT <- rep(szUT, length(lambda1))
szRT <- 1-pnorm(zRT-(lambda2[16]*xbar)/sigm)
for (i in lambda1) {szRT1 <- 1-pnorm(zRT-(lambda2[16]*xbar)/sigm)}
sizeRT <- rep(szRT, length(lambda1))
rho <- 0.1; cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep(zUT, length(lambda1)); b <- rep(zUT, length(lambda1))
dd <- dmvnorm(a,b);
c <- rep(zRT-(lambda2[16]*xbar)/sigm, length(lambda1))
szPTT <- pnorm(a)*(1-pnorm(c)) + dd

```

Plot

```

plot(lambda1,sizeUT,type='l',col='red',cex=0.01,lwd=1,xlim=c(0,20),
xlab= paste(lambda[1]),ylab='Size', main= paste('(iii)') paste(' ')
paste(lambda[2]) paste('= 1.5') paste(',') paste(rho) paste('= 0.1'),ylim=c(0,1))
lines(lambda1,sizeRT,lty=2,lwd=1,col='blue',cex=0.01)
lines(lambda1,szPTT,lty=3,lwd=1,col='black',cex=0.01)
legend(0.5,0.99,col=c('red','blue','black'),legend=c('UT','RT','PTT'),
lty=1:3,cex=0.75,lwd=c(1,1,1),title='Size of the tests')

```

Size calculation $\lambda_2 = 2$

```

szUT <- 1-pnorm(zUT) for (i in lambda1) {szUT1;-1-pnorm(zUT)}
sizeUT <- rep(szUT, length(lambda1))
szRT <- 1-pnorm(zRT-(lambda2[21]*xbar)/sigm)
for (i in lambda1) {szRT1 <- 1-pnorm(zRT-(lambda2[21]*xbar)/sigm)}
sizeRT <- rep(szRT, length(lambda1))
rho <- 0.1; cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep(zUT, length(lambda1)); b <- rep(zUT, length(lambda1))
dd <- dmvnorm(a,b);

```

```

c <- rep(zRT-(lambda2[21]*xbar)/sigm, length(lambda1))
szPTT <- pnorm(a)*(1-pnorm(c)) + dd

# Plot

plot(lambda1,sizeUT,type='l',col='red',cex=0.01,lwd=1,xlim=c(0,20),
xlab= paste(lambda1),ylab='Size', main= paste('(iv)') paste(' ')
paste(lambda2) paste('= 2') paste(',') paste(rho) paste('= 0.1'),ylim=c(0,1))
lines(lambda1,sizeRT,lty=2,lwd=1,col='blue',cex=0.01)
lines(lambda1,szPTT,lty=3,lwd=1,col='black',cex=0.01)
legend(0.5,0.99,col=c('red','blue','black'),legend=c('UT','RT','PTT'),
lty=1:3,cex=0.75,lwd=c(1,1,1),title='Size of the tests')

```

A.5 Figure 3.3. The power of the PTT and size against ρ & λ_2 of the SRM for known σ^2

- The following R codes are used for producing Figure 3.3.

```

library(mvtnorm); par(mfrow=c(2,2))

n<- 20; beta0 <- 5; beta1 <- 1.7; sigm <- 1
set.seed(555555); x <- runif(n,0,1); set.seed(555555); e <- rnorm(n,0,1)
y <- beta0+beta1*x+e; ybar <- mean(y); xbar <- mean(x)
sy <- sqrt(sum ((y - mean(y))^2)/(n - 1)); ssx <- sum ((x - mean(x))^2)
h <- lm(y ~ x); res <- residuals(h); se <- sqrt(sum ((res^2)/(n - 2)))
k1 <- 1/(sigm*sqrt ((1 + (n * xbar^2)/ssx)))
lambda1 <- seq(0,20,1); lambda2 <- seq(0,2,0.1); alpha <- 0.05;
df <- n-2; zUT <- qnorm(1-alpha); zRT <- qnorm(1-alpha)

# Power calculation of the PTT for  $\lambda_2 = 0$ 

pwrUT <- 1-pnorm(zUT-k1*lambda1)
for (i in lambda1) {pwrUT1 <- 1-pnorm(zUT-k1*lambda1)}
pwrRT <- 1-pnorm(zRT-(lambda1+lambda2[1]*xbar)/sigm)
for (i in lambda1) {
pwrRT1 <- 1-pnorm(zRT-(lambda1+lambda2[1]*xbar)/sigm)}
rho <- seq(-1,1,0.1); cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((zUT-lambda2[1]*sqrt(ssx)/(sqrt(n)*sigm)), length(lambda1))
b <- zUT-k1*lambda1; dd <- dmvtorm(a,b)

```

```

c <- zRT-(lambda1+lambda2[1]*xbar)/sigm
pwrPTT1 <- pnorm(a)*(1-pnorm(c)) + dd

# Power calculation for  $\lambda_2 = 1$ 

pwrUT <- 1-pnorm(zUT-k1*lambda1)
for (i in lambda1) {pwrUT1 <- 1-pnorm(zUT-k1*lambda1)}
pwrRT <- -1-pnorm(zRT-(lambda1+lambda2[11]*xbar)/sigm)
for (i in lambda1) {
pwrRT1 <- 1-pnorm(zRT-(lambda1+lambda2[11]*xbar)/sigm)}
rho <- seq(-1,1,0.1); cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((zUT-lambda2[11]*sqrt(ssx)/(sqrt(n)*sigm)), length(lambda1))
b <- zUT-k1*lambda1; dd <- dmvnorm(a,b)
c <- zRT-(lambda1+lambda2[11]*xbar)/sigm
pwrPTT2 <- pnorm(a)*(1-pnorm(c)) + dd

# Power calculation for  $\lambda_2 = 1.5$ 

pwrUT <- 1-pnorm(zUT-k1*lambda1)
for (i in lambda1) {pwrUT1 <- 1-pnorm(zUT-k1*lambda1)}
pwrRT <- -1-pnorm(zRT-(lambda1+lambda2[16]*xbar)/sigm)
for (i in lambda1) {
pwrRT1 <- 1-pnorm(zRT-(lambda1+lambda2[16]*xbar)/sigm)}
rho <- seq(-1,1,0.1); cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((zUT-lambda2[16]*sqrt(ssx)/(sqrt(n)*sigm)), length(lambda1))
b <- zUT-k1*lambda1; dd <- dmvnorm(a,b)
c <- zRT-(lambda1+lambda2[16]*xbar)/sigm
pwrPTT3 <- pnorm(a)*(1-pnorm(c)) + dd

# Power calculation for  $\lambda_2 = 2$ 

pwrUT <- 1-pnorm(zUT-k1*lambda1)
for (i in lambda1) {pwrUT1 <- 1-pnorm(zUT-k1*lambda1)}
pwrRT <- -1-pnorm(zRT-(lambda1+lambda2[21]*xbar)/sigm)
for (i in lambda1) {
pwrRT1 <- 1-pnorm(zRT-(lambda1+lambda2[21]*xbar)/sigm)}
rho <- seq(-1,1,0.1); cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((zUT-lambda2[21]*sqrt(ssx)/(sqrt(n)*sigm)), length(lambda1))
b <- zUT-k1*lambda1; dd <- dmvnorm(a,b)

```

```

c <- zRT-(lambda1+lambda2[21]*xbar)/sigm
pwrPTT4 <- pnorm(a)*(1-pnorm(c)) + dd

# Plot
plot(rho,pwrPTT1,type="l",lwd=1,col='red',cex=0.01,xlim=c(-1,1),
xlab= paste(rho),ylab='Power',main= paste('(i)') paste(' ') paste(lambda[2])
paste('= 0, 1, 1.5, 2'),ylim=c(0,1))
lines(rho,pwrPTT2,lty=2,lwd=1,col='green',cex=0.01)
lines(rho,pwrPTT3,lty=3,lwd=1,col='Black',cex=0.01)
lines(rho,pwrPTT4,lty=4,lwd=1,col='blue',cex=0.01 )
legend(0.4,0.4,col=c("red", "green", "black", "blue"),legend=
c(expression(lambda[2]==0),expression(lambda[2]==1),
expression(lambda[2]==1.5),expression(lambda[2]==2)),
lty=1:4,cex=0.75,lwd=c(1,1,1,1),title='Power of the PTT')

# 2nd Graph. The power of the PTT versus  $\lambda_2$  with  $\rho = 0, .1, .5, .9$ 
# Power calculation for  $\rho = 0$ 
lambda1 <- seq(0,20,1); lambda2 <- seq(0,2,0.1); alpha <- 0.05;
df <- n-2; zUT <- qnorm(1-alpha); zRT <- qnorm(1-alpha)
pwrUT <- 1-pnorm(zUT-k1*lambda1)
for (i in lambda1) {
pwrUT1 <- 1-pnorm(zUT-k1*lambda1)}
pwrRT <- 1-pnorm(zRT-(lambda1+lambda2[1]*xbar)/sigm)
for (i in lambda1) {
pwrRT1 <- 1-pnorm(zRT-(lambda1+lambda2[1]*xbar)/sigm)}
rho <- 0; cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((zUT-lambda2[1]*sqrt(ssx)/(sqrt(n)*sigm)), length(lambda1))
b <- zUT-k1*lambda1; dd <- dmvnorm(a,b)
c <- zRT-(lambda1+lambda2[1]*xbar)/sigm
pwrPTT1 <- pnorm(a)*(1-pnorm(c)) + dd

# Power calculation for  $\rho = 0.1$ 
pwrUT <- 1-pnorm(zUT-k1*lambda1)
for (i in lambda1) {
pwrUT1 <- 1-pnorm(zUT-k1*lambda1)}

```



```

pwrRT <- 1-pnorm(zRT-(lambda1+lambda2[11]*xbar)/sigm)
for (i in lambda1) {
pwrRT1 <- 1-pnorm(zRT-(lambda1+lambda2[11]*xbar)/sigm)}
rho <- 0.1; cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((zUT-lambda2[11]*sqrt(ssx)/(sqrt(n)*sigm)), length(lambda1))
b <- zUT-k1*lambda1; dd <- dmnorm(a,b)
c <- zRT-(lambda1+lambda2[11]*xbar)/sigm
pwrPTT2 <- pnorm(a)*(1-pnorm(c)) + dd

# Power calculation for  $\rho = 0.5$ 

pwrUT <- 1-pnorm(zUT-k1*lambda1)
for (i in lambda1) {
pwrUT1 <- 1-pnorm(zUT-k1*lambda1)}
pwrRT <- 1-pnorm(zRT-(lambda1+lambda2[16]*xbar)/sigm)
for (i in lambda1) {
pwrRT1 <- 1-pnorm(zRT-(lambda1+lambda2[16]*xbar)/sigm)}
rho <- 0.5; cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((zUT-lambda2[16]*sqrt(ssx)/(sqrt(n)*sigm)), length(lambda1))
b <- zUT-k1*lambda1; dd <- dmnorm(a,b)
c <- zRT-(lambda1+lambda2[16]*xbar)/sigm
pwrPTT3 <- pnorm(a)*(1-pnorm(c)) + dd

# Power calculation for  $\rho = 0.9$ 

pwrUT <- 1-pnorm(zUT-k1*lambda1)
for (i in lambda1) {
pwrUT1 <- 1-pnorm(zUT-k1*lambda1)}
pwrRT <- 1-pnorm(zRT-(lambda1+lambda2[21]*xbar)/sigm)
for (i in lambda1) {
pwrRT1 <- 1-pnorm(zRT-(lambda1+lambda2[21]*xbar)/sigm)}
rho <- 0.9; cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((zUT-lambda2[21]*sqrt(ssx)/(sqrt(n)*sigm)), length(lambda1))
b <- zUT-k1*lambda1; dd <- dmnorm(a,b)
c <- zRT-(lambda1+lambda2[21]*xbar)/sigm
pwrPTT4 <- pnorm(a)*(1-pnorm(c)) + dd

# Plot

```

```

plot(lambda2,pwrPTT1,type="l",lwd=1,col='red',cex=0.01,xlim=c(0,1),
xlab= paste(lambda2),ylab='Power',main= paste('(ii)') paste(' ')
paste(rho) paste('= 0, 0.1, 0.5, 0.9'),ylim=c(0,1))
lines(lambda2,pwrPTT2,lty=2,lwd=1,col='green',cex=0.01)
lines(lambda2,pwrPTT3,lty=3,lwd=1,col='Black',cex=0.01)
lines(lambda2,pwrPTT4,lty=4,lwd=1,col='blue',cex=0.01,
ylab='Power',main='Power of the PTT',ylim=c(0,1))
legend(0.7,0.4,col=c("red", "green", "black", "blue"),
legend=c(expression(rho==0),expression(rho==0.1),expression(rho==0.5),
expression(rho==0.9)),lty=1:4,cex=0.75,lwd=c(1,1,1,1),title='Power of the PTT')

```

3rd Graph. The size of the tests versus ρ

```

szUT <- 1-pnorm(zUT)
for (i in lambda1) {
szUT1 <- 1-pnorm(zUT)}
sizeUT <- rep(szUT, length(lambda1))
szRT <- 1-pnorm(zRT-(lambda2*xbar)/sigm)
for (i in lambda1) {
szRT1 <- 1-pnorm(zRT-(lambda2*xbar)/sigm)}
rho <- seq(-1,1,0.1); cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep(zUT, length(lambda1)); b <- rep(zUT, length(lambda1))
dd <- dmvnorm(a,b); c <- zRT-(lambda2*xbar)/sigm
szPTT <- pnorm(a)*(1-pnorm(c)) + dd

```

Plot

```

plot(rho,sizeUT,type='l',col='red',cex=0.01,lwd=1,xlim=c(-1,1),
xlab= paste(rho),ylab='Size',main= paste('(iii)') paste(' ')
paste('Size against') paste(' ') paste(rho),ylim=c(0,0.3))
lines(rho,szRT,lty=2,lwd=1,col='blue',cex=0.01)
lines(rho,szPTT,lty=3,lwd=1,col='black',cex=0.01)
legend(-0.95,0.3,col=c('red','blue','black'),
legend=c('UT','RT','PTT'),lty=1:3,cex=0.75,lwd=c(1,1,1),title='Size')

```

4th Graph. The size of the tests versus λ_2

```

szUT <- 1-pnorm(zUT)

```

```

for (i in lambda1) {
szUT1 <- 1-pnorm(zUT)}
sizeUT <- rep(szUT, length(lambda1))
szRT <- 1-pnorm(zRT-(lambda2*xbar)/sigm)
for (i in lambda1) {
szRT1 <- 1-pnorm(zRT-(lambda2*xbar)/sigm)}
rho <- 0.1; cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep(zUT, length(lambda1)); b <- rep(zUT, length(lambda1))
dd <- dmvnorm(a,b); c <- zRT-(lambda2*xbar)/sigm
szPTT <- pnorm(a)*(1-pnorm(c)) + dd

# Plot

plot(lambda2,sizeUT,type='l',col='red',cex=0.01,lwd=1,xlim=c(0,2),
xlab= paste(lambda[2]),ylab='Size',main= paste('(iv)') paste(' ')
paste(rho) paste('= 0.1'),ylim=c(0,0.3))
lines(lambda2,szRT,lty=2,lwd=1,col='blue',cex=0.01)
lines(lambda2,szPTT,lty=3,lwd=1,col='black',cex=0.01)
legend(0,0.3,col=c('red','blue','black'),
legend=c('UT','RT','PTT'),lty=1:3,cex=0.75,lwd=c(1,1,1),title='Size')

```

A.6 Figures 3.4, 3.5 and 3.6. The power against λ_1 of the SRM for unknown σ^2

- The following R codes are used for producing Figures 3.4, 3.5 and 3.6.

```

library(mvtnorm); par(mfrow=c(2,2))

n<- 20; beta0 <- 5; beta1 <- 1.7; sigm <- 1
set.seed(555555); x <- runif(n,0,1); set.seed(555555); e <- rnorm(n,0,1)
y <- beta0+beta1*x+e; ybar <- mean(y); xbar <- mean(x)
sy <- sqrt(sum ((y - mean(y))^2)/(n - 1)); ssx <- sum ((x - mean(x))^2)
h <- -lm(y ~ x) ; res <- residuals(h); se <- sqrt(sum ((res^2)/(n - 2)))
k1 <- 1/(sigm*sqrt ((1 + (n * xbar^2)/ssx)))
lambda1 <- seq(0,15,1);lambda2 <- seq(0,5); alpha <- 0.05;
df <- n-2; dfUT <- n-2; dfRT <- n-1;
tUT <- qt(1-alpha, dfUT); tRT <- qt(1-alpha, dfRT)

```

```

# Power calculation for  $\lambda_2 = 1$ 
pwrUT <- 1-pt(tUT-k*lambda1,dfUT)
for (i in lambda1) {pwrUT1 <- 1-pt(tUT-k*lambda1,dfUT)}
pwrRT <- 1-pt(tRT-(lambda1+lambda2[2]*xbar)/sy,dfRT)
for (i in lambda1) {
pwrRT1 <- 1-pt(tRT-(lambda1+lambda2[2]*xbar)/sy,dfRT)}
delta <- rep(0,2); dft <- rep(df,2);
rho <- 0.1; cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((tUT-lambda2[2]*sqrt(ssx)/(sqrt(n)*se)), length(lambda1))
b <- tUT-k1*lambda1;
L <- t(matrix(c(a,b), nrow=length(lambda1))); top <- c(Inf, Inf)
dd <- numeric(length(lambda1)) for(i in 1:length(lambda1)){Li <- L[,i]
dd[i] <- pmvt(lower=Li, upper=top, df=dft,corr=cor,sigma=NULL)}
c1 <- tRT-(lambda1+lambda2[2]*xbar)/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2) dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){LLi <- LL[,i] ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
pwrPTT <- dd1+dd

# Plot
plot(lambda1,pwrUT,type="l",col='red',cex=0.01,lwd=1,xlim=c(0,14),
ylab='Power', xlab= paste(lambda[1]),main= paste('(i)') paste(' ') paste
(lambda[2]) paste('= 1') paste(',') paste(rho) paste('= 0.1'),ylim=c(0,1))
lines(lambda1,pwrRT,lty=2,lwd=1,col='blue',cex=0.01)
lines(lambda1,pwrPTT,lty=3,lwd=1,col='black',cex=0.01)
legend(10,0.4,col=c('red','blue','black'),legend=c('UT','RT','PTT'),
lty=1:3,cex=0.8,lwd=c(1,1,1),title='Power')

# Power calculation for  $\lambda_2 = 2$ 
pwrUT <- 1-pt(tUT-k*lambda1,dfUT)
for (i in lambda1) {pwrUT1 <- 1-pt(tUT-k*lambda1,dfUT)}
pwrRT <- 1-pt(tRT-(lambda1+lambda2[3]*xbar)/sy,dfRT)
for (i in lambda1) {

```

```

pwrRT1 <- 1-pt(tRT-(lambda1+lambda2[3]*xbar)/sy,dfRT)}
delta <- rep(0,2); dft <- rep(df,2);
rho <- 0.1; cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((tUT-lambda2[3]*sqrt(ssx)/(sqrt(n)*se)), length(lambda1))
b <- tUT-k1*lambda1;
L <- t(matrix(c(a,b), nrow=length(lambda1))); top <- c(Inf, Inf)
dd <- numeric(length(lambda1)) for(i in 1:length(lambda1)){Li <- L[,i]
dd[i] <- pmvt(lower=Li, upper=top, df=dft,corr=cor,sigma=NULL)}
c1 <- tRT-(lambda1+lambda2[3]*xbar)/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2) dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){LLi <- LL[,i] ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
pwrPTT <- dd1+dd

```

Plot

```

plot(lambda1,pwrUT,type="l",col='red',cex=0.01,lwd=1,xlim=c(0,14),
ylim=c(0,1),xlab= paste(lambda[1]),ylab='Power',main= paste('(ii)') paste(' ')
paste(lambda[2]) paste('= 2') paste(',') paste(rho) paste('= 0.1'))
lines(lambda1,pwrRT,lty=2,lwd=1,col='blue',cex=0.01)
lines(lambda1,pwrPTT,lty=3,lwd=1,col='black',cex=0.01)
legend(10,0.4,col=c('red','blue','black'),legend=c('UT','RT','PTT'),
lty=1:3,cex=0.8,lwd=c(1,1,1),title='Power')

```

Power calculation for $\lambda_2 = 4$

```

pwrUT <- 1-pt(tUT-k*lambda1,dfUT)
for (i in lambda1) {pwrUT1 <- 1-pt(tUT-k*lambda1,dfUT)}
pwrRT <- 1-pt(tRT-(lambda1+lambda2[5]*xbar)/sy,dfRT)
for (i in lambda1) {
pwrRT1 <- 1-pt(tRT-(lambda1+lambda2[5]*xbar)/sy,dfRT)}
delta <- rep(0,2); dft <- rep(df,2);
rho <- 0.1; cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((tUT-lambda2[5]*sqrt(ssx)/(sqrt(n)*se)), length(lambda1))
b <- tUT-k1*lambda1;
L <- t(matrix(c(a,b), nrow=length(lambda1))); top <- c(Inf, Inf)

```

```

dd <- numeric(length(lambda1)) for(i in 1:length(lambda1)){Li <- L[,i]
dd[i]<- pmvt(lower=Li, upper=top, df=dft,corr=cor,sigma=NULL)}
c1 <- tRT-(lambda1+lambda2[5]*xbar)/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2) dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){LLi <- LL[,i] ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
pwrPTT <- dd1+dd

# Plot
plot(lambda1,pwrUT,type="l",col='red',cex=0.01,lwd=1,xlim=c(0,14),
ylim=c(0,1),xlab= paste(lambda[1]),ylab='Power',main= paste('(iii)') paste(' ')
paste(lambda[2]) paste('= 4') paste(',') paste(rho) paste('= 0.1'))
lines(lambda1,pwrRT,lty=2,lwd=1,col='blue',cex=0.01)
lines(lambda1,pwrPTT,lty=3,lwd=1,col='black',cex=0.01)
legend(10,0.4,col=c('red','blue','black'),legend=c('UT','RT','PTT'),
lty=1:3,cex=0.8,lwd=c(1,1,1),title='Power')

# Power calculation for  $\lambda_2 = 5$ 
pwrUT <- 1-pt(tUT-k*lambda1,dfUT)
for (i in lambda1) {pwrUT1 <- 1-pt(tUT-k*lambda1,dfUT)}
pwrRT <- 1-pt(tRT-(lambda1+lambda2[6]*xbar)/sy,dfRT)
for (i in lambda1) {
pwrRT1 <- 1-pt(tRT-(lambda1+lambda2[6]*xbar)/sy,dfRT)}
delta <- rep(0,2); dft <- rep(df,2);
rho <- 0.1; cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((tUT-lambda2[6]*sqrt(ssx)/(sqrt(n)*se)), length(lambda1))
b <- tUT-k1*lambda1;
L <- t(matrix(c(a,b), nrow=length(lambda1))); top <- c(Inf, Inf)
dd <- numeric(length(lambda1)) for(i in 1:length(lambda1)){Li <- L[,i]
dd[i]<- pmvt(lower=Li, upper=top, df=dft,corr=cor,sigma=NULL)}
c1 <- tRT-(lambda1+lambda2[6]*xbar)/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2) dd1 <- numeric(length(lambda1))

```

```
for(i in 1:length(lambda1)){LLi <- LL[,i] ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
pwrPTT <- dd1+dd
```

```
# Plot
```

```
plot(lambda1,pwrUT,type="l",col='red',cex=0.01,lwd=1,xlim=c(0,14),
ylim=c(0,1),xlab= paste(lambda[1]),ylab='Power',main= paste('(iv)') paste(' ')
paste(lambda[2]) paste('= 5') paste(',') paste(rho) paste('= 0.1'))
lines(lambda1,pwrRT,lty=2,lwd=1,col='blue',cex=0.01)
lines(lambda1,pwrPTT,lty=3,lwd=1,col='black',cex=0.01)
legend(10,0.4,col=c('red','blue','black'),legend=c('UT','RT','PTT'),
lty=c(1,2,3),cex=0.8,lwd=c(1,1,1),title='Power')
```

Similarly, Figures 3.5 and 3.6 are plotted by replacing the values of ρ with 0.5 and 0.9, respectively.

A.7 Figure 3.7. The power of the PTT and size against λ_2 and ρ of the SRM for unknown σ^2

- The following R codes are used for producing Figure 3.7.

```
library(mvtnorm); par(mfrow=c(2,2))

n<- 20; beta0 <- - 5; beta1 <- - 2.5; sigm <- -1
set.seed(555555); x <- runif(n,0,1); set.seed(555555); e <- rnorm(n,0,1)
y <- beta0+beta1*x+e; ybar <- mean(y); xbar <- mean(x)
sy <- sqrt(sum ((y - mean(y))^2)/(n - 1)); ssx <- sum ((x - mean(x))^2)
h <- -lm(y ~ x) ; res <- residuals(h); se <- sqrt(sum ((res^2)/(n - 2)))
k1 <- - 1/(sigm*sqrt ((1 + (n * xbar^2)/ssx)))

# Power calculation for  $\rho = 0.1$ 

lambda1 <- seq(0,20,1);lambda2 <- seq(0,20,1); alpha <- - 0.05;
df <- - n-2; dfUT <- - n-2; dfRT <- - n-1;
tUT <- - qt(1-alpha, dfUT); tRT <- - qt(1-alpha, dfRT)

pwrUT <- 1-pt(tUT-k*lambda1,dfUT)
for (i in lambda1) {
pwrUT1 <- 1-pt(tUT-k*lambda1,dfUT)}
```

```

pwrRT <- 1-pt(tRT-(lambda1+lambda2[10]*xbar)/sy,dfRT)
for (i in lambda1) {
pwrRT1 <- 1-pt(tRT-(lambda1+lambda2[10]*xbar)/sy,dfRT)}
delta <- rep(0,2); dft <- rep(df,2); rho <- 0.1
cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((tUT-lambda2[10]*sqrt(ssx)/(sqrt(n)*se)), length(lambda1))
b <- tUT-k*lambda1; L <- t(matrix(c(a,b), nrow=length(lambda1)))
top <- c(Inf, Inf) ; dd <- numeric(length(lambda1))
for(i in 1:length(lambda1)){ Li <- L[,i]
dd[i] <- pmvt(lower=Li, upper=top, df=dft,corr=cor,sigma=NULL)}
c1 <- tRT-(lambda1+lambda2[10]*xbar)/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2)
dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){
LLi <- LL[,i]; ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
pwrPTT1 <- dd1+dd

# Power calculation for  $\rho = 0.5$ 

pwrUT <- 1-pt(tUT-k*lambda1,dfUT)
for (i in lambda1) {
pwrUT1 <- 1-pt(tUT-k*lambda1,dfUT)}
pwrRT <- 1-pt(tRT-(lambda1+lambda2[10]*xbar)/sy,dfRT)
for (i in lambda1) {
pwrRT1 <- 1-pt(tRT-(lambda1+lambda2[10]*xbar)/sy,dfRT)}
delta <- rep(0,2); dft <- rep(df,2); rho <- 0.5
cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((tUT-lambda2[10]*sqrt(ssx)/(sqrt(n)*se)), length(lambda1))
b <- tUT-k*lambda1; L <- t(matrix(c(a,b), nrow=length(lambda1)))
top <- c(Inf, Inf) ; dd <- numeric(length(lambda1))
for(i in 1:length(lambda1)){ Li <- L[,i]
dd[i] <- pmvt(lower=Li, upper=top, df=dft,corr=cor,sigma=NULL)}
c1 <- tRT-(lambda1+lambda2[10]*xbar)/sy

```



```

LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2)
dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){
LLi <- LL[,i]; ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
pwrPTT2 <- dd1+dd

# Power calculation for  $\rho = 0.7$ 

pwrUT <- 1-pt(tUT-k*lambda1,dfUT)
for (i in lambda1) {
pwrUT1 <- 1-pt(tUT-k*lambda1,dfUT)}
pwrRT <- 1-pt(tRT-(lambda1+lambda2[10]*xbar)/sy,dfRT)
for (i in lambda1) {
pwrRT1 <- 1-pt(tRT-(lambda1+lambda2[10]*xbar)/sy,dfRT)}
delta <- rep(0,2); dft <- rep(df,2); rho <- 0.7
cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((tUT-lambda2[10]*sqrt(ssx)/(sqrt(n)*se)), length(lambda1))
b <- tUT-k*lambda1; L <- t(matrix(c(a,b), nrow=length(lambda1)))
top <- c(Inf, Inf) ; dd <- numeric(length(lambda1))
for(i in 1:length(lambda1)){ Li <- L[,i]
dd[i] <- pmvt(lower=Li, upper=top, df=dft,corr=cor,sigma=NULL)}
c1 <- tRT-(lambda1+lambda2[10]*xbar)/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2)
dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){
LLi <- LL[,i]; ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
pwrPTT3 <- dd1+dd

# Power calculation for  $\rho = 0.9$ 

pwrUT <- 1-pt(tUT-k*lambda1,dfUT)
for (i in lambda1) {

```

```

pwrUT1 <- 1-pt(tUT-k*lambda1,dfUT)}
pwrRT <- 1-pt(tRT-(lambda1+lambda2[10]*xbar)/sy,dfRT)
for (i in lambda1) {
pwrRT1 <- 1-pt(tRT-(lambda1+lambda2[10]*xbar)/sy,dfRT)}
delta <- rep(0,2); dft <- rep(df,2); rho <- 0.9
cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((tUT-lambda2[10]*sqrt(ssx)/(sqrt(n)*se)), length(lambda1))
b <- tUT-k*lambda1; L <- t(matrix(c(a,b), nrow=length(lambda1)))
top <- c(Inf, Inf) ; dd <- numeric(length(lambda1))
for(i in 1:length(lambda1)){ Li <- L[,i]
dd[i] <- pmvt(lower=Li, upper=top, df=dft,corr=cor,sigma=NULL)}
c1 <- tRT-(lambda1+lambda2[10]*xbar)/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2)
dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){
LLi <- LL[,i]; ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
pwrPTT4 <- dd1+dd

# Plot
plot(lambda2,pwrPTT1,type="l",lwd=1,col='red',cex=0.01,xlim=c(0,21),
xlab= paste(lambda2),ylab='Power',main= paste('i')
paste('Power of the PTT'),ylim=c(0,1))
lines(lambda2,pwrPTT2,lty=2,lwd=1,col='green',cex=0.01,xlim=c(0,21),
ylab='Power',ylim=c(0,1))
lines(lambda2,pwrPTT3,lty=3,lwd=1,col='Black',cex=0.01,xlim=c(0,21),
ylab='Power',ylim=c(0,1))
lines(lambda2,pwrPTT4,lty=4,lwd=1,col='blue',cex=0.01,xlim=c(0,21),
ylab='Power',main='Power of the PTT',ylim=c(0,1))
legend(14,0.4,col=c("red", "green", "black", "blue"),legend=c(
expression(rho==0.1),expression(rho==0.5),expression(rho==0.7),
expression(rho==0.9)),lty=1:4,cex=0.75,lwd=c(1,1,1,1),title='Power of the PTT')

```

2nd Graph. The power of the PTT vs λ_2 with different sign of ρ

Power calculation for $\rho = 0.5$

```

pwrUT <- 1-pt(tUT-k*lambda1,dfUT)
for (i in lambda1) {
pwrUT1 <- 1-pt(tUT-k*lambda1,dfUT)}
pwrRT <- 1-pt(tRT-(lambda1+lambda2[10]*xbar)/sy,dfRT)
for (i in lambda1) {
pwrRT1 <- 1-pt(tRT-(lambda1+lambda2[10]*xbar)/sy,dfRT)}
delta <- rep(0,2); dft <- rep(df,2); rho <- 0.5
cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((tUT-lambda2[10]*sqrt(ssx)/(sqrt(n)*se)), length(lambda1))
b <- tUT-k*lambda1; L <- t(matrix(c(a,b), nrow=length(lambda1)))
top <- c(Inf, Inf) ; dd <- numeric(length(lambda1))
for(i in 1:length(lambda1)){ Li <- L[,i]
dd[i] <- pmvt(lower=Li, upper=top, df=dft,corr=cor,sigma=NULL)}
c1 <- tRT-(lambda1+lambda2[10]*xbar)/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2)
dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){
LLi <- LL[,i]; ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
pwrPTT1 <- dd1+dd

```

Power calculation for $\rho = 0.1$

```

pwrUT <- 1-pt(tUT-k*lambda1,dfUT)
for (i in lambda1) {
pwrUT1 <- 1-pt(tUT-k*lambda1,dfUT)}
pwrRT <- 1-pt(tRT-(lambda1+lambda2[10]*xbar)/sy,dfRT)
for (i in lambda1) {
pwrRT1 <- 1-pt(tRT-(lambda1+lambda2[10]*xbar)/sy,dfRT)}
delta <- rep(0,2); dft <- rep(df,2); rho <- 0.1
cor <- matrix(c(1, rho, rho, 1), ncol=2)

```

```

a <- rep((tUT-lambda2[10]*sqrt(ssx)/(sqrt(n)*se)), length(lambda1))
b <- tUT-k*lambda1; L <- t(matrix(c(a,b), nrow=length(lambda1)))
top <- c(Inf, Inf) ; dd <- numeric(length(lambda1))
for(i in 1:length(lambda1)){ Li <- L[,i]
dd[i] <- pmvt(lower=Li, upper=top, df=dft,corr=cor,sigma=NULL)}
c1 <- tRT-(lambda1+lambda2[10]*xbar)/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2)
dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){
LLi <- LL[,i]; ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
pwrPTT2 <- dd1+dd

```

Power calculation for $\rho = -0.1$

```

pwrUT <- 1-pt(tUT-k*lambda1,dfUT)
for (i in lambda1) {
pwrUT1 <- 1-pt(tUT-k*lambda1,dfUT)}
pwrRT <- 1-pt(tRT-(lambda1+lambda2[10]*xbar)/sy,dfRT)
for (i in lambda1) {
pwrRT1 <- 1-pt(tRT-(lambda1+lambda2[10]*xbar)/sy,dfRT)}
delta <- rep(0,2); dft <- rep(df,2); rho <- -0.1
cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((tUT-lambda2[10]*sqrt(ssx)/(sqrt(n)*se)), length(lambda1))
b <- tUT-k*lambda1; L <- t(matrix(c(a,b), nrow=length(lambda1)))
top <- c(Inf, Inf) ; dd <- numeric(length(lambda1))
for(i in 1:length(lambda1)){ Li <- L[,i]
dd[i] <- pmvt(lower=Li, upper=top, df=dft,corr=cor,sigma=NULL)}
c1 <- tRT-(lambda1+lambda2[10]*xbar)/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2)
dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){

```

```

LLi <- LL[,i]; ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
pwrPTT3 <- dd1+dd

# Power calculation for  $\rho = -0.5$ 

pwrUT <- 1-pt(tUT-k*lambda1,dfUT)
for (i in lambda1) {
pwrUT1 <- 1-pt(tUT-k*lambda1,dfUT)}
pwrRT <- 1-pt(tRT-(lambda1+lambda2[10]*xbar)/sy,dfRT)
for (i in lambda1) {
pwrRT1 <- 1-pt(tRT-(lambda1+lambda2[10]*xbar)/sy,dfRT)}
delta <- rep(0,2); dft <- rep(df,2); rho <- -0.5
cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((tUT-lambda2[10]*sqrt(ssx)/(sqrt(n)*se)), length(lambda1))
b <- tUT-k*lambda1; L <- t(matrix(c(a,b), nrow=length(lambda1)))
top <- c(Inf, Inf) ; dd <- numeric(length(lambda1))
for(i in 1:length(lambda1)){ Li <- L[,i]
dd[i] <- pmvt(lower=Li, upper=top, df=dft,corr=cor,sigma=NULL)}
c1 <- tRT-(lambda1+lambda2[10]*xbar)/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2)
dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){
LLi <- LL[,i]; ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
pwrPTT4 <- dd1+dd

# Plot
plot(lambda2,pwrPTT1,type="l",lwd=1,col='red',cex=0.01,xlim=c(0,21),
xlab= paste(lambda2),ylab='Power',main= paste('(ii)') paste(' ')
paste('Power of the PTT'),ylim=c(0,1))
lines(lambda2,pwrPTT2,lty=2,lwd=1,col='green',cex=0.01,xlim=c(0,21),
ylab='Power',ylim=c(0,1))
lines(lambda2,pwrPTT3,lty=3,lwd=1,col='Black',cex=0.01,xlim=c(0,21),
ylab='Power',ylim=c(0,1))

```

```

lines(lambda2,pwrPTT4,lty=4,lwd=1,col='blue',cex=0.01,xlim=c(0,21),
ylab='Power',main='Power of the PTT',ylim=c(0,1))
legend(14,0.4,col=c("red","green","black","blue"),legend=c(
expression(rho==0.1),expression(rho==0.5),expression(rho==-0.1),
expression(rho==-0.5)),lty=1:4,cex=0.75,lwd=c(1,1,1,1),title='Power of the PTT')

```

3rd Graph. The power of the PTT vs ρ with $\lambda_2 = 1, 2, 4, 5$

Power calculation for $\lambda_2 = 1$

```

pwrUT <- 1-pt(tUT-k*lambda1,dfUT)
for (i in lambda1) {
pwrUT1 <- 1-pt(tUT-k*lambda1,dfUT)}
pwrRT <- 1-pt(tRT-(lambda1+lambda2[2]*xbar)/sy,dfRT)
for (i in lambda1) {
pwrRT1 <- 1-pt(tRT-(lambda1+lambda2[2]*xbar)/sy,dfRT)}
delta <- rep(0,2); dft <- rep(df,2); rho <- seq(0,1,0.05)
cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((tUT-lambda2[10]*sqrt(ssx)/(sqrt(n)*se)), length(lambda1))
b <- tUT-k*lambda1; L <- t(matrix(c(a,b), nrow=length(lambda1)))
top <- c(Inf, Inf) ; dd <- numeric(length(lambda1))
for(i in 1:length(lambda1)){ Li <- L[,i]
dd[i] <- pmvt(lower=Li, upper=top, df=dft,corr=cor0,sigma=NULL)}
c1 <- tRT-(lambda1+lambda2[10]*xbar)/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2)
dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){
LLi <- LL[,i]; ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
pwrPTT1 <- dd1+dd

```

Power calculation for $\lambda_2 = 2$

```

pwrUT <- 1-pt(tUT-k*lambda1,dfUT)
for (i in lambda1) {

```

```

pwrUT1 <- 1-pt(tUT-k*lambda1,dfUT)}
pwrRT <- 1-pt(tRT-(lambda1+lambda2[3]*xbar)/sy,dfRT)
for (i in lambda1) {
pwrRT1 <- 1-pt(tRT-(lambda1+lambda2[3]*xbar)/sy,dfRT)}
delta <- rep(0,2); dft <- rep(df,2); rho <- seq(0,1,0.05)
cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((tUT-lambda2[3]*sqrt(ssx)/(sqrt(n)*se)), length(lambda1))
b <- tUT-k*lambda1; L <- t(matrix(c(a,b), nrow=length(lambda1)))
top <- c(Inf, Inf) ; dd <- numeric(length(lambda1))
for(i in 1:length(lambda1)){ Li <- L[,i]
dd[i] <- pmvt(lower=Li, upper=top, df=dft,corr=cor0,sigma=NULL)}
c1 <- tRT-(lambda1+lambda2[3]*xbar)/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2)
dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){
LLi <- LL[,i]; ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
pwrPTT2 <- dd1+dd

# Power calculation for  $\lambda_2 = 4$ 

pwrUT <- 1-pt(tUT-k*lambda1,dfUT)
for (i in lambda1) {
pwrUT1 <- 1-pt(tUT-k*lambda1,dfUT)}
pwrRT <- 1-pt(tRT-(lambda1+lambda2[5]*xbar)/sy,dfRT)
for (i in lambda1) {
pwrRT1 <- 1-pt(tRT-(lambda1+lambda2[5]*xbar)/sy,dfRT)}
delta <- rep(0,2); dft <- rep(df,2); rho <- seq(0,1,0.05)
cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((tUT-lambda2[5]*sqrt(ssx)/(sqrt(n)*se)), length(lambda1))
b <- tUT-k*lambda1; L <- t(matrix(c(a,b), nrow=length(lambda1)))
top <- c(Inf, Inf) ; dd <- numeric(length(lambda1))
for(i in 1:length(lambda1)){ Li <- L[,i]
dd[i] <- pmvt(lower=Li, upper=top, df=dft,corr=cor0,sigma=NULL)}

```

```

c1 <- tRT-(lambda1+lambda2[5]*xbar)/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2)
dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){
LLi <- LL[,i]; ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
pwrPTT3 <- dd1+dd

# Power calculation for  $\lambda_2 = 5$ 

pwrUT <- 1-pt(tUT-k*lambda1,dfUT)
for (i in lambda1) {
pwrUT1 <- 1-pt(tUT-k*lambda1,dfUT)}
pwrRT <- 1-pt(tRT-(lambda1+lambda2[6]*xbar)/sy,dfRT)
for (i in lambda1) {
pwrRT1 <- 1-pt(tRT-(lambda1+lambda2[6]*xbar)/sy,dfRT)}
delta <- rep(0,2); dft <- rep(df,2); rho <- seq(0,1,0.05)
cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep((tUT-lambda2[6]*sqrt(ssx)/(sqrt(n)*se)), length(lambda1))
b <- tUT-k*lambda1; L <- t(matrix(c(a,b), nrow=length(lambda1)))
top <- c(Inf, Inf) ; dd <- numeric(length(lambda1))
for(i in 1:length(lambda1)){ Li <- L[,i]
dd[i] <- pmvt(lower=Li, upper=top, df=dft,corr=cor0,sigma=NULL)}
c1 <- tRT-(lambda1+lambda2[6]*xbar)/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2)
dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){
LLi <- LL[,i]; ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
pwrPTT4 <- dd1+dd

# Plot
plot(rho,pwrPTT1,type="l",lwd=1,col='red',cex=0.01,xlim=c(0,1),

```



```

xlab= paste(rho),ylab='Power',main= paste('(iii)') paste(' ')
paste('Power of the PTT'),ylim=c(0,1))
lines(rho,pwrPTT2,lty=2,lwd=1,col='blue',cex=0.01,xlim=c(0,1),ylim=c(0,1))
lines(rho,pwrPTT3,lty=3,lwd=1,col='black',cex=0.01,xlim=c(0,1),ylim=c(0,1))
lines(rho,pwrPTT4,lty=4,lwd=1,col='green',cex=0.01,xlim=c(0,1),ylim=c(0,1))
legend(0.65,0.4,col=c("red", "blue", "black", "green"),legend=c(expression
(lambda[2]==1),expression(lambda[2]==2),expression(lambda[2]==4),
expression(lambda[2]==5)),lty=1:4,cex=0.75,lwd=c(1,1,1,1),title='Power of the
PTT')

```

4th Graph. The size of the PTT vs λ_2

Size calculation for $\rho = 0$

```

delta <- rep(0,2); dft <- rep(df,2); rho <- 0
cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep(tUT,length(lambda1)); b <- rep(tUT,length(lambda1))
L <- t(matrix(c(a,b), nrow=length(lambda1)))
top <- c(Inf, Inf) ; dd <- numeric(length(lambda1))
for(i in 1:length(lambda1)){ Li <- L[,i]
dd[i] <- pmvt(lower=Li, upper=top, df=dft,corr=cor,sigma=NULL)}
c1 <- tRT-lambda2*xbar/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2)
dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){
LLi <- LL[,i]; ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
szPTT1 <- dd1+dd

```

Size calculation for $\rho = 0.5$

```

delta <- rep(0,2); dft <- rep(df,2); rho <-0.5
cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep(tUT,length(lambda1)); b <- rep(tUT,length(lambda1))
L <- t(matrix(c(a,b), nrow=length(lambda1)))
top <- c(Inf, Inf) ; dd <- numeric(length(lambda1))

```

```

for(i in 1:length(lambda1)){ Li <- L[,i]
dd[i] <- pmvt(lower=Li, upper=top, df=dft,corr=cor,sigma=NULL)}
c1 <- tRT-lambda2*xbar/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2)
dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){
LLi <- LL[,i]; ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
szPTT2 <- dd1+dd

# Size calculation for  $\rho = 0.9$ 
delta <- rep(0,2); dft <- rep(df,2); rho <- 0.9
cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep(tUT,length(lambda1)); b <- rep(tUT,length(lambda1))
L <- t(matrix(c(a,b), nrow=length(lambda1)))
top <- c(Inf, Inf) ; dd <- numeric(length(lambda1))
for(i in 1:length(lambda1)){ Li <- L[,i]
dd[i] <- pmvt(lower=Li, upper=top, df=dft,corr=cor,sigma=NULL)}
c1 <- tRT-lambda2*xbar/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2)
dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){
LLi <- LL[,i]; ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
szPTT3 <- dd1+dd

# Size calculation for  $\rho = 1$ 
delta <- rep(0,2); dft <- rep(df,2); rho <- 1
cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep(tUT,length(lambda1)); b <- rep(tUT,length(lambda1))
L <- t(matrix(c(a,b), nrow=length(lambda1)))
top <- c(Inf, Inf) ; dd <- numeric(length(lambda1))

```

```

for(i in 1:length(lambda1)){ Li <- L[,i]
dd[i] <- pmvt(lower=Li, upper=top, df=dft,corr=cor,sigma=NULL)}
c1 <- tRT-lambda2*xbar/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2)
dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){
LLi <- LL[,i]; ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
szPTT4 <- dd1+dd

# Plot
plot(lambda2,szPTT1,type="l",lwd=1,col='red',cex=0.01,xlim=c(0,20),
xlab= paste(lambda2),ylab='Size',main= paste('(iv)') paste(' ')
paste('Size of the PTT'),ylim=c(0,1))
lines(lambda2,szPTT2,lty=2,lwd=1,col='blue',cex=0.01,xlim=c(0,1),ylim=c(0,1))
lines(lambda2,szPTT3,lty=3,lwd=1,col='black',cex=0.01,xlim=c(0,1),ylim=c(0,1))
lines(lambda2,szPTT4,lty=4,lwd=1,col='green',cex=0.01,xlim=c(0,1),ylim=c(0,1))
legend(14,0.4,col=c("red", "blue", "black", "green"),legend=c(expression(rho==0),
expression(rho==0.5),expression(rho==0.9),expression(rho==1)),lty=1:4,cex=0.75,
lwd=c(1,1,1,1),title='Size of the PTT')

```

A.8 Figure 3.8. The size against λ_2 and ρ of the SRM for unknown σ^2

- The following R codes are used for producing Figure 3.8.

```

library(mvtnorm); par(mfrow=c(2,2))

n <- 20; beta0 <- 5; beta1 <- 2.5; sigm <- 1
set.seed(555555); x <- runif(n,0,1); set.seed(555555); e <- rnorm(n,0,1)
y <- beta0+beta1*x+e; ybar <- mean(y); xbar <- mean(x)
sy <- sqrt(sum ((y - mean(y))^2)/(n - 1)); ssx <- sum ((x - mean(x))^2)
h <- -lm(y ~ x) ; res <- residuals(h); se <- sqrt(sum ((res^2)/(n - 2)))
k1 <- 1/(sigm*sqrt ((1 + (n * xbar^2)/ssx)))

```

```

# Size calculation  $\lambda_2$  and  $\rho = 0.1, 0.5, 0.9$ 
# Size calculation for  $\rho = 0.1$ 
lambda1 <- seq(0,20,1);lambda2 <- seq(0,20); alpha <- 0.05;
df <- n-2; dfUT <- n-2; dfRT <- n-1;
tUT <- qt(1-alpha, dfUT); tRT <- qt(1-alpha, dfRT)
szUT <- 1-pt(tUT,dfUT)
for (i in lambda1) {szUT1 <- 1-pt(tUT,dfUT)}
szRT <- 1-pt(tRT-lambda2*xbar/sy,dfRT)
for (i in lambda1) {
szRT1 <- 1-pt(tRT-lambda2*xbar/sy,dfRT)}
delta <- rep(0,2); dft <- rep(df,2); rho <- seq(0,1,0.05)
rho <- 0.10
cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep(tUT,length(lambda1)); b <- rep(tRT,length(lambda1))
L <- t(matrix(c(a,b), nrow=length(lambda1)))
top <- c(Inf, Inf); dd <- numeric(length(lambda1))
for(i in 1:length(lambda1)) { Li <- L[,i]
dd[i] <- pmvt(lower=Li, upper=top, df=dft,corr=cor,sigma=NULL) }
c1 <- tRT-lambda2*xbar/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2)
dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){
LLi <- LL[,i]; ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL) }
sUT <- rep(szUT,length(lambda1)); szPTT <- dd1+dd

# Plot
plot(lambda2,sUT,type="l",col='red',cex=0.01,lwd=1,xlim=c(0,20),
xlab= paste(lambda2),ylab='Size',main= paste('i') paste(' ')
paste(rho) paste('= 0.1'),ylim=c(0,1))
lines(lambda2,szRT,lty=2,lwd=1,col='blue',cex=0.01)
lines(lambda2,szPTT,lty=3,lwd=1,col='violetred3',cex=0.01)
legend(0.2,1,col=c('red','blue','violetred3'),legend=c('UT','RT','PTT'),

```

```
lty=1:3,cex=0.75,lwd=c(1,1,1),title='Size')
```

Figures 3.8(ii) and 3.8(iii) are plotted by replacing the values of ρ with 0.5 and 0.9, respectively.

```
# Size calculation with correlation  $\rho$ 
```

```
df <- n-2; dfUT <- n-2; dfRT <- n-1;
tUT <- qt(1-alpha, dfUT); tRT <- qt(1-alpha, dfRT)
szUT <- 1-pt(tUT,dfUT)
for (i in lambda1) {szUT1 <- 1-pt(tUT,dfUT)}
szRT <- 1-pt(tRT-lambda2*xbar/sy,dfRT)
for (i in lambda1) {
szRT1 <- 1-pt(tRT-lambda2*xbar/sy,dfRT)}
delta <- rep(0,2); dft <- rep(df,2); rho <- seq(0,1,0.05)
#rho <- 0.10
cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep(tUT,length(lambda1)); b <- rep(tRT,length(lambda1))
L <- t(matrix(c(a,b), nrow=length(lambda1)))
top <- c(Inf, Inf); dd <- numeric(length(lambda1))
for(i in 1:length(lambda1)) { Li <- L[,i]
dd[i] <- pmvt(lower=Li, upper=top, df=dft,corr=cor,sigma=NULL) }
c1 <- tRT-lambda2*xbar/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2)
dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){
LLi <- LL[,i]; ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL) }
sUT <- rep(szUT,length(lambda1)); szPTT <- dd1+dd

# Plot
plot(rho,sUT,type="l",col='red',cex=0.01,lwd=1,xlim=c(0,1),
xlab= paste(rho),ylab='Size',main= paste('(iv)') paste(' ')
paste('Size of the tests'),ylim=c(0,1))
lines(rho,szRT,lty=2,lwd=1,col='blue',cex=0.01)
```

```
lines(rho,szPTT,lty=3,lwd=1,col='violetred3',cex=0.01)
legend(0,1,col=c('red','blue','violetred3'),legend=c('UT','RT','PTT'),
lty=1:3,cex=0.75,lwd=c(1,1,1),title='Size')
```

A.9 Figure 3.9. The size against λ_1 of the SRM for unknown σ^2

- The following R codes are used for producing Figure 3.9.

```
library(mvtnorm); par(mfrow=c(2,2))

n <- 20; beta0 <- -5; beta1 <- -1.7; sigm <- 1
set.seed(555555); x <- runif(n,0,1); set.seed(555555); e <- rnorm(n,0,1)
y <- beta0+beta1*x+e; ybar <- mean(y); xbar <- mean(x)
sy <- sqrt(sum((y - mean(y))^2)/(n - 1)); ssx <- sum((x - mean(x))^2)
h <- lm(y ~ x); res <- residuals(h); se <- sqrt(sum((res^2)/(n - 2)))
k1 <- 1/(sigm*sqrt((1 + (n * xbar^2)/ssx)))

# Size calculation  $\lambda_2 = 1$  and  $\rho = 0.5$ 

lambda1 <- seq(0,20,1); lambda2 <- seq(0,20,1); alpha <- 0.05;
df <- n-2; dfUT <- n-2; dfRT <- n-1;
tUT <- qt(1-alpha, dfUT); tRT <- qt(1-alpha, dfRT)
szUT <- 1-pt(tUT,dfUT) for (i in lambda1) {szUT1 <- 1-pt(tUT,dfUT)}
szRT <- 1-pt(tRT-lambda2[2]*xbar/sy,dfRT)
for (i in lambda1) {szRT1 <- 1-pt(tRT-lambda2[2]*xbar/sy,dfRT)}
delta <- rep(0,2); dft <- rep(df,2);
rho <- 0.1; cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep(tUT,length(lambda1)); b <- rep(tRT,length(lambda1))
L <- t(matrix(c(a,b), nrow=length(lambda1))); top <- c(Inf, Inf)
dd <- numeric(length(lambda1)) for(i in 1:length(lambda1)){Li <- L[,i]
dd[i] <- pmvt(lower=Li, upper=top, df=dft,corr=cor,sigma=NULL)}
c1 <- tRT-lambda2[2]*xbar/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2) dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){LLi <- LL[,i] ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
```

```

sUT <- rep(szUT,length(lambda1));
sRT <- rep(szRT,length(lambda1)) ; szPTT <- dd1+dd

# Plot

plot(lambda1,sUT,type="l",col='red',cex=0.01,lwd=1,xlim=c(0,20),
xlab= paste(lambda[1]),ylab='Size',main= paste('(i)') paste(' ')
paste(lambda[2]) paste('= 1'),ylim=c(0,0.8))
lines(lambda1,sRT,lty=2,lwd=1,col='blue',cex=0.01)
lines(lambda1,szPTT,lty=3,lwd=1,col='black',cex=0.01)
legend(14.8,0.8,col=c('red','blue','black'),legend=c('UT','RT','PTT'),
lty=1:3,cex=0.75,lwd=c(1,1,1),title='Size')

# Size calculation  $\lambda_2 = 2$ 

tUT <- qt(1-alpha, dfUT); tRT <- qt(1-alpha, dfRT)
szUT <- 1-pt(tUT,dfUT) for (i in lambda1) {szUT1 <- 1-pt(tUT,dfUT)}
szRT <- 1-pt(tRT-lambda2[3]*xbar/sy,dfRT)
for (i in lambda1) {szRT1 <- 1-pt(tRT-lambda2[3]*xbar/sy,dfRT)}
delta <- rep(0,2); dft <- rep(df,2);
rho <- 0.1; cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep(tUT,length(lambda1)); b <- rep(tRT,length(lambda1))
L <- t(matrix(c(a,b), nrow=length(lambda1))); top <- c(Inf, Inf)
dd <- numeric(length(lambda1)) for(i in 1:length(lambda1)){Li <- L[,i]
dd[i]<- pmvt(lower=Li, upper=top, df=dft,corr=cor,sigma=NULL)}
c1 <- tRT-lambda2[3]*xbar/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2) dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){LLi <- LL[,i] ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
sUT <- rep(szUT,length(lambda1));
sRT <- rep(szRT,length(lambda1)) ; szPTT <- dd1+dd

# Plot

plot(lambda1,sUT,type="l",col='red',cex=0.01,lwd=1,xlim=c(0,20),
xlab= paste(lambda[1]),ylab='Size',main= paste('(ii)') paste(' ')
paste(lambda[2]) paste('= 2'),ylim=c(0,0.8))

```

```

lines(lambda1,sRT,lty=2,lwd=1,col='blue',cex=0.01)
lines(lambda1,szPTT,lty=3,lwd=1,col='black',cex=0.01)
legend(14.8,0.8,col=c('red','blue','black'),legend=c('UT','RT','PTT'),
lty=1:3,cex=0.75,lwd=c(1,1,1),title='Size')

# Size calculation  $\lambda_2 = 4$ 

tUT <- qt(1-alpha, dfUT); tRT <- qt(1-alpha, dfRT)
szUT <- - 1-pt(tUT,dfUT) for (i in lambda1) {szUT1 <- - 1-pt(tUT,dfUT)}
szRT <- - 1-pt(tRT-lambda2[5]*xbar/sy,dfRT)
for (i in lambda1) {szRT1 <- - 1-pt(tRT-lambda2[5]*xbar/sy,dfRT)}
delta <- rep(0,2); dft <- rep(df,2);
rho <- 0.1; cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep(tUT,length(lambda1)); b <- rep(tRT,length(lambda1))
L <- t(matrix(c(a,b), nrow=length(lambda1))); top <- c(Inf, Inf)
dd <- numeric(length(lambda1)) for(i in 1:length(lambda1)){Li <- L[,i]
dd[i] <- pmvt(lower=Li, upper=top, df=dft,corr=cor,sigma=NULL)}
c1 <- tRT-lambda2[5]*xbar/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2) dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){LLi <- LL[,i] ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
sUT <- rep(szUT,length(lambda1));
sRT <- rep(szRT,length(lambda1)) ; szPTT <- dd1+dd

# Plot

plot(lambda1,sUT,type="l",col='red',cex=0.01,lwd=1,xlim=c(0,20),
xlab= paste(lambda[1]),ylab='Size',main= paste('(iii)') paste(' ')
paste(lambda[2]) paste('= 4'),ylim=c(0,0.8))
lines(lambda1,sRT,lty=2,lwd=1,col='blue',cex=0.01)
lines(lambda1,szPTT,lty=3,lwd=1,col='black',cex=0.01)
legend(14.8,0.8,col=c('red','blue','black'),legend=c('UT','RT','PTT'),
lty=1:3,cex=0.75,lwd=c(1,1,1),title='Size')

# Size calculation  $\lambda_2 = 5$ 

tUT <- qt(1-alpha, dfUT); tRT <- qt(1-alpha, dfRT)

```



```

szUT <- 1-pt(tUT,dfUT) for (i in lambda1) {szUT1 <- 1-pt(tUT,dfUT)}
szRT <- 1-pt(tRT-lambda2[6]*xbar/sy,dfRT)
for (i in lambda1) {szRT1 <- 1-pt(tRT-lambda2[6]*xbar/sy,dfRT)}
delta <- rep(0,2); dft <- rep(df,2);
rho <- 0.1; cor <- matrix(c(1, rho, rho, 1), ncol=2)
a <- rep(tUT,length(lambda1)); b <- rep(tRT,length(lambda1))
L <- t(matrix(c(a,b), nrow=length(lambda1))); top <- c(Inf, Inf)
dd <- numeric(length(lambda1)) for(i in 1:length(lambda1)){Li <- L[,i]
dd[i] <- pmvt(lower=Li, upper=top, df=dft,corr=cor,sigma=NULL)}
c1 <- tRT-lambda2[6]*xbar/sy
LL <- t(matrix(c(rep(-Inf, length(lambda1)),c1), nrow=length(lambda1)))
UL <- t(matrix(c(a, rep(Inf, length(lambda1))), nrow=length(lambda1)))
cor0 <- matrix(c(1, 0, 0, 1), ncol=2) dd1 <- numeric(length(lambda1))
for(i in 1:length(lambda1)){LLi <- LL[,i] ULi <- UL[,i]
dd1[i] <- pmvt(lower=LLi, upper=ULi, df=dft, corr=cor0,sigma=NULL)}
sUT <- rep(szUT,length(lambda1));
sRT <- rep(szRT,length(lambda1)) ; szPTT <- dd1+dd

# Plot

plot(lambda1,sUT,type="l",col='red',cex=0.01,lwd=1,xlim=c(0,20),
xlab= paste(lambda[1]),ylab='Size',main= paste('(iv)') paste(' ' )
paste(lambda[2]) paste('= 5'),ylim=c(0,0.8))
lines(lambda1,sRT,lty=2,lwd=1,col='blue',cex=0.01)
lines(lambda1,szPTT,lty=3,lwd=1,col='black',cex=0.01)
legend(14.8,0.8,col=c('red','blue','black'),legend=c('UT','RT','PTT'),
lty=1:3,cex=0.75,lwd=c(1,1,1),title='Size')?

```

A.10 Figure 4.1. The power against ϕ_1 of the MSRM

- The following R codes are used for producing Figure 4.1.

```

library(mvtnorm); par(mfrow=c(2,2));

#Generate beta in term of matrix

k <- 30; p <- 5; v1 <- m <- p; v2 <- n <- k-p;
v3 <- n <- k-p; alpha <- 0.05; beta01 <- 3; beta02 <- 5;
beta03 <- 6; beta04 <- 2; beta05 <- 4; beta11 <- 1.5;

```

```

beta12 <- 2.5; beta13 <- 3; beta14 <- 2; beta15 <- 1;
bii <- c(beta01,beta02,beta03,beta04,beta05,beta11,beta12,
beta13,beta14,beta15); beta <- matrix(bii,nrow=2,byrow=TRUE)

#Generate X and Z
set.seed(555555); x <- runif(n,0,1)
ssx <- sum((x-mean(x))^2); x.matrix <- matrix(x,nrow=n,ncol=1);
z <- matrix(c(rep(1,n),x.matrix),nrow=n,ncol=2)
xbar <- mean(x.matrix); xbarc <- matrix(rep(xbar,n),ncol=1)

#Generate errors
set.seed(555555); e1 <- rnorm(n,0,1); e2 <- rnorm(n,0,1);
e3 <- rnorm(n,0,1); e4 <- rnorm(n,0,1);e5 <- rnorm(n,0,1);
ei <- c(e1,e2,e3,e4,e5); e <- matrix(ei,nrow=n,ncol=5)

#Setting matrix y
zj <- z%%beta ; y <- zj+e

# Estimate the regression parameters in term of matrix
yt <- t(y); tz <- t(z); xtx <- tz %% z;
inve.xtx <- solve(xtx); beta.hat <- inve.xtx%%(tz %% y)

#Determining cov (b00,b01) and sigma term of matrix
b00 <- seq(0,20); b10 <- seq(0,20);
b0 <- beta.hat[1,]; b1 <- beta.hat[2,];
b0i <- rep(b00[2],5); b1i <- rep(b10[2],5);
beta00i <- matrix(c(b0i,b1i),nrow=2,ncol=5);
sse <- t(y-z%%beta.hat) %% (y-z%%beta.hat);
mse <- sse/(n-p); inv.mse <- solve(mse)
fUT <- qf(1-alpha,v1,v3); fRT <- qf(1-alpha,v1,v2);
fPT <- qf(1-alpha,v1,v3); phi1 <- seq(0,4,0.1);
phi2 <- seq(0,4,0.1); omega1 <- seq(0,10,0.1);
omega2 <- seq(0,10,0.1); K <- 11; Phi2.sel <- phi2[K]
Phi1.seq <- phi1[K]; nctseq1 <- ssx*phi1;
nct1 <- nctseq1[3]; nctseq2 <- ssx*phi2; nct2 <- nctseq2[3]
a <- fUT-nct1; b <- fPT-nct2;

# Determining power of the UT, RT, PT
pwrUT <- 1-pf(fUT-phi1,v1,v3);

```

```

pwrRT <- 1-pf(fRT-(phi1+phi2[K]*xbar2+omega1[1]*xbar+omega2[1]*xbar),v1,v2)
pwrPT <- 1-pf(fPT-phi2[K],v1,v3)

# Determining power of the PTT

# rho = 0.1
library(cubature); library(zipfR)
d1a <- (1-pwrPT)*pwrRT
bI2 <- function(alpha,cc,beta)
{ u <- - cc[1]/(1+cc[1]); y <- - cc[2]/(1+cc[1]+cc[2]); v <- - cc[2]/(1+cc[2])
sum_k <- - 0; h <- - alpha[1]-1;
for (k in 0:h) {
sum_k <- - sum_k+(Cgamma(beta+k)/(Cgamma(beta)*factorial(k)))*
(uk)*Rbeta(y,alpha[2], k+beta,lower=TRUE) }
ans <- - Rbeta(v,alpha[2],beta,lower=TRUE)-((1 - u)beta)*sum_k
ans
}
bI2 <- function(alpha,cc,beta,len.k)
{ u <- - cc[1]/(1+cc[1]); y <- - cc[2]/(1+cc[1]+cc[2])
sum_k <- - 0
for (k in 0:len.k)
{ sum_k <- - sum_k+Cgamma(beta+alpha[1]+k)/Cgamma(beta+alpha[1])/
(Cgamma(1+alpha[1]+k)/Cgamma(1+alpha[1]))* u(k+alpha[1]) *
Rbeta(1-y,k+beta+alpha[1],alpha[2],lower=TRUE)
}
ans <- - Rbeta(u,alpha[1],beta,lower=TRUE)-((1 - u)beta)/alpha[1]*
Cgamma(beta+alpha[1])/ (Cgamma(beta)*Cgamma(alpha[1]))*sum_k
ans }
sim_ncbivF <- function(rho,m,n,f1,f2,th1,th2,len.w1,len.w2,len.i,len.k)
{
beta <- - n/2; cc <- - 0; alpha <- - 0
cc[1] <- - f1*m/(n*(1 - rho2)); cc[2] <- - f2*m/(n*(1 - rho2))
sum.I <- - 0
for(w1 in 0:len.w1)
{
for(w2 in 0:len.w2)

```

```

{
for(i in 0:len.i)
{ alpha[1] <- m/2 + i + w1; alpha[2] <- m/2 + i + w2
sum.I <- sum.I + (exp(-th1/2)*(th1/2)^w1)/factorial(w1)*
(exp(-th2/2)*(th2/2)^w2)/factorial(w2)*
(Cgamma(m/2+i)*rho^(2*i))/factorial(i)*bI2(alpha,cc,beta,len.k)
} } }
prob <- ((1 - rho^2)^(m/2))/Cgamma(m/2)*sum.I
list(prob=prob)
}
d <- 3 cdf <- sim_ncbivF(rho=0.1,m=5,n=25,f1=d,f2=d,th1=1,th2=1.5,
len.w1=10, len.w2=10,len.i=30,len.k=40)
cdf1 <- cdf$prob
d1r <- 1-cdf1
pwrPTT <- d1a+d1r

# rho=0.5

d1a <- (1-pwrPT)*pwrRT
library(cubature); library(zipfR)
bI2 <- function(alpha,cc,beta)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2]); v <- cc[2]/(1+cc[2])
sum.k <- 0; h <- alpha[1]-1;
for (k in 0:h) {
sum.k <- sum.k+(Cgamma(beta+k)/(Cgamma(beta)*factorial(k)))*
(u^k)*Rbeta(y,alpha[2], k+beta,lower=TRUE) }
ans <- Rbeta(v,alpha[2],beta,lower=TRUE)-((1 - u)^beta)*sum.k
ans
}
bI2 <- function(alpha,cc,beta,len.k)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2])
sum.k <- 0
for (k in 0:len.k)
{ sum.k <- sum.k+Cgamma(beta+alpha[1]+k)/Cgamma(beta+alpha[1])/
(Cgamma(1+alpha[1]+k)/Cgamma(1+alpha[1]))* u^(k+alpha[1]) *
Rbeta(1-y,k+beta+alpha[1],alpha[2],lower=TRUE)

```

```

}
ans <- Rbeta(u,alpha[1],beta,lower=TRUE)-((1-u)beta)/alpha[1]*
Cgamma(beta+alpha[1])/(Cgamma(beta)*Cgamma(alpha[1]))*sum_k
ans }
sim_ncbivF <- function(rho,m,n,f1,f2,th1,th2,len.w1,len.w2,len.i,len.k)
{
beta <- n/2; cc <- 0; alpha <- 0
cc[1] <- f1*m/(n*(1-rho2)); cc[2] <- f2*m/(n*(1-rho2))
sum.I <- 0
for(w1 in 0:len.w1)
{
for(w2 in 0:len.w2)
{
for(i in 0:len.i)
{ alpha[1] <- m/2 + i + w1; alpha[2] <- m/2 + i + w2
sum.I <- sum.I + (exp(-th1/2)*(th1/2)w1)/factorial(w1)*
(exp(-th2/2)*(th2/2)w2)/factorial(w2)*
(Cgamma(m/2+i)*rho(2*i))/factorial(i)*bI2(alpha,cc,beta,len.k)
} } }
prob <- ((1-rho2)(m/2))/Cgamma(m/2)*sum.I
list(prob=prob)
}
d <- 3 cdf <- sim_ncbivF(rho=0.5,m=5,n=25,f1=d,f2=d,th1=1,th2=1.5,
len.w1=10, len.w2=10,len.i=30,len.k=40)
cdf1 <- cdf$prob
pwrPTT1 <- d1a+d1r

# rho=0.9

library(cubature); library(zipfR);
d1a <- (1-pwrPT)*pwrRT
bI2 <- function(alpha,cc,beta)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2]); v <- cc[2]/(1+cc[2])
sum.k <- 0; h <- alpha[1]-1;
for (k in 0:h) {
sum.k <- sum.k+(Cgamma(beta+k)/(Cgamma(beta)*factorial(k)))*

```

```

( $u^k$ )*Rbeta(y,alpha[2], k+beta,lower=TRUE) }
ans <- Rbeta(v,alpha[2],beta,lower=TRUE)-((1 -  $u$ )beta)*sum_k
ans
}
bI2 <- function(alpha,cc,beta,len.k)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2])
sum_k <- 0
for (k in 0:len.k)
{ sum_k <- sum_k+Cgamma(beta+alpha[1]+k)/Cgamma(beta+alpha[1])/
(Cgamma(1+alpha[1]+k)/Cgamma(1+alpha[1]))*  $u^{(k+alpha[1])}$  *
Rbeta(1-y,k+beta+alpha[1],alpha[2],lower=TRUE)
}
ans <- Rbeta(u,alpha[1],beta,lower=TRUE)-((1 -  $u$ )beta)/alpha[1]*
Cgamma(beta+alpha[1])/ (Cgamma(beta)*Cgamma(alpha[1]))*sum_k
ans }
sim_ncbivF <- function(rho,m,n,f1,f2,th1,th2,len.w1,len.w2,len.i,len.k)
{
beta <- n/2; cc <- 0; alpha <- 0
cc[1] <- f1*m/(n*(1 - rho2)); cc[2] <- f2*m/(n*(1 - rho2))
sum.I <- 0
for(w1 in 0:len.w1)
{
for(w2 in 0:len.w2)
{
for(i in 0:len.i)
{ alpha[1] <- m/2 + i +w1; alpha[2] <- m/2 + i +w2
sum.I <- sum.I + (exp(-th1/2)*(th1/2)w1)/factorial(w1)*
(exp(-th2/2)*(th2/2)w2)/factorial(w2)*
(Cgamma(m/2+i)*rho(2*i)/factorial(i))*bI2(alpha,cc,beta,len.k)
} } }
prob <- ((1 - rho2)(m/2))/Cgamma(m/2)*sum.I
list(prob=prob)
}
d <- 3 cdf <- sim_ncbivF(rho=0.9,m=5,n=25,f1=d,f2=d,th1=1,th2=1.5,
len.w1=10, len.w2=10,len.i=30,len.k=40)

```

```

cdf1 <- cdf$prob
pwrPTT2 <- d1a+d1r

# Plot

plot(phi1,pwrUT,type="l",lwd=1,col='red',cex=0.01,xlim=c(0,3.3),
xlab= paste(phi[1]), ylab='Power',main= paste('i') paste(phi[2])
paste('= 1') paste(',') paste(nu[1]) paste('=5') paste(',')
paste(nu[2]) paste('=25') paste(',') paste(theta[1]) paste('=1')
paste(',') paste(theta[2]) paste('=1.5'),ylim=c(0,1))
lines(phi1,pwrRT,lty=2,lwd=1,col='blue',cex=0.01,ylim=c(0,1))
lines(phi1,pwrPTT,lty=3,lwd=1,col='black',cex=0.01,ylim=c(0,1))
lines(phi1,pwrPTT1,lty=4,lwd=1,col='green',cex=0.01,ylim=c(0,1))
lines(phi1,pwrPTT2,lty=5,lwd=1,col='violet',cex=0.01,ylim=c(0,1))
legend(1.8,0.4,col=c('red','blue','black','green','violet'),
legend=c( expression( paste('UT')), expression( paste('RT')),
expression( paste('PTT') paste(',') paste(rho) paste('= +/- 0.1')),
expression( paste('PTT') paste(',') paste(rho) paste('= +/- 0.5')),
expression( paste('PTT') paste(',') paste(rho) paste('= +/- 0.9'))),
lty=1:5,cex=0.7,lwd=c(1,1,1,1,1), title='Power of the tests')

```

Figure 4.1(ii) is plotted for $\phi_1 = 1.5$, Figure 4.1(iii) is plotted for different d.f. (m, n) and Figure 4.1(iv) is plotted for different noncentrality parameters (θ_1, θ_2) .

A.11 Figure 4.2. The size against ϕ_1 of the MSRM

- The following R codes are used for producing Figure 4.2.

```

library(mvtnorm); par(mfrow=c(2,2));

#Generate beta matrix

k <- 30; p <- 5; v1 <- m <- p; v2 <- n <- k-p;
v3 <- n <- k-p; alpha <- 0.05; beta01 <- 3; beta02 <- 5;
beta03 <- 6; beta04 <- 2; beta05 <- 4; beta11 <- 1.5;
beta12 <- 2.5; beta13 <- 3; beta14 <- 2; beta15 <- 1;
bii <- c(beta01,beta02,beta03,beta04,beta05,beta11,beta12,
beta13,beta14,beta15); beta <- matrix(bii,nrow=2,byrow=TRUE)

#Generate X and Z

```

```

set.seed(555555); x <- runif(n,0,1)
ssx <- sum((x-mean(x))^2); x.matrix <- matrix(x,nrow=n,ncol=1);
z <- matrix(c(rep(1,n),x.matrix),nrow=n,ncol=2)
xbar <- mean(x.matrix); xbarc <- matrix(rep(xbar,n),ncol=1)
#Generate errors
set.seed(555555); e1 <- rnorm(n,0,1); e2 <- rnorm(n,0,1);
e3 <- rnorm(n,0,1); e4 <- rnorm(n,0,1);e5 <- rnorm(n,0,1);
ei <- c(e1,e2,e3,e4,e5); e <- matrix(ei,nrow=n,ncol=5)
#Setting matrix y
zj <- z%%beta ; y <- zj+e
# Estimate the regression parameters in term of matrix
yt <- t(y); tz <- t(z); xtx <- tz %% z;
inve.xtx <- solve(xtx); beta.hat <- inve.xtx%%(tz %% y)
#Determining cov (b00,b01) and sigma in term of matrix
b00 <- seq(0,20); b10 <- seq(0,20);
b0 <- beta.hat[1,]; b1 <- beta.hat[2,];
b0i <- rep(b00[2],5); b1i <- rep(b10[2],5);
beta00i <- matrix(c(b0i,b1i),nrow=2,ncol=5);
sse <- t(y-z%%beta.hat) %% (y-z%%beta.hat);
mse <- sse/(n-p); inv.mse <- solve(mse)
fUT <- qf(1-alpha,v1,v3); fRT <- qf(1-alpha,v1,v2);
fPT <- qf(1-alpha,v1,v3); phi1 <- seq(0,4,0.1);
phi2 <- seq(0,4,0.1); omega1 <- seq(0,10,0.1);
omega2 <- seq(0,10,0.1); K <- 11; Phi2.sel <- phi2[K]
Phi1.seq <- phi1[K]; nctseq1 <- ssx*phi1;
nct1 <- nctseq1[3]; nctseq2 <- ssx*phi2; nct2 <- nctseq2[3]
a <- fUT-nct1; b <- fPT-nct2;
# Graph for rho = 0.1, 0.5, 0.9, phi2= 2
K <- 22; Phi2.sel <- phi2[K]
# Determining size of the UT, RT, PT
szUT <- 1-pf(fUT,v1,v3); sizeUT <- rep(szUT,length(phi1))
szRT <- 1-pf(fRT-(phi2[K]*xbar^2),v1,v2); sizeRT <- rep(szRT,length(phi1))
szPT <- 1-pf(fPT-phi2[K],v1,v3); sizePT <- rep(szPT,length(phi1))

```


Determining size of the PTT

```

d1a <- (1-szPT)*szRT
library(cubature); library(zipfR)
bI2 <- function(alpha,cc,beta)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2]); v <- cc[2]/(1+cc[2])
sum_k <- 0; h <- alpha[1]-1;
for (k in 0:h) {
sum_k <- sum_k+(Cgamma(beta+k)/(Cgamma(beta)*factorial(k))) *
(u^k)*Rbeta(y,alpha[2], k+beta,lower=TRUE) }
ans <- Rbeta(v,alpha[2],beta,lower=TRUE)-((1 - u)^beta)*sum_k
ans
}
bI2 <- function(alpha,cc,beta,len.k)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2])
sum_k <- 0
for (k in 0:len.k)
{ sum_k <- sum_k+Cgamma(beta+alpha[1]+k)/Cgamma(beta+alpha[1])/
(Cgamma(1+alpha[1]+k)/Cgamma(1+alpha[1])) * u^(k+alpha[1]) *
Rbeta(1-y,k+beta+alpha[1],alpha[2],lower=TRUE)
}
ans <- Rbeta(u,alpha[1],beta,lower=TRUE)-((1 - u)^beta)/alpha[1]*
Cgamma(beta+alpha[1])/ (Cgamma(beta)*Cgamma(alpha[1]))*sum_k
ans }
sim_ncbivF <- function(rho,m,n,f1,f2,th1,th2,len.w1,len.w2,len.i,len.k)
{
beta <- n/2; cc <- 0; alpha <- 0
cc[1] <- f1*m/(n*(1 - rho^2)); cc[2] <- f2*m/(n*(1 - rho^2))
sum.I <- 0
for(w1 in 0:len.w1)
{
for(w2 in 0:len.w2)
{
for(i in 0:len.i)
{ alpha[1] <- m/2 + i +w1; alpha[2] <- m/2 + i +w2
sum.I <- sum.I + (exp(-th1/2)*(th1/2)^w1)/factorial(w1)*

```

```

(exp(-th2/2)*(th2/2)w2)/factorial(w2)*
(Cgamma(m/2+i)*rho(2*i)/factorial(i))*bI2(alpha,cc,beta,len.k)
} } }
prob <- ((1 - rho2)(m/2))/Cgamma(m/2)*sum_I
list(prob=prob)
}
d <- 3 cdf <- sim_ncbivF(rho=0.5,m=5,n=25,f1=d,f2=d,th1=1,th2=1.5,
len.w1=10, len.w2=10,len.i=30,len.k=40)
cdf1 <- cdf$prob
pwrPTT1 <- d1a+d1r

# rho=0.9

library(cubature); library(zipfR);
d1a <- (1-pwrPT)*pwrRT
bI2 <- function(alpha,cc,beta)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2]); v <- cc[2]/(1+cc[2])
sum_k <- 0; h <- alpha[1]-1;
for (k in 0:h) {
sum_k <- sum_k+(Cgamma(beta+k)/(Cgamma(beta)*factorial(k)))*
(uk)*Rbeta(y,alpha[2], k+beta,lower=TRUE) }
ans <- Rbeta(v,alpha[2],beta,lower=TRUE)-((1 - u)beta)*sum_k
ans
}
bI2 <- function(alpha,cc,beta,len.k)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2])
sum_k <- 0
for (k in 0:len.k)
{ sum_k <- sum_k+Cgamma(beta+alpha[1]+k)/Cgamma(beta+alpha[1])/
(Cgamma(1+alpha[1]+k)/Cgamma(1+alpha[1]))* u(k+alpha[1]) *
Rbeta(1-y,k+beta+alpha[1],alpha[2],lower=TRUE)
}
ans <- Rbeta(u,alpha[1],beta,lower=TRUE)-((1 - u)beta)/alpha[1]*
Cgamma(beta+alpha[1])/ (Cgamma(beta)*Cgamma(alpha[1]))*sum_k
ans }
sim_ncbivF <- function(rho,m,n,f1,f2,th1,th2,len.w1,len.w2,len.i,len.k)

```

```

{
beta <- n/2; cc <- 0; alpha <- 0
cc[1] <- f1*m/(n*(1 - rho^2)); cc[2] <- f2*m/(n*(1 - rho^2))
sum.I <- 0
for(w1 in 0:len.w1)
{
for(w2 in 0:len.w2)
{
for(i in 0:len.i)
{ alpha[1] <- - m/2 + i +w1; alpha[2] <- - m/2 + i +w2
sum.I <- - sum.I + (exp(-th1/2)*(th1/2)^w1)/factorial(w1)*
(exp(-th2/2)*(th2/2)^w2)/factorial(w2)*
(Cgamma(m/2+i)*rho^(2*i)/factorial(i))*bI2(alpha,cc,beta,len.k)
} } }
prob <- - ((1 - rho^2)^(m/2))/Cgamma(m/2)*sum.I
list(prob=prob)
}
d <- 3 cdf <- sim.ncbivF(rho=0.1,m=5,n=25,f1=d,f2=d,th1=1,th2=1.5,
len.w1=10, len.w2=10,len.i=30,len.k=40)
cdf1 <- cdf$prob
d1r <- 1-cdf1
szPTT <- d1a+d1r; sizePTT <- rep(szPTT,length(phi1))

# Plot
plot(phi1,sizeUT,type="l",lwd=1,col='red',cex=0.01,xlim=c(0,4),
xlab= paste(phi[1]),main= paste('(i)') paste(' ') paste(phi[2])
paste('= 2.1') paste(',') paste(rho) paste('= 0.5') paste(',')
paste('d = 3') paste(',') paste(m) paste('= 5') paste(',')
paste(n) paste('= 25'),ylab='Size',ylim=c(0,1))
lines(phi1,sizeRT,lty=2,lwd=1,col='blue',cex=0.01,ylim=c(0,1))
lines(phi1,sizePTT,lty=3,lwd=1,col='black',cex=0.01,ylim=c(0,1))
legend(0.1,0.95,col=c('red','blue','black'),legend=c('UT','RT','PTT'),
lty=1:3,cex=0.7,lwd=c(1,1,1),title='Size of the tests')

```

Figure 4.2(ii) is plotted for $\rho = 0.7$, Figure 4.2(iii) is plotted for values of $\rho = 0.9$ with $\phi_2 = 2$, and Figure 4.2(iv) is plotted for $\phi_2 = 2.1$ with $\rho = 0.9$.

A.12 Figure 5.1. The power against ζ_1 of the MRM

- The following R codes are used for producing Figure 5.1.

```
library(mvtnorm); library(copula); par(mfrow=c(2,2))
```

Generate beta matrix

```
n <- 100; k <- 3; r <- 2; s <- k-(r-1); v1 <- k; v2 <- n-s;
v3 <- n-r; v4 <- n-(k+1); alpha <- 0.05; beta0 <- 3; beta1 <- 1.5;
beta2 <- 5; beta3 <- 2.5; b1 <- c(beta0,beta1); b2 <- c(beta2,beta3);
betar <- matrix(b1,nrow=2,byrow=TRUE);
betas <- matrix(b2,nrow=2,byrow=TRUE)
bii <- c(beta0,beta1,beta2,beta3); beta <- matrix(bii,nrow=4,byrow=TRUE)
```

Generate X and Y

```
set.seed(555555); x1 <- runif(n,0,10); x2 <- runif(n,0,10);
x3 <- runif(n,0,10); one <- matrix(rep(1,length(n)),nrow=n,ncol=1)
cx1 <- c(one,x1); cx2 <- c(x2,x3); X1 <- matrix(cx1,nrow=n,ncol=2)
X2 <- matrix(cx2,nrow=n,ncol=2); Xm <- matrix(c(cx1,cx2),nrow=n,ncol=4)
X1t <- t(X1)%*% X1; X2t <- t(X2)%*% X2; Xmt <- t(Xm)%*% Xm;
invX1 <- solve(X1t); invX2 <- solve(X2t); invXm <- solve(Xmt)
```

Generate errors

```
set.seed(555); e1 <- rnorm(n,0,1); ei <- c(e1)
e <- matrix(ei,nrow=n,byrow=TRUE)
```

Setting matrix y

```
Y1 <- X1%*%b1+e; Y2 <- X2%*%b2+e;
Y <- Xm %*% beta+e; Yt <- t(Y)
```

Determine the regression parameters in term of matrix

```
beta1.est <- invX1 %*%(t(X1)%*% Y)
beta2.est <- invX2 %*%(t(X2)%*% Y)
beta.est <- invX1 %*%(t(X1)%*% Y)
```

Generate matrix H

```
set.seed(555); h1 <- rnorm(n,0,1); h2 <- rnorm(n,0,1)
h3 <- rnorm(n,0,1); h4 <- rnorm(n,0,1); h1m <- c(h1,h2);
```

```

h2m <- c(h3,h4); H1 <- matrix(h1m,nrow=n,ncol=2);
H2<- matrix(h2m,nrow=n,ncol=2); H<- matrix(c(H1,H2),nrow=n,ncol=4)
H1t <- t(H1); H2t <- t(H2); Ht <- t(H); h1 <- H1%%betar;
h2 <- H2 %%betas; h <- H %%beta

```

Determine Se and F of the UT, RT and PT

```

q <- 1; se1 <- 1/(n-r)*(t(Y-X1%%beta1.est)%%(Y-X1%%beta1.est))
se2 <- 1/(n-s)*(t(Y-X2%%beta2.est)%%(Y-X2%%beta2.est))
se <- 1/(n-r)*(t(Y-X1%%beta.est)%%(Y-X1%%beta.est))
F1 <- (1/(q*se1))*(t(H1%%beta1.est-h1)%%(H1%%invX1%%t(H1))
%%((H1%%beta1.est)-h1))
F2 <- (1/(q*se2))*(t(H2%%beta2.est-h2)%%(H2%%invX2%%t(H2))
%%((H2%%beta2.est)-h2))
F <- (1/(n*se))*(t(H%%beta.est-h)%%(H%%invXm%%t(H))
%%((H%%beta.est)-h))

```

Determine delta

```

sigma <- 5
delta1 <- (1/sigma^2)*(t(H1%%betar-h1)%%(H1%%invX1%%t(H1))
%%((H1%%betar)-h1))
delta2 <- (1/sigma^2)*(t(H2%%betas-h2)%%(H2%%invX2%%t(H2))
%%((H2%%betas)-h2))
delta <- (1/sigma^2)*(t(H%%beta-h)%%(H%%invXm%%t(H))
%%((H%%beta)-h))

```

Determine qut, qrt

```

qut <- (sigma^2/q*se1)*(t(H1%%betar-h1)%%(H1%%invX1%%t(H1))
%%((H1%%betar)-h1))
qrt <- (sigma^2/q*se)*(t(H1%%betar-h1)%%(H1%%invX1%%t(H1))
%%((H%%beta)-h))
qpt <- (sigma^2/q*se2)*(t(H2%%betas-h2)%%(H2%%invX2%%t(H2))
%%((H2%%betas)-h2))
k1 <- (sigma^2/q*se1); k2 <- (sigma^2/q*se); k3 <- (sigma^2/q*se2)
fUT<- qf(1-alpha,q,v3); fRT<- qf(1-alpha,q,v3); fPT<- qf(1-alpha,q,v2)
zeta1<- seq(0,220,0.1); zeta2<- seq(0,100,0.1); zeta3<- seq(0,100,0.1);
zeta4<- seq(0,100,0.1); zeta11<- seq(0,100,0.1); zeta22<- seq(0,100,0.1)

```

Determine power of the UT, RT and PT

```

K <- 11 ; zeta2.sel <- zeta2[K] ;
pwrUT <- 1-pf(fUT-(1/k1)*zeta1,q,v3);
pwrRT <- 1-pf(fRT-(1/k2)*(zeta1+zeta2[K]),q,v3);
pwrPT <- 1-pf(fPT-(1/k3)*zeta2[K],q,v2);

```

Determine power of the PTT

```

library(cubature); library(zipfR)
# rho = 0.1
d1a <- (1-pwrPT)*pwrRT
bI2 <- function(alpha,cc,beta)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2]); v <- cc[2]/(1+cc[2])
sum.k <- 0; h <- alpha[1]-1;
for (k in 0:h) {
sum.k <- sum.k+(Cgamma(beta+k)/(Cgamma(beta)*factorial(k)))*
(uk)*Rbeta(y,alpha[2], k+beta,lower=TRUE) }
ans <- Rbeta(v,alpha[2],beta,lower=TRUE)-((1 - u)beta)*sum.k
ans
}
bI2 <- function(alpha,cc,beta,len.k)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2])
sum.k <- 0
for (k in 0:len.k)
{ sum.k <- sum.k+Cgamma(beta+alpha[1]+k)/Cgamma(beta+alpha[1])/
(Cgamma(1+alpha[1]+k)/Cgamma(1+alpha[1]))* u(k+alpha[1]) *
Rbeta(1-y,k+beta+alpha[1],alpha[2],lower=TRUE)
}
ans <- Rbeta(u,alpha[1],beta,lower=TRUE)-((1 - u)beta)/alpha[1]*
Cgamma(beta+alpha[1])/ (Cgamma(beta)*Cgamma(alpha[1]))*sum.k
ans }
sim_ncbivF <- function(rho,m,n,f1,f2,th1,th2,len.w1,len.w2,len.i,len.k)
{
beta <- n/2; cc <- 0; alpha <- 0
cc[1] <- f1*m/(n*(1 - rho2)); cc[2] <- f2*m/(n*(1 - rho2))
sum.I <- 0

```

```

for(w1 in 0:len.w1)
{
for(w2 in 0:len.w2)
{
for(i in 0:len.i)
{ alpha[1] <- - m/2 + i +w1; alpha[2] <- - m/2 + i +w2
sum.I <- - sum.I + (exp(-th1/2)*(th1/2)w1)/factorial(w1)*
(exp(-th2/2)*(th2/2)w2)/factorial(w2)*
(Cgamma(m/2+i)*rho(2*i)/factorial(i))*bI2(alpha,cc,beta,len.k)
} } }
prob <- - ((1 - rho2)(m/2))/Cgamma(m/2)*sum.I
list(prob=prob)
}
d <- - 3 cdf <- - sim_ncbivF(rho=0.1,m=5,n=25,f1=d,f2=d,th1=1,th2=2,
len.w1=10, len.w2=10,len.i=30,len.k=40)
cdf1 <- - cdf$prob
d1r <- - 1-cdf1
pwrPTT <- - d1a+d1r

# rho = 0.5

library(cubature); library(zipfR)
# rho = 0.1
d1a <- - (1-pwrPT)*pwrRT
bI2 <- - function(alpha,cc,beta)
{ u <- - cc[1]/(1+cc[1]); y <- - cc[2]/(1+cc[1]+cc[2]); v <- - cc[2]/(1+cc[2])
sum.k <- - 0; h <- - alpha[1]-1;
for (k in 0:h) {
sum.k <- - sum.k+(Cgamma(beta+k)/(Cgamma(beta)*factorial(k)))*
(uk)*Rbeta(y,alpha[2], k+beta,lower=TRUE) }
ans <- - Rbeta(v,alpha[2],beta,lower=TRUE)-((1 - u)beta)*sum.k
ans
}
bI2 <- - function(alpha,cc,beta,len.k)
{ u <- - cc[1]/(1+cc[1]); y <- - cc[2]/(1+cc[1]+cc[2])
sum.k <- - 0

```

```

for (k in 0:len.k)
{ sum_k <- sum_k+Cgamma(beta+alpha[1]+k)/Cgamma(beta+alpha[1])/
(Cgamma(1+alpha[1]+k)/Cgamma(1+alpha[1]))* u^(k+alpha[1]) *
Rbeta(1-y,k+beta+alpha[1],alpha[2],lower=TRUE)
}
ans <- Rbeta(u,alpha[1],beta,lower=TRUE)-((1 - u)^beta)/alpha[1]*
Cgamma(beta+alpha[1])/ (Cgamma(beta)*Cgamma(alpha[1]))*sum_k
ans }
sim_ncbivF <- function(rho,m,n,f1,f2,th1,th2,len.w1,len.w2,len.i,len.k)
{
beta <- n/2; cc <- 0; alpha <- 0
cc[1] <- f1*m/(n*(1 - rho^2)); cc[2] <- f2*m/(n*(1 - rho^2))
sum.I <- 0
for(w1 in 0:len.w1)
{
for(w2 in 0:len.w2)
{
for(i in 0:len.i)
{ alpha[1] <- m/2 + i +w1; alpha[2] <- m/2 + i +w2
sum.I <- sum.I + (exp(-th1/2)*(th1/2)^w1)/factorial(w1)*
(exp(-th2/2)*(th2/2)^w2)/factorial(w2)*
(Cgamma(m/2+i)*rho^(2*i)/factorial(i))*bI2(alpha,cc,beta,len.k)
} } }
prob <- ((1 - rho^2)^(m/2))/Cgamma(m/2)*sum.I
list(prob=prob)
}
d <- 3 cdf <- sim_ncbivF(rho=0.5,m=5,n=25,f1=d,f2=d,th1=1,th2=2,
len.w1=10, len.w2=10,len.i=30,len.k=40)
cdf1 <- cdf$prob
d1r <- 1-cdf1
pwrPTT1 <- d1a+d1r

library(cubature); library(zipfR)
# rho = 0.9
d1a <- (1-pwrPT)*pwrRT
bI2 <- function(alpha,cc,beta)

```



```

{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2]); v <- cc[2]/(1+cc[2])
sum.k <- 0; h <- alpha[1]-1;
for (k in 0:h) {
sum.k <- sum.k+(Cgamma(beta+k)/(Cgamma(beta)*factorial(k)))*
(u^k)*Rbeta(y,alpha[2], k+beta,lower=TRUE) }
ans <- Rbeta(v,alpha[2],beta,lower=TRUE)-((1 - u)^beta)*sum.k
ans
}
bI2 <- function(alpha,cc,beta,len.k)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2])
sum.k <- 0
for (k in 0:len.k)
{ sum.k <- sum.k+Cgamma(beta+alpha[1]+k)/Cgamma(beta+alpha[1])/
(Cgamma(1+alpha[1]+k)/Cgamma(1+alpha[1]))* u^(k+alpha[1]) *
Rbeta(1-y,k+beta+alpha[1],alpha[2],lower=TRUE)
}
ans <- Rbeta(u,alpha[1],beta,lower=TRUE)-((1 - u)^beta)/alpha[1]*
Cgamma(beta+alpha[1])/ (Cgamma(beta)*Cgamma(alpha[1]))*sum.k
ans }
sim_ncbivF <- function(rho,m,n,f1,f2,th1,th2,len.w1,len.w2,len.i,len.k)
{
beta <- n/2; cc <- 0; alpha <- 0
cc[1] <- f1*m/(n*(1 - rho^2)); cc[2] <- f2*m/(n*(1 - rho^2))
sum.I <- 0
for(w1 in 0:len.w1)
{
for(w2 in 0:len.w2)
{
for(i in 0:len.i)
{ alpha[1] <- m/2 + i +w1; alpha[2] <- m/2 + i +w2
sum.I <- sum.I + (exp(-th1/2)*(th1/2)^w1)/factorial(w1)*
(exp(-th2/2)*(th2/2)^w2)/factorial(w2)*
(Cgamma(m/2+i)*rho^(2*i)/factorial(i))*bI2(alpha,cc,beta,len.k)
} } }
prob <- ((1 - rho^2)^(m/2))/Cgamma(m/2)*sum.I

```

```

list(prob=prob)
}
d <- 3 cdf <- sim_ncbivF(rho=0.9,m=5,n=25,f1=d,f2=d,th1=1,th2=2,
len.w1=10, len.w2=10,len.i=30,len.k=40)
cdf1 <- cdf$prob
d1r <- 1-cdf1
pwrPTT2 <- d1a+d1r

```

Plot

```

plot(zeta1,pwrUT,type="l",lwd=1,col='red',cex=0.01,xlim=c(0,207),
xlab= paste(zeta[1]),ylab='Power',main= paste('i') paste(zeta[2])
paste('= 1') paste(',') paste(nu[1]) paste('=5') paste(',')
paste(nu[2]) paste('=25') paste(',') paste(theta[1]) paste('=1')
paste(',') paste(theta[2]) paste('=2'),ylim=c(0,1))
lines(zeta1,pwrRT,lty=2,lwd=1,col='blue',cex=0.01,ylim=c(0,1))
lines(zeta1,pwrPTT,lty=3,lwd=1,col='black',cex=0.01,ylim=c(0,1))
lines(zeta1,pwrPTT1,lty=4,lwd=1,col='green',cex=0.01,ylim=c(0,1))
lines(zeta1,pwrPTT2,lty=5,lwd=1,col='violet',cex=0.01,ylim=c(0,1))
legend(0.5,1,col=c('red','blue','black','green','violet'),
legend=c( expression( paste('UT')), expression( paste('RT')),
expression( paste('PTT') paste(',') paste(rho) paste('= 0.1')),
expression( paste('PTT') paste(',') paste(rho) paste('= 0.5')),
expression( paste('PTT') paste(',') paste(rho) paste('= 0.9'))),
lty=1:5,cex=0.8,lwd=c(1,1,1,1,1),title='Power of the tests')

```

Figure 5.1(ii) is plotted for $\zeta_2 = 1$ and $\theta_1 = 2$, Figure 5.1(iii) is plotted for $\zeta_2 = 5$ $\nu_1 = 5$, and Figure 5.1(iv) is plotted for $\zeta_2 = 5$ and $\nu_1 = 7$ for some selected $\rho = 0.1, 0.5, 0.9$.

A.13 Figure 5.2. The size against ζ_1 of the MRM

- The following R codes are used for producing Figure 5.2.

```
library(mvtnorm); library(copula); library(zipfR); par(mfrow=c(2,2))
```

Generate beta in term of matrix

```
n <- 100; k <- 3; r <- 2; s <- k-(r-1); v1 <- k; v2 <- n-s;
```

```
v3 <- n-r; alpha <- 0.05; beta0 <- 3; beta1 <- 1.5; beta2 <- 5;
beta3 <- 2.5; b1 <- c(beta0,beta1); b2 <- c(beta2,beta3);
betar <- matrix(b1,nrow=2,byrow=TRUE);
betas <- matrix(b2,nrow=2,byrow=TRUE)
bii <- c(beta0,beta1,beta2,beta3); beta <- matrix(bii,nrow=4,byrow=TRUE)
```

Generate X and Y

```
set.seed(555555); x1 <- runif(n,0,10); x2 <- runif(n,0,10);
x3 <- runif(n,0,10); one <- matrix(rep(1,length(n)),nrow=n,ncol=1)
cx1 <- c(one,x1); cx2 <- c(x2,x3); X1 <- matrix(cx1,nrow=n,ncol=2)
X2 <- matrix(cx2,nrow=n,ncol=2); Xm <- matrix(c(cx1,cx2),nrow=n,ncol=4)
X1t <- t(X1)%*% X1; X2t <- t(X2)%*%X2; Xmt <- t(Xm)%*% Xm;
invX1 <- solve(X1t); invX2 <- solve(X2t); invXm <- solve(Xmt)
```

Generate errors

```
set.seed(555); e1 <- rnorm(n,0,1); ei <- c(e1)
e <- matrix(ei,nrow=n,byrow=TRUE)
```

Setting matrix y

```
Y1 <- X1%*%b1+e; Y2 <- X2%*%b2+e;
Y <- Xm %*% beta+e; Yt <- t(Y)
```

Determine the regression parameters in term of matrix

```
beta1.est <- invX1 %*%(t(X1)%*% Y)
beta2.est <- invX2 %*%(t(X2)%*% Y)
beta.est <- invXm %*%(t(Xm)%*% Y)
```

Generate matrix H

```
set.seed(555); h1 <- rnorm(n,0,1); h2 <- rnorm(n,0,1)
h3 <- rnorm(n,0,1); h4 <- rnorm(n,0,1); h1m <- c(h1,h2);
h2m <- c(h3,h4); H1 <- matrix(h1m,nrow=n,ncol=2);
H2 <- matrix(h2m,nrow=n,ncol=2); H <- matrix(c(H1,H2),nrow=n,ncol=4)
H1t <- t(H1); H2t <- t(H2); Ht <- t(H); h1 <- H1%*% betar;
h2 <- H2 %*% betas; h <- H %*% beta
```

Determine Se and F of the UT, RT and PT

```
q <- 1; se1 <- 1/(n-r)*(t(Y-X1%*%beta1.est)%*%(Y-X1%*%beta1.est))
```

```

se2 <- 1/(n-s)*(t(Y-X2%%beta2.est)%%(Y-X2%%beta2.est))
se <- 1/(n-(k+1))*(t(Y-Xm%%beta.est)%%(Y-Xm%%beta.est))
F1 <- (1/(q*se1))*(t(H1%%beta1.est-h1)%%(H1%%invX1%%t(H1))
%%((H1%%beta1.est)-h1))
F2 <- (1/(q*se2))*(t(H2%%beta2.est-h2)%%(H2%%invX2%%t(H2))
%%((H2%%beta2.est)-h2))
F <- (1/(n*se))*(t(H%%beta.est-h)%%(H%%invXm%%t(H))
%%((H%%beta.est)-h))

```

Determine delta

```

sigma <- 5
delta1 <- (1/sigma^2)*(t(H1%%betar-h1)%%(H1%%invX1%%t(H1))
%%((H1%%betar)-h1))
delta2 <- (1/sigma^2)*(t(H2%%betas-h2)%%(H2%%invX2%%t(H2))
%%((H2%%betas)-h2))
delta <- (1/sigma^2)*(t(H%%beta-h)%%(H%%invXm%%t(H))
%%((H%%beta)-h))

```

Determine qut, qrt

```

qut <- (sigma^2/q*se1)*(t(H1%%betar-h1)%%(H1%%invX1%%t(H1))
%%((H1%%betar)-h1))
qrt <- (sigma^2/q*se)*(t(H%%beta-h)%%(H%%invXm%%t(H))
%%((H%%beta)-h))
qpt <- (sigma^2/q*se2)*(t(H2%%betas-h2)%%(H2%%invX2%%t(H2))
%%((H2%%betas)-h2))
k1 <- (sigma^2/q*se1); k2 <- (sigma^2/q*se); k3 <- (sigma^2/q*se2)
fUT<- qf(1-alpha,q,v4); fRT<- qf(1-alpha,q,v3); fPT<- qf(1-alpha,q,v5)
zeta1<- seq(0,100,0.1); zeta2<- seq(0,100,0.1); zeta3<- seq(0,100,0.1);
zeta4<- seq(0,100,0.1); zeta11<- seq(0,100,0.1); zeta22<- seq(0,100,0.1)

```

Determine size of the UT, RT and PT

```

K <- 11 ; zeta2.sel <- zeta2[K] ;
szUT <- 1-pf(fUT,q,v3);
sizeUT <- rep(szUT,length(zeta1));
szRT <- 1-pf(fRT,q,v3);
sizeRT <- rep(szRT,length(zeta1));

```

```

szPT <- 1-pf(fPT,q,v2);
sizePT <- rep(szPT,length(zeta1)) ;

# rho = 0.1

library(cubature);library(zipfR);
d1a <- (1-szPT)*szRT ;
bI2 <- function(alpha,cc,beta)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2]); v <- cc[2]/(1+cc[2])
sum_k <- 0; h <- alpha[1]-1;
for (k in 0:h) {
sum_k <- sum_k+(Cgamma(beta+k)/(Cgamma(beta)*factorial(k)))*
(u^k)*Rbeta(y,alpha[2], k+beta,lower=TRUE) }
ans <- Rbeta(v,alpha[2],beta,lower=TRUE)-((1 - u)^beta)*sum_k
ans
}
bI2 <- function(alpha,cc,beta,len.k)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2])
sum_k <- 0
for (k in 0:len.k)
{ sum_k <- sum_k+Cgamma(beta+alpha[1]+k)/Cgamma(beta+alpha[1])/
(Cgamma(1+alpha[1]+k)/Cgamma(1+alpha[1]))* u^(k+alpha[1]) *
Rbeta(1-y,k+beta+alpha[1],alpha[2],lower=TRUE)
}
ans <- Rbeta(u,alpha[1],beta,lower=TRUE)-((1 - u)^beta)/alpha[1]*
Cgamma(beta+alpha[1])/ (Cgamma(beta)*Cgamma(alpha[1]))*sum_k
ans }
sim_ncbivF <- function(rho,m,n,f1,f2,th1,th2,len.w1,len.w2,len.i,len.k)
{
beta <- n/2; cc <- 0; alpha <- 0
cc[1] <- f1*m/(n*(1 - rho^2)); cc[2] <- f2*m/(n*(1 - rho^2))
sum.I <- 0
for(w1 in 0:len.w1)
{
for(w2 in 0:len.w2)
{

```

```

for(i in 0:len.i)
{ alpha[1] <- m/2 + i +w1; alpha[2] <- m/2 + i +w2
sum.I <- sum.I + (exp(-th1/2)*(th1/2)w1)/factorial(w1)*
(exp(-th2/2)*(th2/2)w2)/factorial(w2)*
(Cgamma(m/2+i)*rho(2*i)/factorial(i))*bI2(alpha,cc,beta,len.k)
} } }
prob <- ((1 - rho2)(m/2))/Cgamma(m/2)*sum.I
list(prob=prob)
}
d <- 3 cdf <- sim_ncbivF(rho=0.1,m=5,n=25,f1=d,f2=d,th1=1,th2=2,
len.w1=10, len.w2=10,len.i=30,len.k=40)
cdf1 <- cdf$prob
d1r <- 1-cdf1
szPTT <- d1a+d1r;
sizePTT1 <- rep(szPTT,length(zeta1))

# rho = 0.1

library(cubature);library(zipfR);
d1a <- (1-szPT)*szRT ;
bI2 <- function(alpha,cc,beta)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2]); v <- cc[2]/(1+cc[2])
sum.k <- 0; h <- alpha[1]-1;
for (k in 0:h) {
sum.k <- sum.k+(Cgamma(beta+k)/(Cgamma(beta)*factorial(k)))*
(uk)*Rbeta(y,alpha[2], k+beta,lower=TRUE) }
ans <- Rbeta(v,alpha[2],beta,lower=TRUE)-((1 - u)beta)*sum.k
ans
}
bI2 <- function(alpha,cc,beta,len.k)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2])
sum.k <- 0
for (k in 0:len.k)
{ sum.k <- sum.k+Cgamma(beta+alpha[1]+k)/Cgamma(beta+alpha[1])/
(Cgamma(1+alpha[1]+k)/Cgamma(1+alpha[1]))* u(k+alpha[1]) *
Rbeta(1-y,k+beta+alpha[1],alpha[2],lower=TRUE)

```

```

}
ans <- Rbeta(u,alpha[1],beta,lower=TRUE)-((1-u)beta)/alpha[1]*
Cgamma(beta+alpha[1]) / (Cgamma(beta)*Cgamma(alpha[1]))*sum_k
ans }
sim_ncbivF <- function(rho,m,n,f1,f2,th1,th2,len.w1,len.w2,len.i,len.k)
{
beta <- n/2; cc <- 0; alpha <- 0
cc[1] <- f1*m/(n*(1-rho2)); cc[2] <- f2*m/(n*(1-rho2))
sum.I <- 0
for(w1 in 0:len.w1)
{
for(w2 in 0:len.w2)
{
for(i in 0:len.i)
{ alpha[1] <- m/2 + i + w1; alpha[2] <- m/2 + i + w2
sum.I <- sum.I + (exp(-th1/2)*(th1/2)w1)/factorial(w1)*
(exp(-th2/2)*(th2/2)w2)/factorial(w2)*
(Cgamma(m/2+i)*rho(2*i))/factorial(i)*bI2(alpha,cc,beta,len.k)
} } }
prob <- ((1-rho2)(m/2))/Cgamma(m/2)*sum.I
list(prob=prob)
}
d <- 3 cdf <- sim_ncbivF(rho=0.5,m=5,n=25,f1=d,f2=d,th1=1,th2=2,
len.w1=10, len.w2=10,len.i=30,len.k=40)
cdf1 <- cdf$prob
d1r <- 1-cdf1
szPTT <- d1a+d1r;
sizePTT2 <- rep(szPTT,length(zeta1))

# rho = 0.1

library(cubature);library(zipfR);
d1a <- (1-szPT)*szRT ;
bI2 <- function(alpha,cc,beta)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2]); v <- cc[2]/(1+cc[2])
sum.k <- 0; h <- alpha[1]-1;

```

```

for (k in 0:h) {
sum_k <- sum_k+(Cgamma(beta+k)/(Cgamma(beta)*factorial(k)))*
(u^k)*Rbeta(y,alpha[2], k+beta,lower=TRUE) }
ans <- Rbeta(v,alpha[2],beta,lower=TRUE)-((1 - u)^beta)*sum_k
ans
}
bI2 <- function(alpha,cc,beta,len.k)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2])
sum_k <- 0
for (k in 0:len.k)
{ sum_k <- sum_k+Cgamma(beta+alpha[1]+k)/Cgamma(beta+alpha[1])/
(Cgamma(1+alpha[1]+k)/Cgamma(1+alpha[1]))* u^(k+alpha[1]) *
Rbeta(1-y,k+beta+alpha[1],alpha[2],lower=TRUE)
}
ans <- Rbeta(u,alpha[1],beta,lower=TRUE)-((1 - u)^beta)/alpha[1]*
Cgamma(beta+alpha[1])/ (Cgamma(beta)*Cgamma(alpha[1]))*sum_k
ans }
sim_ncbivF <- function(rho,m,n,f1,f2,th1,th2,len.w1,len.w2,len.i,len.k)
{
beta <- n/2; cc <- 0; alpha <- 0
cc[1] <- f1*m/(n*(1 - rho^2)); cc[2] <- f2*m/(n*(1 - rho^2))
sum.I <- 0
for(w1 in 0:len.w1)
{
for(w2 in 0:len.w2)
{
for(i in 0:len.i)
{ alpha[1] <- m/2 + i +w1; alpha[2] <- m/2 + i +w2
sum.I <- sum.I + (exp(-th1/2)*(th1/2)^w1)/factorial(w1)*
(exp(-th2/2)*(th2/2)^w2)/factorial(w2)*
(Cgamma(m/2+i)*rho^(2*i)/factorial(i))*bI2(alpha,cc,beta,len.k)
} } }
prob <- ((1 - rho^2)^(m/2))/Cgamma(m/2)*sum.I
list(prob=prob)
}

```



```

d <- 3 cdf <- sim_ncbivF(rho=0.9,m=5,n=25,f1=d,f2=d,th1=1,th2=2,
len.w1=10, len.w2=10,len.i=30,len.k=40)
cdf1 <- cdf$prob
d1r <- 1-cdf1
szPTT <- d1a+d1r;
sizePTT3 <- rep(szPTT,length(zeta1))
# Plot
plot(zeta1,sizeUT,type="l",lwd=1,col='red',cex=0.01,xlim=c(0,50),
xlab= paste(zeta[1]),ylab='Size',main= paste('(iii)') paste(' ')
paste(zeta[2]) paste('= 1') paste(',') paste(nu[1]) paste('=5')
paste(',') paste(nu[2]) paste('=25') paste(',') paste(theta[1])
paste('=1') paste(',') paste(theta[2]) paste('=2'),ylim=c(0,1))
lines(zeta1,sizeRT,lty=2,lwd=1,col='blue',cex=0.01,ylim=c(0,1))
lines(zeta1,sizePTT1,lty=3,lwd=1,col='black',cex=0.01,ylim=c(0,1))
lines(zeta1,sizePTT2,lty=3,lwd=1,col='green',cex=0.01,ylim=c(0,1))
lines(zeta1,sizePTT3,lty=3,lwd=1,col='violet',cex=0.01,ylim=c(0,1))
legend(0.1,1,col=c('red','blue','black','green','violet'),
legend=c(expression( paste('UT')), expression( paste('RT')),
expression( paste('PTT') paste(',') paste(rho) paste('= 0.1')),
expression( paste('PTT') paste(',') paste(rho) paste('= 0.5')),
expression( paste('PTT') paste(',') paste(rho) paste('= 0.9'))),
lty=1:3,cex=0.8,lwd=c(1,1,1,1,1), title='Size of the tests')

```

Figure 5.2(ii) is plotted for $\zeta_2 = 1$ and $\theta_1 = 2$, Figure 5.2(iii) is plotted for $\zeta_2 = 5$ $\nu_1 = 5$, and Figure 5.2(iv) is plotted for $\zeta_2 = 5$ and $\nu_1 = 7$ for some selected $\rho = 0.1, 0.5, 0.9$.

A.14 Figure 6.1. The power against δ_1 of the PRM

- The following R codes are used for producing Figure 6.1.

```
library(mvtnorm); library(copula); par(mfrow=c(2,2));
```

Generate beta in term of matrix

```
n<- 100; p<- 3; v1<- p-1; v2<- (n-2*p); alpha<- 0.05
theta01<- 3; beta11<- 2; theta02<- 3.6
```

```

beta12<- 2; theta03<- 4; beta13<- 2
b0i<- c(theta01,theta02,theta03); bli<- c(beta11, beta12,beta13)
b0m<- matrix(b0i,nrow=3,byrow=TRUE)
b1m<- matrix(b1i,nrow=3,byrow=TRUE)

```

Generate X and Y

```

set.seed(55555); x1<- runif(n,0,1); x2<- runif(n,0,5)
x3<- runif(n,0,8); X1<- matrix(x1,ncol=1,nrow=n)
X2<- matrix(x2,ncol=1,nrow=n); X3<- matrix(x3,ncol=1,nrow=n)
MX<- c(X1,X2,X3); X<- matrix(MX,nrow=n,ncol=3)
data1<- c(mean(x1),mean(x2),mean(x3))
p<- 3; DiagT<- matrix(data=data1,nrow=p,ncol=p); DiagT
onep<- matrix(rep(1,length(n)),nrow=n,ncol=p)
one1<- matrix(rep(1,length(n)),nrow=n,ncol=1)
one2<- matrix(rep(1,length(n)),nrow=n,ncol=1)
one3<- matrix(rep(1,length(n)),nrow=n,ncol=1)

```

Generate errors

```

set.seed(555555); e1<- rnorm(n,0,1); e2<- rnorm(n,0,1)
e3<- rnorm(n,0,1); ei<- c(e1,e2,e3)
e<- matrix(ei, nrow=n, byrow=TRUE)

```

Setting matrix y

```

Y1<- theta01+beta11*x1+e1; Y2<- theta02+beta12*x2+e2
Y3<- theta03+beta13*x3+e3

```

Determine the regression parameters in term of matrix

```

theta01.est<- mean(Y1)-beta11*mean(x1)
theta02.est<- mean(Y2)-beta12*mean(x2)
theta03.est<- mean(Y3)-beta13*mean(x3)
theta.est<- matrix(c(theta01,theta02,theta03),ncol=1,nrow=3)
Q1<- t(X1)%*%X1-(1/n)*(t(one1)%*%X1)
Q2<- t(X2)%*%X2-(1/n)*(t(one2)%*%X2)
Q3<- t(X3)%*%X3-(1/n)*(t(one3)%*%X3)
beta11.est<- (t(X1)%*%Y1)-(1/n)*((t(one1)%*%X1)%*%
(t(one1)%*%Y1)/n*Q1)

```

```

beta12.est <- (t(X2)%*%Y2)-(1/n)*((t(one2)%*%X2)%*%
(t(one2)%*%Y1)/n*Q2)
beta13.est <- (t(X3)%*%Y3)-(1/n)*((t(one3)%*%X3)%*%
(t(one3)%*%Y3)/n*Q3)

```

Determine identity, diagonal and H matrices

```

Q <- n*(Q1+Q2+Q3); g <- n*Q
n <- c(20); Ip <- matrix(0,nrow=n,ncol=n)
Ip[row(Ip)==col(Ip)] <- 1
data3 <- c(n*Q1,n*Q2,n*Q3); h <- 20
inv.D22 <- matrix(data=data3,nrow=h,ncol=h); inv.D22
H <- Ip - ((1/385650.5)*((onep)%*%t(onep))%*%(inv.D22))

```

Determine k1, k2, k3

```

Y1 <- theta01+beta11*x1+e1; Ybar1 <- mean(Y1);
xbar1 <- mean(X1); Sy1 <- sqrt(sum((Y1 - mean(Y1))^2)/(n-1))
Ssx1 <- sum((X1 - mean(X1))^2); Fit1 <- lm(Y1 X1)
Res1 <- residuals(Fit1); mse1 <- sqrt(sum((Res1^2)/(n-2)))
se1 <- sqrt(sum((Res1^2)))
Y2 <- theta02+beta12*x2+e2; Ybar2 <- mean(Y2);
xbar1 <- mean(X2); Sy2 <- sqrt(sum((Y2 - mean(Y2))^2)/(n-1))
Ssx2 <- sum((X2 - mean(X2))^2); Fit2 <- lm(Y2 X2)
Res2 <- residuals(Fit2); mse2 <- sqrt(sum((Res2^2)/(n-2)))
se2 <- sqrt(sum((Res2^2)))
Y3 <- theta03+beta13*x3+e3; Ybar3 <- mean(Y3);
xbar1 <- mean(X3); Sy3 <- sqrt(sum((Y3 - mean(Y3))^2)/(n-1))
Ssx3 <- sum((X3 - mean(X3))^2); Fit3 <- lm(Y3 X3)
Res3 <- residuals(Fit3); mse3 <- sqrt(sum((Res3^2)/(n-2)))
se3 <- sqrt(sum((Res3^2)))
Setot <- (1/(n-2*p))*(se1+se2+se3); Setor <- (1/(n-p))*(se1+se2+se3)
k1 <- (1/(p-1))*Setot; k2 <- (1/(p-1))*Setor; k3 <- (1/(p-1))*Setot

```

Determining power of the UT, RT, PT and PTT

```

fUT <- qf(1-alpha,v1,v2); fRT <- qf(1-alpha,v1,v2);
fPT <- qf(1-alpha,v1,v2); delta1 <- seq(0,2.9,0.1); delta2 <- seq(0,2.9,0.1)

```

Power of the UT

```

K <- 9 (11,21,26); delta2.sel <- delta2[K]
pwrUT <- 1-pf(fUT-k1*delta1,v1,v2)

Power of the RT

pwrRT <- 1-pf(fRT-k2*(delta1+delta2[K]),v1,v2)

Power of the PT

pwrPT <- 1-pf(fPT-k1*delta2[K],v1,v2)

# Power of the PTT

# rho = 0.1
library(cubature); library(zipfR)
d1a <- (1-pwrPT)*pwrRT
bI2 <- function(alpha,cc,beta)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2]); v <- cc[2]/(1+cc[2])
sum.k <- 0; h <- alpha[1]-1;
for (k in 0:h) {
sum.k <- sum.k+(Cgamma(beta+k)/(Cgamma(beta)*factorial(k)))*
(uk)*Rbeta(y,alpha[2], k+beta,lower=TRUE) }
ans <- Rbeta(v,alpha[2],beta,lower=TRUE)-((1 - u)beta)*sum.k
ans
}
bI2 <- function(alpha,cc,beta,len.k)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2])
sum.k <- 0
for (k in 0:len.k)
{ sum.k <- sum.k+Cgamma(beta+alpha[1]+k)/Cgamma(beta+alpha[1])/
(Cgamma(1+alpha[1]+k)/Cgamma(1+alpha[1]))* u(k+alpha[1]) *
Rbeta(1-y,k+beta+alpha[1],alpha[2],lower=TRUE)
}
ans <- Rbeta(u,alpha[1],beta,lower=TRUE)-((1 - u)beta)/alpha[1]*
Cgamma(beta+alpha[1])/ (Cgamma(beta)*Cgamma(alpha[1]))*sum.k
ans }
sim_ncbivF <- function(rho,m,n,f1,f2,th1,th2,len.w1,len.w2,len.i,len.k)
{
beta <- n/2; cc <- 0; alpha <- 0

```

```

cc[1] <- f1*m/(n*(1 - rho^2)); cc[2] <- f2*m/(n*(1 - rho^2))
sum.I <- 0
for(w1 in 0:len.w1)
{
for(w2 in 0:len.w2)
{
for(i in 0:len.i)
{ alpha[1] <- - m/2 + i +w1; alpha[2] <- - m/2 + i +w2
sum.I <- - sum.I + (exp(-th1/2)*(th1/2)^w1)/factorial(w1)*
(exp(-th2/2)*(th2/2)^w2)/factorial(w2)*
(Cgamma(m/2+i)*rho^(2*i))/factorial(i)*bI2(alpha,cc,beta,len.k)
} } }
prob <- - ((1 - rho^2)^(m/2))/Cgamma(m/2)*sum.I
list(prob=prob)
}
d <- 3 cdf <- sim.ncbivF(rho=0.1,m=5,n=25,f1=d,f2=d,th1=1,th2=1.5,
len.w1=10, len.w2=10,len.i=30,len.k=40)
cdf1 <- cdf$prob
d1r <- 1-cdf1
pwrPTT <- d1a+d1r

# rho=0.5

d1a <- (1-pwrPT)*pwrRT
library(cubature); library(zipfR)
bI2 <- function(alpha,cc,beta)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2]); v <- cc[2]/(1+cc[2])
sum.k <- 0; h <- alpha[1]-1;
for (k in 0:h) {
sum.k <- sum.k+(Cgamma(beta+k)/(Cgamma(beta)*factorial(k)))*
(u^k)*Rbeta(y,alpha[2], k+beta,lower=TRUE) }
ans <- Rbeta(v,alpha[2],beta,lower=TRUE)-((1 - u)^beta)*sum.k
ans
}
bI2 <- function(alpha,cc,beta,len.k)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2])

```

```

sum.k <- 0
for (k in 0:len.k)
{ sum.k <- sum.k+Cgamma(beta+alpha[1]+k)/Cgamma(beta+alpha[1])/
(Cgamma(1+alpha[1]+k)/Cgamma(1+alpha[1]))* u^(k+alpha[1]) *
Rbeta(1-y,k+beta+alpha[1],alpha[2],lower=TRUE)
}
ans <- Rbeta(u,alpha[1],beta,lower=TRUE)-((1 - u)^beta)/alpha[1]*
Cgamma(beta+alpha[1])/ (Cgamma(beta)*Cgamma(alpha[1]))*sum.k
ans }
sim_ncbivF <- function(rho,m,n,f1,f2,th1,th2,len.w1,len.w2,len.i,len.k)
{
beta <- n/2; cc <- 0; alpha <- 0
cc[1] <- f1*m/(n*(1 - rho^2)); cc[2] <- f2*m/(n*(1 - rho^2))
sum.I <- 0
for(w1 in 0:len.w1)
{
for(w2 in 0:len.w2)
{
for(i in 0:len.i)
{ alpha[1] <- m/2 + i +w1; alpha[2] <- m/2 + i +w2
sum.I <- sum.I + (exp(-th1/2)*(th1/2)^w1)/factorial(w1)*
(exp(-th2/2)*(th2/2)^w2)/factorial(w2)*
(Cgamma(m/2+i)*rho^(2*i)/factorial(i))*bI2(alpha,cc,beta,len.k)
} } }
prob <- ((1 - rho^2)^(m/2))/Cgamma(m/2)*sum.I
list(prob=prob)
}
d <- 3 cdf <- sim_ncbivF(rho=0.5,m=5,n=25,f1=d,f2=d,th1=1,th2=1.5,
len.w1=10, len.w2=10,len.i=30,len.k=40)
cdf1 <- cdf$prob
pwrPTT1 <- d1a+d1r

# rho=0.9

library(cubature); library(zipfR);
d1a <- (1-pwrPT)*pwrRT

```

```

bI2 <- function(alpha,cc,beta)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2]); v <- cc[2]/(1+cc[2])
sum.k <- 0; h <- alpha[1]-1;
for (k in 0:h) {
sum.k <- sum.k+(Cgamma(beta+k)/(Cgamma(beta)*factorial(k))) *
(uk)*Rbeta(y,alpha[2], k+beta,lower=TRUE) }
ans <- Rbeta(v,alpha[2],beta,lower=TRUE)-((1 - u)beta)*sum.k
ans
}
bI2 <- function(alpha,cc,beta,len.k)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2])
sum.k <- 0
for (k in 0:len.k)
{ sum.k <- sum.k+Cgamma(beta+alpha[1]+k)/Cgamma(beta+alpha[1])/
(Cgamma(1+alpha[1]+k)/Cgamma(1+alpha[1])) * u(k+alpha[1]) *
Rbeta(1-y,k+beta+alpha[1],alpha[2],lower=TRUE)
}
ans <- Rbeta(u,alpha[1],beta,lower=TRUE)-((1 - u)beta)/alpha[1]*
Cgamma(beta+alpha[1])/ (Cgamma(beta)*Cgamma(alpha[1]))*sum.k
ans }
sim_ncbivF <- function(rho,m,n,f1,f2,th1,th2,len.w1,len.w2,len.i,len.k)
{
beta <- n/2; cc <- 0; alpha <- 0
cc[1] <- f1*m/(n*(1 - rho2)); cc[2] <- f2*m/(n*(1 - rho2))
sum.I <- 0
for(w1 in 0:len.w1)
{
for(w2 in 0:len.w2)
{
for(i in 0:len.i)
{ alpha[1] <- m/2 + i +w1; alpha[2] <- m/2 + i +w2
sum.I <- sum.I + (exp(-th1/2)*(th1/2)w1)/factorial(w1)*
(exp(-th2/2)*(th2/2)w2)/factorial(w2)*
(Cgamma(m/2+i)*rho(2*i)/factorial(i))*bI2(alpha,cc,beta,len.k)
} } }
} } }

```

```

prob <- ((1 - rho^2)^(m/2))/Cgamma(m/2)*sum_I
list(prob=prob)
}
d <- 3 cdf <- sim_ncbivF(rho=0.9,m=5,n=25,f1=d,f2=d,th1=1,th2=1.5,
len.w1=10, len.w2=10,len.i=30,len.k=40)
cdf1 <- cdf$prob
pwrPTT2 <- d1a+d1r

# Plot
plot(delta1,pwrUT,type="l",lwd=1,col='red',cex=0.01,xlim=c(0,3.3),
xlab= paste(delta[1]),ylab='Power',main= paste('(i)') paste("
paste(delta[2]) paste('=1') paste(',') paste(nu[1]) paste('=5')
paste(',') paste(nu[2]) paste('=25') paste(',') paste(theta[1])
paste('=1') paste(',') paste(theta[2]) paste('=1.5'),ylim=c(0,1))
lines(delta1,pwrRT,lty=2,lwd=1,col='blue',cex=0.01,ylim=c(0,1))
lines(delta1,pwrPTT,lty=3,lwd=1,col='black',cex=0.01,ylim=c(0,1))
lines(delta1,pwrPTT1,lty=4,lwd=1,col='green',cex=0.01,ylim=c(0,1))
lines(delta1,pwrPTT2,lty=5,lwd=1,col='violet',cex=0.01,ylim=c(0,1))
legend(1.55,0.48,col=c('red','blue','black','green','violet'),
legend=c( expression( paste('UT')), expression( paste('RT')),
expression( paste('PTT') paste(',') paste(rho) paste('= +/- 0.1')),
expression( paste('PTT') paste(',') paste(rho) paste('= +/- 0.5')),
expression( paste('PTT') paste(',') paste(rho) paste('= +/- 0.9'))),
lty=1:5,cex=0.8,lwd=c(1,1,1,1,1),title='Power of the tests')

```

Figure 6.1(i) is plotted for $\delta_1 = 1.1$, Figure 6.1(iii) is plotted for different degrees of freedom (m, n) and Figure 6.1(iv) is plotted for different noncentrality parameters (θ_1, θ_2).

A.15 Figure 6.2. The size against δ_1 of the PRM

- The following R codes are used for producing Figure 6.2.

```
library(mvtnorm); library(copula); par(mfrow=c(2,2));
```

Generate beta matrix

```
n <- 100; p <- 3; v1 <- p-1; v2 <- (n-2*p); alpha <- 0.05
```



```
theta01<- 3; beta11<- 2; theta02<- 3.6
beta12<- 2; theta03<- 4; beta13<- 2
b0i<-c(theta01,theta02,theta03); bli<-c(beta11, beta12,beta13)
b0m<-matrix(b0i,nrow=3,byrow=TRUE)
b1m<-matrix(b1i,nrow=3,byrow=TRUE)
```

Generate X and Y

```
set.seed(55555); x1<- runif(n,0,1); x2<- runif(n,0,5)
x3<- runif(n,0,8); X1<- matrix(x1,ncol=1,nrow=n)
X2<- matrix(x2,ncol=1,nrow=n); X3<- matrix(x3,ncol=1,nrow=n)
MX<- c(X1,X2,X3); X<- matrix(MX,nrow=n,ncol=3)
data1<- c(mean(x1),mean(x2),mean(x3))
p<- 3; DiagT<- matrix(data=data1,nrow=p,ncol=p); DiagT
onep<- matrix(rep(1,length(n)),nrow=n,ncol=p)
one1<- matrix(rep(1,length(n)),nrow=n,ncol=1)
one2<- matrix(rep(1,length(n)),nrow=n,ncol=1)
one3<- matrix(rep(1,length(n)),nrow=n,ncol=1)
```

Generate errors

```
set.seed(555555); e1<- rnorm(n,0,1); e2<- rnorm(n,0,1)
e3<- rnorm(n,0,1); ei<- c(e1,e2,e3)
e<- matrix(ei, nrow=n, byrow=TRUE)
```

Setting matrix y

```
Y1<- theta01+beta11*x1+e1; Y2<- theta02+beta12*x2+e2
Y3<- theta03+beta13*x3+e3
```

Determine the regression parameters

```
theta01.est<- mean(Y1)-beta11*mean(x1)
theta02.est<- mean(Y2)-beta12*mean(x2)
theta03.est<- mean(Y3)-beta13*mean(x3)
theta.est<- matrix(c(theta01,theta02,theta03),ncol=1,nrow=3)
Q1<- t(X1)%*%X1-(1/n)*(t(one1)%*%X1)
Q2<- t(X2)%*%X2-(1/n)*(t(one2)%*%X2)
Q3<- t(X3)%*%X3-(1/n)*(t(one3)%*%X3)
beta11.est<- (t(X1)%*%Y1)-(1/n)*((t(one1)%*%X1)%*%
```

```

(t(one1)%*%Y1)/n*Q1)
beta12.est<- (t(X2)%*%Y2)-(1/n)*((t(one2)%*%X2)%*%
(t(one2)%*%Y1)/n*Q2)
beta13.est<- (t(X3)%*%Y3)-(1/n)*((t(one3)%*%X3)%*%
(t(one3)%*%Y3)/n*Q3)

```

Determine identity, diagonal and H matrices

```

Q<- n*(Q1+Q2+Q3); g<- n*Q
n<- c(20); Ip<- matrix(0,nrow=n,ncol=n)
Ip[row(Ip)==col(Ip)] <- 1
data3<- c(n*Q1,n*Q2,n*Q3); h<- 20
inv.D22<- matrix(data=data3,nrow=h,ncol=h); inv.D22
H<- Ip - ((1/385650.5)*((onep)%*%t(onep))%*%(inv.D22))

```

Determine k1, k2, k3

```

Y1<- theta01+beta11*x1+e1; Ybar1<- mean(Y1);
xbar1<- mean(X1); Sy1<- sqrt(sum((Y1 - mean(Y1))^2)/(n-1))
Ssx1<- sum((X1 - mean(X1))^2); Fit1 <- lm(Y1 X1)
Res1<- residuals(Fit1); mse1<- sqrt(sum((Res1^2)/(n-2)))
se1<- sqrt(sum((Res1^2)))
Y2<- theta02+beta12*x2+e2; Ybar2<- mean(Y2);
xbar1<- mean(X2); Sy2<- sqrt(sum((Y2 - mean(Y2))^2)/(n-1))
Ssx2 <- sum((X2 - mean(X2))^2); Fit2 <- lm(Y2 X2)
Res2<- residuals(Fit2); mse2<- sqrt(sum((Res2^2)/(n-2)))
se2<- sqrt(sum((Res2^2)))
Y3<- theta03+beta13*x3+e3; Ybar3<- mean(Y3);
xbar1<- mean(X3); Sy3<- sqrt(sum((Y3 - mean(Y3))^2)/(n-1))
Ssx3<- sum((X3 - mean(X3))^2); Fit3<- lm(Y3 X3)
Res3<- residuals(Fit3); mse3<- sqrt(sum((Res3^2)/(n-2)))
se3<- sqrt(sum((Res3^2)))
Setot<- (1/(n-2*p))*(se1+se2+se3); Setor<- (1/(n-p))*(se1+se2+se3)
k1<- (1/(p-1))*Setot; k2<- (1/(p-1))*Setor;k3<- (1/(p-1))*Setot

```

Plot of the size of the UT, RT, PT and PTT

```

fUT<- qf(1-alpha,v1,v2); fRT<- qf(1-alpha,v1,v2);
fPT<- qf(1-alpha,v1,v2); delta1<- seq(0,2.9,0.1); delta2<- seq(0,2.9,0.1)

```

Determining size of the UT, RT and PT

```

K <- 16; Phi2.sel <- phi2[K];
szUT <- 1-pf(fUT,v1,v3);
sizeUT <- rep(szUT,length(phi1));
szRT <- 1-pf(fRT-(phi2[K]*xbar^2),v1,v2);
sizeRT <- rep(szRT,length(phi1));
szPT <- 1-pf(fPT-phi2[K],v1,v3);
sizePT <- rep(szPT,length(phi1));

```

Determining size of the PTT**# rho = 0.1**

```

library(cubature); library(zipfR);
d1a <- (1-szPT)*szRT ;
bI2 <- function(alpha,cc,beta)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2]); v <- cc[2]/(1+cc[2])
sum_k <- 0; h <- alpha[1]-1;
for (k in 0:h) {
sum_k <- sum_k+(Cgamma(beta+k)/(Cgamma(beta)*factorial(k))) *
(u^k)*Rbeta(y,alpha[2], k+beta,lower=TRUE) }
ans <- Rbeta(v,alpha[2],beta,lower=TRUE)-((1 - u)^beta)*sum_k
ans
}
bI2 <- function(alpha,cc,beta,len.k)
{ u <- cc[1]/(1+cc[1]); y <- cc[2]/(1+cc[1]+cc[2])
sum_k <- 0
for (k in 0:len.k)
{ sum_k <- sum_k+Cgamma(beta+alpha[1]+k)/Cgamma(beta+alpha[1])/
(Cgamma(1+alpha[1]+k)/Cgamma(1+alpha[1])) * u^(k+alpha[1]) *
Rbeta(1-y,k+beta+alpha[1],alpha[2],lower=TRUE)
}
ans <- Rbeta(u,alpha[1],beta,lower=TRUE)-((1 - u)^beta)/alpha[1]*
Cgamma(beta+alpha[1])/ (Cgamma(beta)*Cgamma(alpha[1]))*sum_k
ans }
sim_ncbivF <- function(rho,m,n,f1,f2,th1,th2,len.w1,len.w2,len.i,len.k)
{

```

```

beta <- n/2; cc <- 0; alpha <- 0
cc[1] <- f1*m/(n*(1 - rho^2)); cc[2] <- f2*m/(n*(1 - rho^2))
sum.I <- 0
for(w1 in 0:len.w1)
{
for(w2 in 0:len.w2)
{
for(i in 0:len.i)
{ alpha[1] <- - m/2 + i +w1; alpha[2] <- - m/2 + i +w2
sum.I <- - sum.I + (exp(-th1/2)*(th1/2)^w1)/factorial(w1)*
(exp(-th2/2)*(th2/2)^w2)/factorial(w2)*
(Cgamma(m/2+i)*rho^(2*i))/factorial(i)*bI2(alpha,cc,beta,len.k)
} } }
prob <- - ((1 - rho^2)^(m/2))/Cgamma(m/2)*sum.I
list(prob=prob)
}
d <- 3 cdf <- sim.ncbivF(rho=0.1,m=5,n=25,f1=d,f2=d,th1=1,th2=1.5,
len.w1=10, len.w2=10,len.i=30,len.k=40)
cdf1 <- cdf$prob; d1rj-1-cdf1
szPTT <- d1a+d1r; sizePTT <- rep(szPTT,length(delta1))

# Plot
plot(delta1,sizeUT,type="l",lwd=1,col='red',cex=0.01,xlim=c(0,3),
xlab= paste(delta[1]),,ylab='Size',main= paste('(i)') paste(delta[2])
paste('= 1.5') paste(',') paste(rho) paste('= 0.1') paste(',')
paste(m) paste('= 5') paste(',') paste(n) paste('= 25'),ylim=c(0,1))
lines(delta1,sizeRT,lty=2,lwd=1,col='blue',cex=0.01,ylim=c(0,1))
lines(delta1,sizePTT,lty=3,lwd=1,col='black',cex=0.01,ylim=c(0,1))
legend(0.1,0.95,col=c('red','blue','black'),legend=c('UT','RT','PTT'),
lty=1:3,cex=0.7,lwd=c(1,1,1),title='Size of the tests')

```

Figures 6.2(i) and 6.2(iii) are plotted for $\rho = 0.5$ and 0.9 , respectively, and Figure 6.2(iv) is plotted $\rho = 0.5$ and $\delta_2 = 2$.

Appendix B

List of Publications and Seminars

B.1 Publications

1. Khan, S. and Pratikno, B. (2012). Testing Base Load with Non-Sample Prior Information on Process Load (Simple Regression), *Statistical Papers*, In Press.
2. Khan, S. and Pratikno, B. (2012). Testing Base Load with Non-Sample Prior Information on Process Load (Multivariate Model), Chapter of the Book on *Statistics in Scientific Investigations*, edited by Maman Djauhari.
3. Khan, S., Pratikno, B., Ibrahim, A. I. N. and Yunus, R. M. (2012). The Correlated Bivariate Noncentral F Distribution with Applications, submitted to *Computational Statistics and Data Analysis*.
4. Pratikno, B. and Khan, S. (2011). Testing of Intercept when Slope is Under Suspicion, Proceeding of 11th Islamic Countries Conference on Statistical Science (ICCS-11), Vol.21, p.19-29, Lahore-Pakistan, December 2011.

B.2 Seminars

1. Test of Hypotheses for Linear Models with Non-Sample Prior Information, USQ Australia, 30 May 2012.
2. Testing the Intercept of Simple Regression Model with Non-Sample Prior Information on Slope (*seminar*), Annual Seminar of Postgraduate Students, USQ Australia, November 2011.

3. Testing the Intercept of a Multivariate Simple Regression Model with Non-Sample Prior Information on the Slope (*poster*), USQ Research Evening, Australia, November 2011.
4. Testing the Intercept of a Simple Regression Model with Non-Sample Prior Information on the Slope for Unknown variance (*poster*), USQ Research Evening, Australia, November 2011.
5. Testing the Intercept of a Simple Regression Model with Non-Sample Prior Information on the Slope (*poster*), USQ Research Evening, Australia, November 2010.
6. Test of Hypotheses with Non-Sample Prior Information (*seminar of PhD proposal*), USQ Australia, 12 August 2010.

Appendix C

Curriculum Vitae

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