

# INTERACTION OF GASEOUS BUBBLES UNDER THE ACTION OF RADIATION MODIFIED BJERKNES FORCE

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**Summary** Interaction of two floating-up bubbles is studied under the action of buoyancy and radiation modified BjerKNes forces in viscous fluid. Governing equations are studied analytically and numerically in the creeping flow approximation taking into account both quasi-stationary Stokes drag force and history drag force. Interesting features of the bubble mechanistic behavior are revealed.

## INTRODUCTION: BACKGROUND INFORMATION AND MAIN ASSUMPTIONS

Numerous investigative studies on bubble dynamics in viscous liquid have been carried out. The complete understanding of their fundamental behaviors remains however far from the resolution. This concerns not only a collective behavior of large bubble ensembles but also on pair interaction of two bubbles and even on individual bubble behavior in shear flows. The importance of such investigations is motivated by many practical applications where gas-liquid mixtures occur, e.g., nucleate boiling in cooling systems of nuclear reactors, gas-liquid mixtures in a pharmaceutical industry, cavitations in technological processes, mixing phenomena in the upper ocean due to the collapse of wind waves, etc.

The behavior of two air bubbles in water is an intriguing problem where observations have shown that for practically the same initial conditions two bubbles may: i) attract each other until they coalesce into one bubble, ii) approach each other only until certain distance and then, begin repelling and go away from each other, or iii) periodically approach to certain distance and move away. From theoretical viewpoint, the adequate qualitative and quantitative description of complex bubble dynamics in real liquids is still required. In this paper, a theory based on the exploitation of a modified formula for BjerKNes force exerting on oscillating bubbles [1] is proposed. The scope of the present study considers the air bubbles to be of sufficiently small radii, of the order of  $R \sim 0.1$  mm or less. For such bubble sizes the surface tension retains the spherical shape of the bubbles and does not contribute significantly to the total pressure within the bubbles [2] and can therefore be safely neglected.

In the developed theory we consider two bubbles of equal radii  $R$  oscillating with an arbitrary phase shift with respect to each other but with the same eigenfrequency [3]. It is assumed that the bubbles are floating side-by-side in a vertical direction under the action of buoyancy force in a viscous compressible liquid. In general, their center of mass simultaneously moves horizontally under the action of a radiative force caused by sound wave radiation from the coupled system of two oscillating bubbles, and the bubbles also move with respect to each other in the center-of-mass coordinate system. The BjerKNes force is normally calculated in the quasi-static approximation which presumes that the bubbles are located so close to each other that the pressure field produced by oscillation of one bubble immediately exerts on another bubble. This occurs when the distance between bubbles is far less than the wavelength of sound wave generated by oscillating bubbles. In some cases where time delay occurs for the pressure perturbation produced by one bubble to reach another bubble, this leads however to the requirement of modifying the conventional BjerKNes formula [1]. Due to such modification some new features arise. Firstly, the interaction force between bubbles decays more slowly with distance when the radiative effect is taken into account; it is now proportional to  $r^{-1}$  rather than  $r^{-2}$  in the incompressible fluid. Secondly, the forces acting on the bubbles are not equal in general. Thirdly, due to sound wave radiation, the bubbles center of mass may travel in the opposite direction.

The interaction of two bubbles and their joint motion is studied under the action of buoyancy, viscous and radiation modified BjerKNes forces. Viscous drag force is accounted for in the unsteady creeping flow approximation assuming that the Reynolds number for each bubble is small enough,  $Re \ll 1$ . Two components of the total drag force are considered: i) quasi-steady Stokes drag (SD) force and ii) history or memory integral drag (MID) force [4].

## EQUATIONS OF MOTION AND RESULTS OBTAINED

Following Nemtsov [1], consider two bubbles whose radii oscillate in time with a frequency  $\omega$  and amplitudes  $a$ :

$$r_1(t) = R + a \sin(\omega t + \varphi_1), \quad r_2(t) = R + a \sin(\omega t + \varphi_2), \quad a \ll R \quad (1)$$

where  $R$  is unperturbed radius of bubbles,  $\varphi_{1,2}$  are the initial phases of bubble oscillations.

The time averaged forces (over the period of bubble oscillations) exerted from one bubble onto another bubble when the radiative effects are taken into account are:

$$\mathbf{F}_{1,2} = -\frac{2\pi\rho a^2 R^4 \omega^2}{r_{1,2}^3} \left[ \cos(\varphi_1 - \varphi_2 - kr_{1,2}) - kr_{1,2} \sin(\varphi_1 - \varphi_2 - kr_{1,2}) \right] \mathbf{r}_{1,2}, \quad (2)$$

where  $\rho$  is water density (air density within the bubbles is assumed to be negligibly small in comparison with the water density),  $k = \omega/c$ ,  $c$  is the sound speed in water,  $\mathbf{r}_{1,2}$  is the radius-vector connecting centers of first and second bubbles  $r_{1,2} = r_{2,1} = |\mathbf{r}_{1,2}|$ . When  $k \rightarrow 0$ , Eq. (2) reduces to the conventional formula for the Bjerknes force. Taking into account this force as well as other forces, *viz.*, the buoyancy force, viscous drag forces, and added mass effect, the equation of motion for one of the bubbles can be written in the form:

$$\frac{d^2 \mathbf{r}_1}{dt^2} = -3a^2 R \omega^2 \left[ \cos(\varphi_1 - \varphi_2 - k|\mathbf{r}_1 - \mathbf{r}_2|) - k|\mathbf{r}_1 - \mathbf{r}_2| \sin(\varphi_1 - \varphi_2 - k|\mathbf{r}_1 - \mathbf{r}_2|) \right] \frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3} - 2\mathbf{g} - \frac{9\nu}{R^2} \left\{ f \left( R, \mathbf{r}_1, \mathbf{r}_2, \frac{d\mathbf{r}_2}{dt} \right) \frac{d\mathbf{r}_1}{dt} + \frac{4}{3} \int_0^t \frac{d^2 \mathbf{r}_1}{d\tau^2} \exp[9\nu(t-\tau)/R^2] \operatorname{erfc} \left( \sqrt{9\nu(t-\tau)/R^2} \right) d\tau \right\}, \quad (3)$$

where  $\mathbf{g}$  is the acceleration due to gravity, function  $f$  describes a correction to the SD force due to influence of another bubble [5]; its specific form is determined by the bubble positional relationship; the last integral term represents the MID force [4];  $\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$  is the complimentary error function. Equation similar to Eq. (3) with the indices interchange holds for the second bubble. Taking the difference and sum of equations describing each of two bubbles, one can readily obtain the final equations of motion in scalar form for bubbles floating-up vertically side-by-side:

$$\frac{d^2 \Delta x}{d\theta^2} = -\cos(\varphi_1 - \varphi_2) \frac{\cos(\kappa|\Delta x|) + \kappa|\Delta x| \sin(\kappa|\Delta x|)}{|\Delta x|^2} - 9\eta \left[ f_1(\Delta x) \frac{d\Delta x}{d\theta} + \frac{4}{3} \int_0^\theta e^{9\eta(\theta-\theta')} \operatorname{erfc} \sqrt{9\eta(\theta-\theta')} \frac{d^2 \Delta x}{d\theta'^2} d\theta' \right], \quad (4)$$

$$\frac{d^2 X}{d\theta^2} = -\frac{1}{2} \sin(\varphi_1 - \varphi_2) \frac{\sin(\kappa|\Delta x|) + \kappa|\Delta x| \cos(\kappa|\Delta x|)}{|\Delta x|^2} - 9\eta \left[ f_2(\Delta x) \frac{dX}{d\theta} + \frac{4}{3} \int_0^\theta e^{9\eta(\theta-\theta')} \operatorname{erfc} \sqrt{9\eta(\theta-\theta')} \frac{d^2 X}{d\theta'^2} d\theta' \right], \quad (5)$$

$$\frac{d^2 Z}{d\theta^2} = G - 9\eta \left[ f_3(\Delta x) \frac{dZ}{d\theta} + \frac{4}{3} \int_0^\theta e^{9\eta(\theta-\theta')} \operatorname{erfc} \sqrt{9\eta(\theta-\theta')} \frac{d^2 Z}{d\theta'^2} d\theta' \right], \quad (6)$$

where  $\Delta x = (x_1 - x_2)/R$  is the distance between bubbles,  $X = (x_1 + x_2)/(2R)$  and  $Z = z/R$ ,  $\theta = \omega a t \sqrt{6}/R$  are the coordinates of their center of mass,  $\kappa = kR \equiv \omega R/c$ ,  $\eta = \nu/(\omega R a \sqrt{6})$ ,  $G = gR/(3a^2 \omega^2)$ , and functions  $f_1(\Delta x)$ ,  $f_2(\Delta x)$  and  $f_3(\Delta x)$  can be presented in the form [5]:

$$f_1(\xi) \approx 1 + 3\xi + 9\xi^2 + 19\xi^3 + 93\xi^4 + 387\xi^5 + 1197\xi^6 + 5331\xi^7 + 19821\xi^8 + 76115\xi^9 + \frac{2048}{3} \frac{\xi^{10}}{1-4\xi}, \quad (7)$$

$$f_2(\xi) \approx 1 - 3\xi + 9\xi^2 - 19\xi^3 + 93\xi^4 - 387\xi^5 + 1197\xi^6 - 5331\xi^7 + 19821\xi^8 - 76115\xi^9 + \frac{2048}{3} \frac{\xi^{10}}{1+4\xi}, \quad (8)$$

$$f_3(\xi) \approx 1 - \frac{3}{2}\xi + \frac{9}{4}\xi^2 - \frac{59}{8}\xi^3 + \frac{273}{16}\xi^4 - \frac{1107}{32}\xi^5 + 2^6 \frac{\xi^6}{1+2\xi}, \quad \text{where } \xi = 1/(2\Delta x). \quad (9)$$

Eqs. (4)–(6) are analysed analytically and numerically for different cases: i) when viscosity is neglected, ii) when fluid is incompressible and compressible, iii) when viscous drag is accounted for by only SD force and iv) by the total drag force (SD + MID forces). The results allowed the explanation of all possible regimes of the interaction of two bubbles.

Figure 1 shows a phase plane of Eq. (4) when the viscosity is neglected,  $\eta = 0$ ,  $\kappa = 0.014$  and bubbles oscillate in anti-phase. Closed trajectories correspond to periodic bubble oscillations around some stable positions. Trajectories lying outside of the first separatrix and in-between of any other separatrices correspond to mutual bubble rapprochement, which replaces then, by their scatter. Bubble coalescence may occur when they oscillate in phase. When the radiative effect is neglected, bubbles either monotonically draw together or move away from each other.

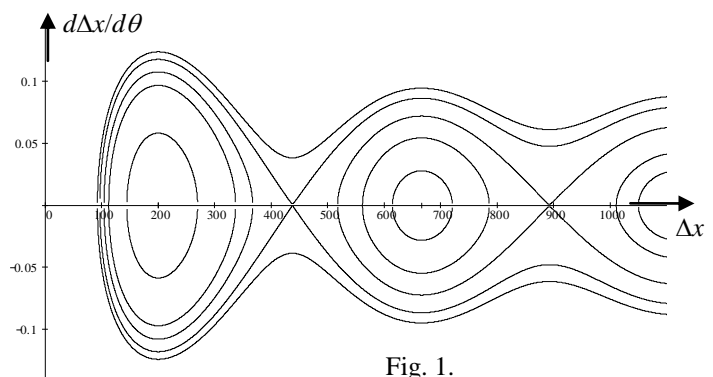


Fig. 1.

## References

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